

# A Bilevel Edge Computing Architecture and Collaborative Offloading Mechanism for Offshore Buoys Network (Appendix)

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## III. PROBLEM SOLVING

### B. Joint Iterative Mechanism

2) *Solving for Decision Variables  $\{\mathcal{P}^b\}$* : In this subsection, the derivation of the decision variable  $\mathcal{P}^b = \{p_{ij}\}_{i \in \mathcal{N}, j \in \mathcal{N}}$  is proposed, when other decision variables are fixed, and the optimization problem  $P_3$  is given as follows.

$$\mathbf{P}_3^{\text{Sub}} : \min_{\mathcal{P}^b} \sum_{m=1}^M \sum_{q=1}^Q T_{qm}^{c1}(\mathcal{P}^b) \quad (32a)$$

$$s.t. \quad C_3, C'_4, C_5, C_7, C_9 \quad (32b)$$

The objective function in (32a), regarded as a composition function, is illustrated based on the formula (11), where the outer function of (32a) can be regarded as an inverse proportional function, and the inner function of (32a) is a monotonically increasing function. According to the composition rules, it is easy to know that this objective function is also a convex function with respect to  $\{\mathcal{P}^b\}$ .

In addition, for  $\forall i \in \mathcal{N}$ , constraints  $C_5$  and  $C_9$  can be combined and expressed as follows.

$$E_i^{c1} \leq c_5, \quad \forall i \in \mathcal{N} = \mathcal{N}^M \cup \mathcal{N}^Q \quad (33)$$

where constant  $c_5$  is calculated as follows.

$$c_5 = \begin{cases} E_0 - E_i^{c2}, & \text{for } C_5 \text{ constraint} \\ E_0 - E_i^s - E_i^{c2}, & \text{for } C_9 \text{ constraint} \end{cases} \quad (34)$$

it is evident that  $E_i^{c1}$  can be regarded as the function form of  $f(x) = \sum_i \frac{x_i}{\log_2(x_i+1)} - c$  based on (13), which is a concave function when  $x > 0$ . The above conclusion implies that the constraint of formula (33) is a non-convex constraint.

To solve such problem, a novel power iterative algorithm, based on Successive Convex Approximation (SCA) method, is proposed to approximate this non-convex constraint. Specifically, the non-convex constraint  $E_i^{c1}$  can be regarded as the sum of two convex functions multiplied together. Then, define a new function  $h_q(p_{i\tilde{\xi}^q})$ , which implies that  $E_i^{c1} = \sum_{q=1}^Q h_q(p_{i\tilde{\xi}^q})$  holds. We can derive the convex upper bound for each non-convex  $h_q(p_{i\tilde{\xi}^q})$  function in (36). Then, (33) can be converted to convex constraint as follows.

$$C'_5, C''_9 : \sum_{q=1}^Q \tilde{h}_q(p_{i\tilde{\xi}^q}; \tilde{p}_{i\tilde{\xi}^q}) \leq c_5, \forall i \in \mathcal{N} \quad (35)$$

which is a convex constraint with respect to  $p_{i\tilde{\xi}^q}$ . In addition, it is worth mentioning that the link  $i \rightarrow \tilde{\xi}^q$  from  $i$  to  $\tilde{\xi}^q$  is

chosen according to the decision variable  $x_{i\tilde{\xi}^q} = 1$ , and  $\tilde{\xi}^q$  is the next hop for  $i$  within the path  $\Gamma_{q\tilde{m}}$ .

Then,  $C'_4$  constraint, as mentioned in (18), is also a convex function. Similarly,  $C_3$  and  $C_7$  constraints can also be combined as follows.

$$C'_3, C'_7 : \sum_{n=1}^N p_{in} \leq c_6, \quad \forall i \in \mathcal{N} = \mathcal{N}^M \cup \mathcal{N}^Q \quad (37)$$

where constant  $c_6$  is calculated as follows.

$$c_6 = \begin{cases} P_0, & \text{for } C_3 \text{ constraint} \\ P_0 - \sum_{\omega=1}^W g_{ms}^\omega, & \text{for } C_7 \text{ constraint} \end{cases} \quad (38)$$

