In the context of time series analysis, yes, we often refer to observations taken at specific time points or dates. The values in the time series represent data points collected sequentially over time, such as daily stock prices, monthly sales figures, or yearly temperature averages.

Clarifying the Demonstration

Let's adjust the demonstration to clarify how you would apply ACF and PACF with an example that is more relevant to time series data, including explicit time points. Here's how it could look:

Example Time Series Data

Let's consider a simple time series where each value corresponds to monthly sales figures over a period of five months:

- **Dates**: January, February, March, April, May
- **Sales Figures**: [200, 250, 300, 350, 400]

In this case, the values represent sales figures at the end of each month.

Calculation of ACF and PACF

Step 1: Calculate the Mean

- The mean ($\bar{Y}$):

$$\bar{Y} = \frac{200 + 250 + 300 + 350 + 400}{5} = 320$$

Step 2: Calculate ACF for Lag 1 ($k = 1$)

Using the ACF formula:

$$\rho(1) = \frac{\sum_{t=2}^{5}(Y_t - \bar{Y})(Y_{t-1} - \bar{Y})}{\sum_{t=1}^{5}(Y_t - \bar{Y})^2}$$

1. **Calculate the Numerator**:

$$\text{Numerator} = (250 - 320)(200 - 320) + (300 - 320)(250 - 320) + (350 - 320)(300 - 320) + (400 - 320)(350 - 320)$$

$$= (-70)(-120) + (-20)(-70) + (30)(30) + (80)(30)$$

$$= 8400 + 1400 + 900 + 2400 = 12100$$

2. **Calculate the Denominator**:

$$\text{Denominator} = (200 - 320)^2 + (250 - 320)^2 + (300 - 320)^2 + (350 - 320)^2 + (400 - 320)^2$$

$$= (-120)^2 + (-70)^2 + (-20)^2 + (30)^2 + (80)^2$$

$$= 14400 + 4900 + 400 + 900 + 6400 = 20400$$

3. **Calculate ACF**:

$$\rho(1) = \frac{12100}{20400} \approx 0.593$$

Step 3: Calculate PACF for Lag 1

Since the PACF at lag 1 is equal to the ACF at lag 1:

$$\phi(1) = \rho(1) \approx 0.593$$

Step 4: Calculate ACF for Lag 2 ($k = 2$)

To calculate ACF for lag 2, use the same formula with lag 2:

1. **Numerator**:

$$= (300 - 320)(200 - 320) + (350 - 320)(250 - 320) + (400 - 320)(300 - 320)$$

$$= (-20)(-120) + (30)(-70) + (80)(-20)$$

$$= 2400 - 2100 - 1600 = -1300$$

2. **Denominator** remains the same as calculated earlier: 20400.
3. **Calculate ACF**:

$$\rho(2) = \frac{-1300}{20400} \approx -0.064$$

Step 5: Calculate PACF for Lag 2

Using the previously calculated values:

$$\phi(2) = \rho(2) - \phi(1) \cdot \rho(1)$$

$$= -0.064 - (0.593 \cdot 0.593)$$

$$= -0.064 - 0.351649 \approx -0.415649$$

Summary of Calculated Values

- ACF at Lag 1: $\rho(1) \approx 0.593$
- PACF at Lag 1: $\phi(1) \approx 0.593$
- ACF at Lag 2: $\rho(2) \approx -0.064$
- PACF at Lag 2: $\phi(2) \approx -0.415649$

Conclusion

In this adjusted example, the time series values are clearly associated with specific dates, and the calculations of ACF and PACF show how to assess the relationships among observations over time. ACF and PACF help identify the appropriate lag structure for time series models like ARIMA.