Assignment - 9

Task 1

1. You survey households in your area to find the average rent they are paying. Find the standard deviation from the following data:

$1550, $1700, $900, $850, $1000, $950.

Solution:

Formula of Mean:

$$\bar{X} = \frac{1}{\mathrm{n}} * \sum_{i=1}^{n} \mathrm{x_i}$$

Calculation of Mean:

$\bar{X} = \frac{(X1+X2+X3+X4+X5+X6)}{n}$

The values of $X1$= 1550, $X2$=1700, $X3$=900, $X4$=850, $X5$=1000 & $X6$=950

The number of elements, n=6

$\bar{X} = \frac{(1550+1700+900+850+1000+950)}{6}$

$\bar{X}$ =1158.33

Formula of Variance:

$$S^2 = \frac{\sum(X - \bar{X})^2}{N-1}$$

Calculation of Variance:

	$\boldsymbol{X}$	$\boldsymbol{X - \bar{X}}$	$(\boldsymbol{X} - \boldsymbol{\bar{X}})^2$
1	1550	391.67	153405.38
2	1700	541.67	293406.38
3	900	-258.33	66734.38
4	850	-308.33	95067.38
5	1000	-158.33	25068.38

6	950	-208.33	43401.38
Sum	6950		677083.28
Mean ($\bar{X}$)	1158.33		
N	6		

Variance (S^2) = $\frac{677083.28}{(6-1)}$

Variance (S^2) = 135416.65

<u>Formula of Standard Deviation:</u>

$$S = \sqrt{\frac{\sum(X - \bar{X})^2}{N - 1}}$$

<u>Calculation of Standard Deviation:</u>

Standard Deviation (S) = $\sqrt{135416.65}$

Standard Deviation (S) = 367.99

	X	$X - \bar{X}$	$(X - \bar{X})^2$
1	1550	391.67	153405.38
2	1700	541.67	293406.38
3	900	-258.33	66734.38
4	850	-308.33	95067.38
5	1000	-158.33	25068.38
6	950	-208.33	43401.38
Sum	6950		677083.28
Mean ($\bar{X}$)	1158.33		
N	6		
Variance (S^2)	135416.65		
Standard Deviation (S)	367.99		

2. Find the variance for the following set of data representing trees in California (heights in feet):
3, 21, 98, 203, 17, 9

Solution:

Formula of Mean:

$$\bar{X} = \frac{1}{\mathrm{n}} * \sum_{i=1}^{n} \mathrm{x_i}$$

Calculation of Mean:

$\bar{X} = \frac{(X1+X2+X3+X4+X5+X6)}{n}$

The values of $X1$= 3, $X2$=21, $X3$=98, $X4$=203, $X5$=17 & $X6$=9

The number of elements, n=6

$\bar{X} = \frac{(3+21+98+203+17+9)}{6}$

$\bar{X}$ =58.5

Formula of Variance:

$$S^2 = \frac{\sum(X - \bar{X})^2}{N-1}$$

Calculation of Variance:

	$\boldsymbol{X}$	$\boldsymbol{X - \bar{X}}$	$\boldsymbol{(X - \bar{X})^2}$
1	3	-55.5	3080.25
2	21	-37.5	1406.25
3	98	39.5	1560.25
4	203	144.5	20880.25
5	17	-41.5	1722.25
6	9	-49.5	2450.25
Sum	351		31099.5
Mean ($\bar{X}$)	58.5		
N	6		
Variance (S^2)	6219.9		

Variance (S^2) = $\frac{31099.5}{(6-1)}$

Variance (S^2) = 6219.9

---

3. In a class on 100 students, 80 students passed in all subjects, 10 failed in one subject, 7 failed in two subjects and 3 failed in three subjects. Find the probability distribution of the variable for number of subjects a student from the given class has failed in.

Solution:

The probability of failing in 0 subjects, $P(X = 0) = \frac{80}{100} = 0.8$

The probability of failing in 1 subjects, $P(X = 1) = \frac{10}{100} = 0.1$

The probability of failing in 2 subjects, $P(X = 2) = \frac{7}{100} = 0.07$

The probability of failing in 3 subjects, $P(X = 3) = \frac{3}{100} = 0.03$

The probability distribution can be shown as:

X	0	1	2	3
$P(X)$	0.8	0.1	0.07	0.03

Task 2:

1. A test is conducted which is consisting of 20 MCQs (multiple choices questions) with every MCQ having its four options out of which only one is correct. Determine the probability that a person undertaking that test has answered exactly 5 questions wrong.