Next, the Lagrangian function of  $P_3^{\text{Sub}}$  is expressed as follows.

$$\begin{aligned} L_3(\{\mathcal{P}^b\}, \{\mu_i, \gamma_i\}_{i \in \mathcal{N}}, \{\delta_q\}_{q \in \mathcal{N}^Q}) \\ = \sum_{m=1}^M \sum_{q=1}^Q T_{qm}^{c1}(\mathcal{P}^b) + \sum_{i=1}^N \mu_i \left( \sum_{n=1}^N p_{in} - c_6 \right) \\ + \sum_{i=1}^N \gamma_i \cdot \left( \sum_{q=1}^Q \tilde{h}_q(p_{i\tilde{\xi}^q}; \tilde{p}_{i\tilde{\xi}^q}) - c_5 \right) \\ + \sum_{q=1}^Q \left[ \delta_q^1 \cdot (-R_{q \rightarrow \xi_1} + R_{\min}) + \sum_{k \in \{1, \dots, K_{q\tilde{m}}-1\}} \delta_q^k \cdot (-R_{\xi_k \rightarrow \xi_{k+1}} + R_{\min}) \right. \\ \left. + \delta_q^{K_{q\tilde{m}}} \cdot (-R_{\xi_{K_{q\tilde{m}}} \rightarrow \tilde{m}} + R_{\min}) \right] \\ \triangleq \sum_{m=1}^M L_3^m(\{p_{m\tilde{\xi}^m}\}_{m \in \mathcal{N}^M}) + \sum_{q=1}^Q L_3^q(\{p_{q\tilde{\xi}^q}\}_{q \in \mathcal{N}^Q}) + c_7 \quad (39) \end{aligned}$$

where  $L_3^m(\{p_{m\tilde{\xi}^m}\}_{m \in \mathcal{N}^M})$  is defined as the function of decision variables  $\{p_{m\tilde{\xi}^m}\}_{m \in \mathcal{N}^M}$  for gateway buoys  $\mathcal{N}^M$ ,  $L_3^q(\{p_{q\tilde{\xi}^q}\}_{q \in \mathcal{N}^Q})$  is defined as the function of decision variables  $\{p_{q\tilde{\xi}^q}\}_{q \in \mathcal{N}^Q}$  for detection buoys  $\mathcal{N}^Q$ , and  $c_7$  is the constant that is independent of  $\mathcal{P}^b$ . We use the symbol  $\tilde{\xi}^q$  to refer to the next hop of buoy  $q$ , and the symbol  $\tilde{\xi}^m$  to refer to the next hop of buoy  $m$ .

Then, take the partial derivative of the Lagrangian function  $L_3^m(\{p_{m\tilde{\xi}^m}\}_{m \in \mathcal{N}^M})$  with respect to  $p_{m\tilde{\xi}^m}$  for any  $m \in \mathcal{N}^M$ , take the partial derivative of the Lagrangian function  $L_3^q(\{p_{q\tilde{\xi}^q}\}_{q \in \mathcal{N}^Q})$  with respect to  $p_{q\tilde{\xi}^q}$  for any  $q \in \mathcal{N}^Q$ . We

represent all power variables in terms of  $\{p_{ij}\}_{i \in \mathcal{N}, j \in \mathcal{N}}$ , and a unified representation for updates is formulated as follows:

$$p_{ij}^{(\kappa+1)} = \begin{cases} p_{m\tilde{\xi}^m}^{(\kappa)} - \zeta_{m\tilde{\xi}^m}^{(\kappa)} \cdot \frac{\partial L_3^m(\{p_{m\tilde{\xi}^m}^{(\kappa)}\}_{m \in \mathcal{N}^M})}{\partial p_{m\tilde{\xi}^m}^{(\kappa)}}, & \text{if } i \in \mathcal{N}^M \wedge i \in \tilde{\Gamma} \\ p_{q\tilde{\xi}^q}^{(\kappa)} - \zeta_{q\tilde{\xi}^q}^{(\kappa)} \cdot \frac{\partial L_3^q(\{p_{q\tilde{\xi}^q}^{(\kappa)}\}_{q \in \mathcal{N}^Q})}{\partial p_{q\tilde{\xi}^q}^{(\kappa)}}, & \text{if } i \in \mathcal{N}^Q \wedge i \in \tilde{\Gamma} \\ 0, & \text{if } i \notin \tilde{\Gamma} \end{cases} \quad (40)$$

