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COM 232

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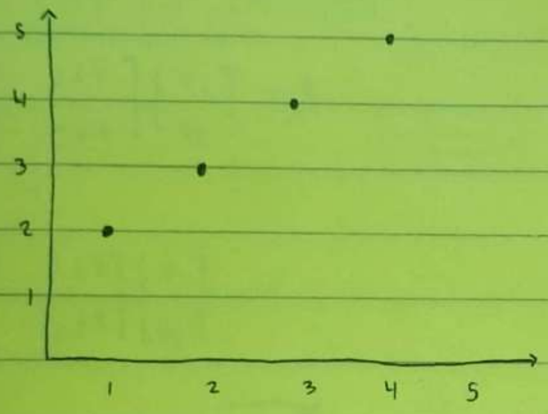
D	4.	5

1. Plot



2. Mean

$$\mu_1 = \frac{1+2+3+4}{4} = 2.5 \qquad \mu_2 = \frac{2+3+4+5}{4} = 3.5 \qquad \mu = \begin{bmatrix} 2.5 & 3.5 \end{bmatrix}$$

3-4.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix} \qquad \mu = \begin{bmatrix} 2.5 & 3.5 \end{bmatrix} \qquad A_{centered} = \begin{bmatrix} 1-2.5 & 2-3.5 \\ 2-2.5 & 3-3.5 \\ 3-2.5 & 4-3.5 \\ 4-2.5 & 5-3.5 \end{bmatrix} = \begin{bmatrix} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{bmatrix}$$

5. Covariance Matrix

$$\Sigma = \frac{1}{n-1} A^T_{centered} \, A_{centered}$$

$$A_c = \begin{bmatrix} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{bmatrix} \qquad A^T_c = \begin{bmatrix} -1.5 & -0.5 & 0.5 & 1.5 \\ -1.5 & -0.5 & 0.5 & 1.5 \end{bmatrix}$$

$$\Sigma = \frac{1}{n-1} A^T_c A_c$$

$$\Sigma = \frac{1}{n-1} \begin{bmatrix} -1.5 & -0.5 & 0.5 & 1.5 \\ -1.5 & -0.5 & 0.5 & 1.5 \end{bmatrix} \begin{bmatrix} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{bmatrix}$$

$$\Sigma = \frac{1}{4-1} \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}_{//}$$

$$\Sigma = \frac{1}{3} \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} = \Sigma = \begin{bmatrix} 1.67 & 1.67 \\ 1.67 & 1.67 \end{bmatrix}_{//}$$

6. eigenvalues of the covariance matrix

$$\det(A - \lambda I) = 0$$

$$\left| \begin{bmatrix} 1.67 & 1.67 \\ 1.67 & 1.67 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1.67 & 1.67 \\ 1.67 & 1.67 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 1.67 - \lambda & 1.67 \\ 1.67 & 1.67 - \lambda \end{vmatrix} = 0$$

$$(1.67 - \lambda)^2 - 2.79 = 0$$

$$(1.67 - \lambda)(1.67 - \lambda) - 2.79 = 0$$

$$\cancel{2.79} - 1.67\lambda - 1.67\lambda + \lambda^2 - \cancel{2.79} = 0$$

$$\lambda^2 - 3.34 = 0$$

$$(\lambda - 0)(\lambda - 3.34) = 0$$

$$\lambda_1 = 3.34_{//}$$

$$\lambda_2 = 0_{//}$$

7. Compute the corresponding eigenvectors

$$(A-\lambda I)v = 0$$

$$A = \begin{bmatrix} 1.67 & 1.67 \\ 1.67 & 1.67 \end{bmatrix}$$

$$v = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \lambda_1 = 3.34$$

$$\left(\begin{bmatrix} 1.67 & 1.67 \\ 1.67 & 1.67 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\left(\begin{bmatrix} 1.67 & 1.67 \\ 1.67 & 1.67 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\left(\begin{bmatrix} 1.67 & 1.67 \\ 1.67 & 1.67 \end{bmatrix} - \begin{bmatrix} 3.34 & 0 \\ 0 & 3.34 \end{bmatrix}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1.67 & 1.67 \\ 1.67 & -1.67 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1.67 & 1.67 \\ 1.67 & -1.67 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1.67v_1 + 1.67v_2 \\ 1.67v_1 + -1.67v_2 \end{bmatrix} \quad \begin{bmatrix} -1.67v_2 = 1.67v_1 \\ 1.67v_1 = 1.67v_2 \end{bmatrix} \quad \begin{bmatrix} -v_2 = v_1 \\ v_1 = v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$ //

8. Normalize the eigenvectors

$$u = \frac{v}{\|v\|} \qquad \|v\| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$u = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$ //

9. Checking $Av = \lambda v$

$$\begin{bmatrix} 1.67 & 1.67 \\ 1.67 & 1.67 \end{bmatrix} v = \lambda v$$

$$\begin{bmatrix} 1.67 & 1.67 \\ 1.67 & 1.67 \end{bmatrix} v = 3.34v$$

$$\begin{bmatrix} 1.67 & 1.67 \\ 1.67 & 1.67 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 3.34\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1.67 + 1.67 \\ 1.67 + 1.67 \end{bmatrix} = \begin{bmatrix} 3.34 \\ 3.34 \end{bmatrix} = \begin{bmatrix} 3.34 \\ 3.34 \end{bmatrix} \checkmark = \begin{bmatrix} 3.34 \\ 3.34 \end{bmatrix}$$ //

10. 3.34 is chosen because it explain the greatest amount of variance in the data compared to the other components.

11. new projection = A centered × u

$$A_c = \begin{bmatrix} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{bmatrix} \qquad u = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\text{new projection} = \begin{bmatrix} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{bmatrix} \times \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$1st = \frac{-1.5-1.5}{\sqrt{2}} = \frac{-3}{\sqrt{2}} = -\frac{3\sqrt{2}}{2}$$

$$2nd = \frac{-0.5-0.5}{\sqrt{2}} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$3rd = \frac{1.5+1.5}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

$$4th = \frac{0.5+0.5}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

12.

$$\text{data in 2d} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix} \qquad \text{new projection} = \begin{bmatrix} -\frac{3\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \\ \frac{3\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} //$$