Solution:

Here, No of Trials = $\boldsymbol{n}$ = 20

Count of success = $\boldsymbol{k}$ = 5,

Count of getting a wrong answer = $\boldsymbol{n} - \boldsymbol{k}$ = $20 - 5$ = 15.

Here the probability of success = probability of giving a right answer = q = $\frac{1}{4}$

Hence, the probability of failure = probability of giving a wrong answer=p = $1 - q$ = $1 - \frac{1}{4} = \frac{3}{4}$

Formula of Binomial Distribution:

$$b(k;\ n, P) \ = \ nCk * P^k \ * \ (1 - P)n - k$$

$$P(k) = \frac{n!}{(n-k)k!} * \ (p)^k * \ (q)^{n-k}$$

When we substitute these values in the formula for Binomial distribution we get,

$$P(k) = \frac{20!}{15! * 5!} * \ (\frac{3}{4})^5 * \ (\frac{1}{4})^{15}$$

$P(k)$= 0.0000034265

Thus, the required probability is $\mathbf{0.0000034265}$ approximately.

---

2. A die marked A to E is rolled 50 times. Find the probability of getting a "D" exactly 5 times. 2. A die marked A to E is rolled 50 times. Find the probability of getting a "D" exactly 5 times.

Solution:

Here, No of Trials = $\boldsymbol{n}$ = 50

Count of success = $\boldsymbol{k}$ = 5,

Count of getting a wrong answer = $\boldsymbol{n} - \boldsymbol{k}$ = $50 - 5$ = 45

The probability of success = probability of getting a "D"= $\boldsymbol{q}$ = $\frac{1}{5}$

Hence, the probability of failure = probability of not getting a "D" = $\boldsymbol{p}$ =$\mathbf{1} - \boldsymbol{q}$ = $1 - \frac{1}{5} = \frac{4}{5}$

Formula of Binomial Distribution:

$$b(k;\ n, P) \ = \ nCk * P^k \ * \ (1 - P)n - k$$

$$P(k) = \frac{n!}{(n-k)k!} * \ (p)^k * \ (q)^{n-k}$$

When we substitute these values in the formula for Binomial distribution we get,

$$P(k) = \ \frac{50!}{45! * 5!} * \ (\frac{1}{5})^5 * \ (\frac{4}{5})^{45}$$

$$P(k) = 0.02953$$

Thus, the required probability is $\mathbf{0.02953}$ approximately.

3. Two balls are drawn at random in succession without replacement from an urn containing 4 red balls and 6 black balls.

Solution:

Find the probabilities of all the possible outcomes.

Red balls (R) = 4 Black balls (B) = 6 Total balls (T) = 10

Total ways to draw 2 balls =$10 * 9$ = 90 ways

Ways to draw both red balls (RR) =$4 * 3$ =12 ways

Ways to draw 1 red and 1 black ball (RB) = $4 * 6$ = 24 ways

Ways to draw 1 black and 1 red ball (BR) = $6 * 4$ = 24 ways

Ways to draw both black balls (BB) = $6 * 5$ = 30 ways

The probability of all possible outcomes is:

Probability of drawing both red balls (RR) = $(\frac{4}{10}) * (\frac{3}{9})$ = $\frac{12}{90}$ = 0.1333 = $\mathbf{13.33}\%$

Probability of drawing 1 red & 1 black ball (RB) = $(\frac{4}{10}) * (\frac{6}{9})$ = $\frac{24}{90}$= 0.2666 = $\mathbf{26.67}\%$

Probability of drawing 1 black & 1 red ball (BR) = $(\frac{6}{10}) * (\frac{4}{9})$ = $\frac{24}{90}$ =0 $.2666$ = $\mathbf{26.67}\%$

Probability of drawing both black balls (BB) = $(\frac{6}{10}) * (\frac{5}{9})$ = $\frac{30}{90}$ = 0.3333 = $\mathbf{33.33}\%$