where the calculate of variable  $\{p_{ij}\}_{i \in \mathcal{N}, j \in \mathcal{N}}$  is divided into three cases. If the node  $i$  serves as a gateway buoy and has an associated forwarding task, (i.e.,  $i \in \mathcal{N}^M \wedge i \in \tilde{\Gamma}$ ) holds, then we can use  $L_3^m(\{p_{m\tilde{\xi}^m}^{(\kappa)}\}_{m \in \mathcal{N}^M})$  to update  $p_{m\tilde{\xi}^m}$ . In addition, if the node  $i$  serves as a detection buoy and has an associated forwarding task, (i.e.,  $i \in \mathcal{N}^Q \wedge i \in \tilde{\Gamma}$ ) holds, then we can use  $L_3^q(\{p_{q\tilde{\xi}^q}^{(\kappa)}\}_{q \in \mathcal{N}^Q})$  to update  $p_{q\tilde{\xi}^q}$ . Otherwise,  $p_{ij} = 0$  when the node  $i$  does not have an associated forwarding task.

Let  $\kappa$  represent the index of dual ascent algorithm for outer loop, and let  $\tilde{\kappa}$  represent the index of SCA algorithm for inner loop. Then, the update of  $\tilde{p}_{ij}^{(\tilde{\kappa}+1)}$  for  $\tilde{\kappa}+1$ -th iteration is given as follows.

$$\tilde{p}_{ij}^{(\kappa, \tilde{\kappa}+1)} = \arg \min_{\mathcal{P}^b} \sum_{m=1}^M \sum_{q=1}^Q T_{qm}^{c1}(\mathcal{P}^b) \quad (41a)$$

$$s.t. \begin{cases} C'_3, C'_7 : \sum_{n=1}^N p_{in} \leq c_6, \quad \forall i \in \mathcal{N} \\ C'_5, C'_9 : \sum_{q=1}^Q \tilde{h}_q(p_{i\tilde{\xi}^q}; \tilde{p}_{i\tilde{\xi}^q}^{(\kappa, \tilde{\kappa}+1)}) \leq c_5, \quad \forall i \\ C'_4 \text{ (refer to (??))} \end{cases} \quad (41b)$$

where the above formula can be used to update the auxiliary variables in SCA.

Then, the Lagrange multipliers are calculated as follows.

$$\begin{aligned} \mu_i^{(\kappa+1)} &= \mu_i^{(\kappa)} + \zeta_{m\mu}^{(\kappa)} \cdot \left( \sum_{n=1}^N p_{in}^{(\kappa)} - c_6 \right) \\ \gamma_i^{(\kappa+1)} &= \gamma_i^{(\kappa)} + \zeta_{m\gamma}^{(\kappa)} \cdot \left( \sum_{q=1}^Q \tilde{h}_q(p_{i\tilde{\xi}^q}; \tilde{p}_{i\tilde{\xi}^q}) - c_5 \right) \\ \delta_q^{k(\kappa+1)} &= \delta_q^{k(\kappa)} + \zeta_{qk\delta}^{(\kappa)} \cdot (-R_{\xi_k \rightarrow \xi_{k+1}} + R_{\min}) \end{aligned} \quad (42)$$

where  $\zeta_{m\mu}^{(\kappa)}, \zeta_{m\gamma}^{(\kappa)}, \zeta_{qk\delta}^{(\kappa)}$  are the step sizes for Lagrange multipliers. Finally, the solution algorithm for decision variables  $\{\mathcal{P}^b\}$  is given in Alg. 3.

3) *Solving for Decision Variables  $\{\mathcal{C}\}$*  : In this subsection, the derivation of the decision variable  $\mathcal{C}$  is proposed, when other decision variables are fixed, and the optimization problem  $P_4^{\text{Sub}}$  is given as follows.

$$P_4^{\text{Sub}} : \min_{\mathcal{C}} \sum_{m=1}^M \sum_{q=1}^Q T_{qm}^{s1}(\mathcal{C}) \quad (43a)$$

$$s.t. \quad C_2, C_9 \quad (43b)$$

where the objective function is an inverse proportional function with respect to  $\mathcal{C} = \{c_{qm}\}_{q \in \mathcal{N}^Q, m \in \mathcal{N}^M}$ , which is also a convex function. Then,  $C_9$  constraint can be rewritten as

$$E_m^s \leq c_8, \quad \forall m \in \mathcal{N}^M \quad (44)$$

where constant  $c_8$  is equals to  $E_0 - E_m^s$ . Then,  $C_2$  constraint and  $C_9$  constraint are linear functions and quadratic functions with respect to  $\mathcal{C}$  respectively, and obviously both satisfy the

$$\begin{aligned} h_q(p_{i\tilde{\xi}^q}) &= \frac{\mathbb{I}_{\{i \in \Gamma_{q\tilde{m}}\}} I_q \cdot p_{i\tilde{\xi}^q}}{B_0 \log_2 \left( 1 + \frac{p_{i\tilde{\xi}^q} |h_{i\tilde{\xi}^q}^{\text{IoT}}|^2}{\sigma_{\tilde{\xi}^q}^2} \right)} = \left( \mathbb{I}_{\{i \in \Gamma_{q\tilde{m}}\}} I_q \cdot p_{i\tilde{\xi}^q} \right) \cdot \left( \frac{1}{B_0 \log_2 \left( 1 + \frac{p_{i\tilde{\xi}^q} |h_{i\tilde{\xi}^q}^{\text{IoT}}|^2}{\sigma_{\tilde{\xi}^q}^2} \right)} \right) \\ &= \frac{1}{2} \left( \mathbb{I}_{\{i \in \Gamma_{q\tilde{m}}\}} I_q p_{i\tilde{\xi}^q} + \frac{1}{B_0 \log_2 \left( 1 + \frac{p_{i\tilde{\xi}^q} |h_{i\tilde{\xi}^q}^{\text{IoT}}|^2}{\sigma_{\tilde{\xi}^q}^2} \right)} \right)^2 - \frac{1}{2} \left( \mathbb{I}_{\{i \in \Gamma_{q\tilde{m}}\}} I_q p_{i\tilde{\xi}^q} \right)^2 - \frac{1}{2} \left[ B_0 \log_2 \left( 1 + \frac{p_{i\tilde{\xi}^q} |h_{i\tilde{\xi}^q}^{\text{IoT}}|^2}{\sigma_{\tilde{\xi}^q}^2} \right) \right]^{-2} \\ &\leq H_q(p_{i\tilde{\xi}^q}) - \frac{1}{2} \left( \mathbb{I}_{\{i \in \Gamma_{q\tilde{m}}\}} I_q \tilde{p}_{i\tilde{\xi}^q} \right)^2 - \frac{1}{2} \left[ B_0 \log_2 \left( 1 + \frac{\tilde{p}_{i\tilde{\xi}^q} |h_{i\tilde{\xi}^q}^{\text{IoT}}|^2}{\sigma_{\tilde{\xi}^q}^2} \right) \right]^{-2} \\ &\quad - \left( \mathbb{I}_{\{i \in \Gamma_{q\tilde{m}}\}} I_q \right)^2 \cdot \tilde{p}_{i\tilde{\xi}^q} \cdot (p_{i\tilde{\xi}^q} - \tilde{p}_{i\tilde{\xi}^q}) - \frac{-\ln 2 \cdot B \cdot \frac{\tilde{p}_{i\tilde{\xi}^q} |h_{i\tilde{\xi}^q}^{\text{IoT}}|^2}{\sigma_{\tilde{\xi}^q}^2 + \tilde{p}_{i\tilde{\xi}^q} |h_{i\tilde{\xi}^q}^{\text{IoT}}|^2}}{\left( B \log_2 \left( 1 + \frac{\tilde{p}_{i\tilde{\xi}^q} |h_{i\tilde{\xi}^q}^{\text{IoT}}|^2}{\sigma_{\tilde{\xi}^q}^2} \right) \right)^3} \cdot (p_{i\tilde{\xi}^q} - \tilde{p}_{i\tilde{\xi}^q}) \\ &\triangleq \tilde{h}_q(p_{i\tilde{\xi}^q}; \tilde{p}_{i\tilde{\xi}^q}) \end{aligned} \quad (36)$$

**Algorithm 3** Solving for Computing Resources  $\{\mathcal{P}^b\}$  for Gateway Guoys

**Input:** Optimization Problem  $P_3^{\text{Sub}}$ .

**Output:**  $\{\mathcal{P}^b\} = \{p_{ij}\}_{i \in \mathcal{N}, j \in \mathcal{N}}$

- 1: **Initialize**  $\{\{p_{ij}^{(0)}\}_{j \in \mathcal{N}}, \mu_i^{(0)}, \gamma_i^{(0)}\}_{i \in \mathcal{N}}, \{\delta_q^{k(0)}\}_{q \in \mathcal{N}^Q}, \xi_k \in \Gamma_{q\bar{m}}$ .
- 2: **while** (1) **do**
- 3:   // **Part(a): Dual-Ascent-based Variable Update**
- 4:   **for**  $m = \{1, 2, \dots, M\}$  **do**
- 5:     Calculate the Lagrangian function based on (39).
- 6:     Calculate the partial derivatives and update decision variable  $p_{ij}^{(\kappa+1)}$  for  $\forall i, j$ , based on (40).
- 7:     Update Lagrange multipliers  $\mu_i^{(\kappa+1)}, \gamma_i^{(\kappa+1)}, \delta_q^{(\kappa+1)}$  for  $\forall i, q$  based on (42).
- 8:   **end for**
- 9:   **if**  $\max_i \{\mu_i^{(\kappa+1)} - \mu_i^{(\kappa)}\} < \varepsilon_\mu \wedge \max_i \{\gamma_i^{(\kappa+1)} - \gamma_i^{(\kappa)}\} < \varepsilon_\gamma \wedge \max_{q,k} \{\delta_q^{(\kappa+1)} - \delta_q^{(\kappa)}\} < \varepsilon_\delta$  **then**
- 10:     **BreakWhile**
- 11:   **end if**
- 12:   Update  $\kappa = \kappa + 1$ .
- 13:   // **Part(b): SCA-based Power Iteration**
- 14:   Initialize  $\{\tilde{p}_{ij}^{(\kappa,0)}\}_{i \in \mathcal{N}, j \in \mathcal{N}}$  variables.
- 15:   **while** (1) **do**
- 16:     Update  $\tilde{p}_{ij}^{(\kappa, \tilde{\kappa}+1)}$  by solving the convex optimization problem (41) with substitute variable  $\tilde{h}_q(p_{ij}^{(\kappa)}; \tilde{p}_{ij}^{(\kappa, \tilde{\kappa})})$  mentioned in (36).
- 17:     **if**  $\{\tilde{p}_{ij}^{(\kappa, \tilde{\kappa}+1)}\}_{i \in \mathcal{N}, j \in \mathcal{N}}$  converge **then**
- 18:       **Break While**
- 19:     **end if**
- 20:     Update  $\tilde{\kappa} = \tilde{\kappa} + 1$ .
- 21:   **end while**
- 22: **end while**
- 23: **return**  $\{\mathcal{P}^b\}$

definition of convex functions. Thus, the Lagrangian function of  $P_4^{\text{Sub}}$  is expressed as follows.

$$\begin{aligned}
L_4(\{\mathcal{C}\}, \{\alpha_m, \beta_m, \eta_m\}_{m \in \mathcal{N}^M}) \\
&= \sum_{m=1}^M \sum_{q=1}^Q \frac{x_{qm} \phi_q(1 - y_{qm})}{c_{qm}} + \sum_{m=1}^M \lambda_m \left( \sum_{q=1}^Q c_{qm} - C_0 \right) \\
&+ \sum_{m=1}^M \theta_m \cdot \left( \epsilon_m \sum_{q=1}^Q c_{qm}^2 x_{qm} \phi_q(1 - y_{qm}) - c_8 \right) \\
&\triangleq \sum_{m=1}^M L_4^m(\{c_{qm}\}_{m \in \mathcal{N}^M}) + c_9 \tag{45}
\end{aligned}$$

where  $L_4^m(\{c_{qm}\}_{m \in \mathcal{N}^M})$  is defined as the function of decision variables  $\{\mathcal{C}\}$ , and  $c_9$  is the constant that is independent of  $\{\mathcal{C}\}$ .

Then, take the partial derivative of the Lagrangian function  $L_4^m(\{c_{qm}\}_{m \in \mathcal{N}^M})$  with respect to  $c_{qm}$  for any  $m \in \mathcal{N}^M$  as

$$\begin{aligned}
\frac{\partial L_4^m(\{c_{qm}\}_{m \in \mathcal{N}^M})}{\partial c_{qm}} &= -\frac{x_{qm} \phi_q(1 - y_{qm})}{c_{qm}^2} + \lambda_m \\
&+ \left( 2\theta_m \epsilon_m x_{qm} \phi_q(1 - y_{qm}) \right) \cdot c_{qm} \tag{46}
\end{aligned}$$

**Algorithm 4** Solving for Computing Resources  $\{\mathcal{C}\}$  for Gateway Guoys

**Input:** Optimization Problem  $P_4^{\text{Sub}}$ .

**Output:**  $\{\mathcal{C}\} = \{c_{qm}\}_{q \in \mathcal{N}^Q, m \in \mathcal{N}^M}$

- 1: **Initialize**  $\{\{c_{qm}^{(0)}\}_{q \in \mathcal{N}^Q}, \lambda_m^{(0)}, \theta_m^{(0)}\}_{m \in \mathcal{N}^M}$
- 2: **while** (1) **do**
- 3:   **for**  $m = \{1, 2, \dots, M\}$  **do**
- 4:     Calculate the Lagrangian function based on (45).
- 5:     Calculate the partial derivatives based on (46).
- 6:     Update decision variable  $c_{qm}^{(\kappa+1)}$  for  $\forall q, m$ , based on (47).
- 7:     Update Lagrange multipliers  $\lambda_m^{(\kappa+1)}$  and  $\theta_m^{(\kappa+1)}$  for  $\forall m$  based on (48).
- 8:   **end for**
- 9:   **if**  $\max_m \{\lambda_m^{(\kappa+1)} - \lambda_m^{(\kappa)}\} < \varepsilon_\lambda \wedge \max_m \{\theta_m^{(\kappa+1)} - \theta_m^{(\kappa)}\} < \varepsilon_\theta$  **then**
- 10:     **BreakWhile**
- 11:   **end if**
- 12: **end while**
- 13: **return**  $\{\mathcal{C}\}$

let  $\frac{\partial L_4^m(\{c_{qm}\}_{m \in \mathcal{N}^M})}{\partial c_{qm}} = 0$ , then the formula (46) can be regarded as the intersection of the linear function  $(2\theta_m \epsilon_m) \cdot c_{qm} + \frac{\lambda_m}{x_{qm} \phi_q(1 - y_{qm})}$  and the power function  $c_{qm}^{-2}$ . Next, let  $x^* = H(a, b)$  denote the intersection (or solution) of function  $ax + b = x^{-2}$ , which implies that the iteration formula can be written as follows.

$$c_{qm}^{(\kappa+1)} = H(2\theta_m \epsilon_m, \frac{\lambda_m^{(\kappa)}}{x_{qm} \phi_q(1 - y_{qm})}) \tag{47}$$

then, the Lagrange multipliers are calculated as follows.

$$\lambda_m^{(\kappa+1)} = \lambda_m^{(\kappa)} + \zeta_{m\lambda} \cdot \left( \sum_{q=1}^Q c_{qm} - C_0 \right) \tag{48}$$

$$\theta_m^{(\kappa+1)} = \theta_m^{(\kappa)} + \zeta_{m\theta} \cdot \left( \epsilon_m \sum_{q=1}^Q c_{qm}^2 x_{qm} \phi_q(1 - y_{qm}) - c_8 \right)$$

Finally, the solution algorithm for decision variables  $\{\mathcal{C}\}$  is given in Alg. 4.

4) *Solving for Decision Variables  $\{\mathcal{Y}\}$* : In this subsection, the solution of the decision variable  $\mathcal{Y}$  is proposed, when other decision variables are fixed, and the optimization problem  $P_5^{\text{Sub}}$  is given as follows.

$$P_5^{\text{Sub}} : \min_{\mathcal{Y}} \sum_{m=1}^M T_m^{s1} + T^{s2} + \sum_{m=1}^M T_{ms}^{c2} \tag{49a}$$

$$s.t. \quad C_5, C_6, C_9 \tag{49b}$$

where both the objective function and the constraints are linear function with respect to  $\mathcal{Y} = \{y_{qm}\}_{q \in \mathcal{N}^Q, m \in \mathcal{N}^M}$ . Thus, we can solve the optimization problem  $P_5^{\text{Sub}}$  using a similar method mentioned above (or using the barrier function interior point method, CVX, and so on).

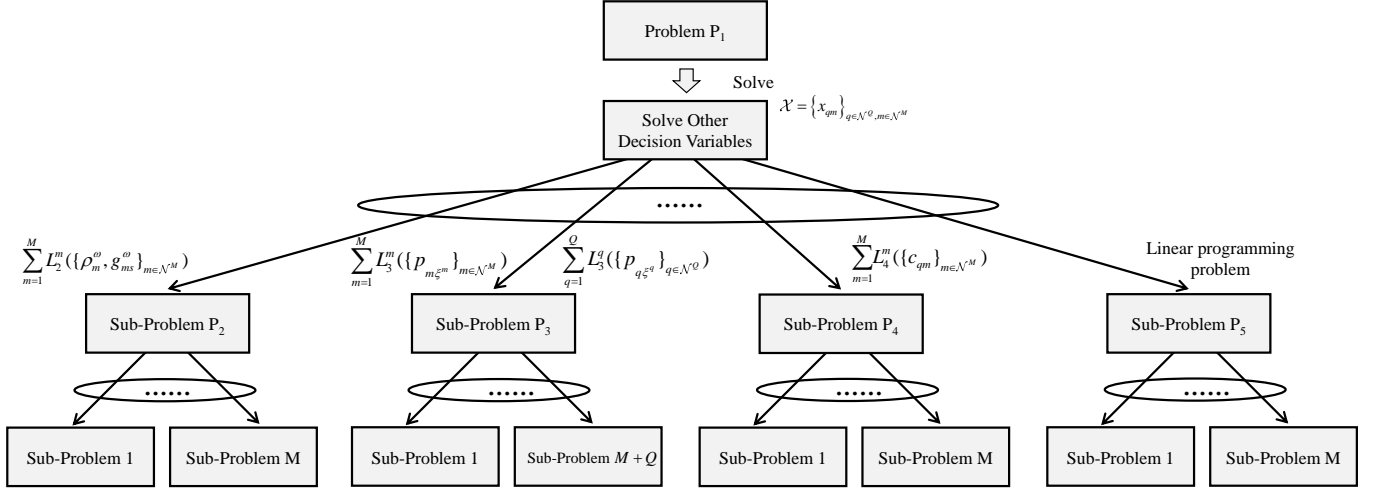


Fig. 7: Detailed decomposition architecture for the optimization problem  $P_1$ .

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**Algorithm 5** Joint Iteration Algorithm (Top Algorithm)

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**Input:** Optimization Problems  $P_2^{\text{Sub}}, P_3^{\text{Sub}}$  and  $P_4^{\text{Sub}}$ .

**Output:**  $\{\varrho, \mathcal{G}, \mathcal{P}^b, \mathcal{C}, \mathcal{Y}\}$ .

- 1: **Initialize**  $\varrho^{(0)}, \mathcal{G}^{(0)}, \mathcal{P}^{b(0)}, \mathcal{C}^{(0)}, \mathcal{Y}^{(0)}, \tau = 0$ .
  - 2: **while** (1) **do**
  - 3:   Fix the decision variables  $\{\mathcal{P}^{b(\tau)}, \mathcal{C}^{(\tau)}, \mathcal{Y}^{(\tau)}\}$ , and solve  $\{\varrho^{(\tau+1)}, \mathcal{G}^{(\tau+1)}\}$  by Alg. 2.
  - 4:   Fix the decision variables  $\{\varrho^{(\tau+1)}, \mathcal{G}^{(\tau+1)}, \mathcal{C}^{(\tau)}, \mathcal{Y}^{(\tau)}\}$ , and solve  $\{\mathcal{P}^{b(\tau+1)}\}$  by Alg. 3.
  - 5:   Fix the decision variables  $\{\varrho^{(\tau+1)}, \mathcal{G}^{(\tau+1)}, \mathcal{P}^{b(\tau+1)}, \mathcal{Y}^{(\tau)}\}$ , and solve  $\{\mathcal{C}^{(\tau+1)}\}$  by Alg. 4.
  - 6:   Fix the decision variables  $\{\varrho^{(\tau+1)}, \mathcal{G}^{(\tau+1)}, \mathcal{P}^{b(\tau+1)}, \mathcal{C}^{(\tau+1)}\}$ , and solve  $\{\mathcal{Y}^{(\tau+1)}\}$  by barrier function interior point method.
  - 7:   **if**  $\{\mathcal{P}^{b(\tau+1)}, \mathcal{C}^{(\tau+1)}, \mathcal{Y}^{(\tau+1)}, \varrho^{(\tau+1)}, \mathcal{G}^{(\tau+1)}\}$  converge **then**
  - 8:     **BreakWhile**
  - 9:   **end if**
  - 10:   Update  $\tau = \tau + 1$ .
  - 11: **end while**
  - 12: **return**  $\{\varrho, \mathcal{G}\}$
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The computational complexity is analyzed in this paper. In particular, the BCD method, as mentioned in Alg. 5, has a computational complexity of  $\mathcal{O}(\tau \cdot C(M, Q, W))$ , where  $\tau$  represents the iteration count for BCD, and  $C(M, Q, W)$  denotes the maximum complexity for solving each optimization problem (Alg. 2, Alg. 3, and Alg. 4) when other decision variables are fixed, which implies that  $C(M, Q, W) = \max\{C_2(M, W), C_3(M, Q), C_4(M, Q), LP\}$ .

5) *Outer Iteration Algorithm and Complexity Analysis :*

It is worth noting that Alg. 2, Alg. 3, and Alg. 4 serve as inner-layer iterative algorithms, each solving one variable while keeping the others fixed. Based on Fig. 3, a joint iterative algorithm based on BCD method for the outer layer is proposed. This algorithm iteratively polls and resolves the different variables in sequence, as detailed in Alg. 5.

Next, we analyze each individual sub-optimization problem. It is worth mentioning that each sub-optimization problem mentioned in this article can be decomposed in the form of  $L^m(\cdot)$  or  $L^q(\cdot)$ , as mentioned in Fig. 7, which implies that distributed parallel computing is feasible, and the algorithm complexity can be effectively reduced. Specifically, for the sub-optimization problem  $P_2$ , the computational complexity  $C_2(M, W) = k'_1 \cdot W$ , where  $M$  is the number of nodes for parallel computing, and  $k'_1$  is the number of iteration index. For the sub-optimization problem  $P_3$ , the computational complexity  $C_3(M, W) = \max\{C_3(M), C_3(Q)\} = \max\{k'_2 \cdot M, k'_3 \cdot Q\}$ . In addition,  $C_4(M, Q) = k'_4 \cdot Q$ , and  $LP$  is the computational complexity of linear programming problem. Thus, based on the decomposition architecture for the optimization problem  $P_1$ , the computational complexity can be effectively reduced