Computational Finance

AMSTERDAM BUSINESS SCHOOL

Business

Preliminaries

General Information

- My name is Simon Broda. You can find me at REC E4.27 (by appointment). Email: s.a.broda@uva.nl.
- Format of this course: 12 lectures of 2h each (2 per week), plus computer labs.
- Final grade based on a group assignment (groups of two; 40%), an indiviual assignment (35%), and a final exam (closed book, 2h; 25%).
- Additional exercises will be made available but not graded.

Material

- These lecture slides. Available on Blackboard, Github, and Microsoft Azure.
- Books:
  - Yves Hilpisch. Python for Finance: Analyze Big Financial Data. O’Reilly, 2014. ISBN 978-1-4919-4528-5 (603 pages, c. EUR 31). Code is available on Github.
  - John C. Hull. Options, Futures and Other Derivatives. 8th Edition (or later), Prentice Hall, 2012. ISBN 978-0273759072 (847 pages, c. EUR 58).
- Further reading:
  - Python documentation
  - Yves Hilpisch. Derivatives Analytics with Python. Wiley, 2015. ISBN 978-1-119-03799-6 (374 pages, c. EUR 72). Code is available on Github.
  - Python for Data Analysis. 2nd Edition, O'Reilly, 2017. ISBN 978-1-4919-5766-0 (544 pages, c. EUR 34). Code is available on Github.

Outline and Reading List

Week	Topic	Read: Hilpisch (2014)	Read: Hull (2012)
1	Introduction to Python	Chs. 2, 4 (pp. 79-95)	
2	Dealing with Data	Chs. 4 (pp. 95-108), 6, App. C	
3	Risk Measures; Plotting	Chs. 5, 10 (pp. 398-301), 11 (pp. 307-322)	Chs. 21.1, 21.2, 21.8, 22.1, 22.2
4	Binomial Trees	Ch. 8 (pp. 218-223)	Chs. 12, 14, 19, 20.1-20.5
5	Monte Carlo Methods	Ch. 10 (pp. 265-287, 290-294)	Chs. 13, 16.3, 20.6, 25.12
6	Advanced Monte Carlo Methods	Ch. 10 (pp. 287-290)	Chs. 18, 20.7, 25.7-25.12

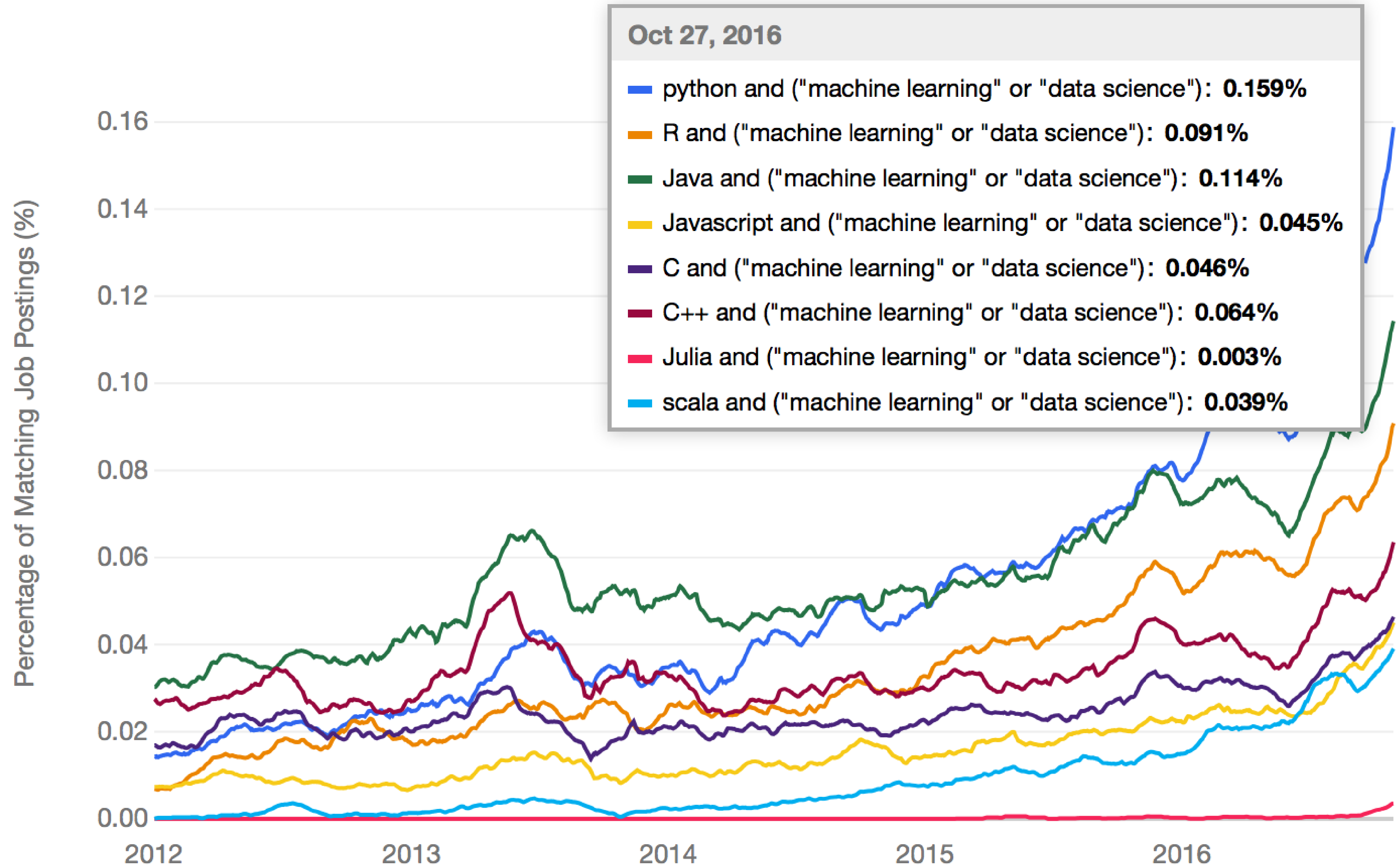
Note: Chapter numbers for Hull refer to the 8th edition. Increment by 1 for the 9th and 10th editions.

Introduction to Python

Why Python?

- General purpose programming language, unlike, e.g., Matlab®.
- High-level language with a simple syntax, interactive (*REPL*: read-eval-print loop). Hence ideal for rapid development.
- Vast array of libraries available, including for scientific computing and finance.
- Native Python is usually slower than compiled languages like C++. Alleviated by highly optimized libraries, e.g. NumPy for calculations with arrays.
- Free and open source software. Cross-platform.
- Python skills are a marketable asset: most popular language for data science.

Job Postings on Indeed.com



[Source]

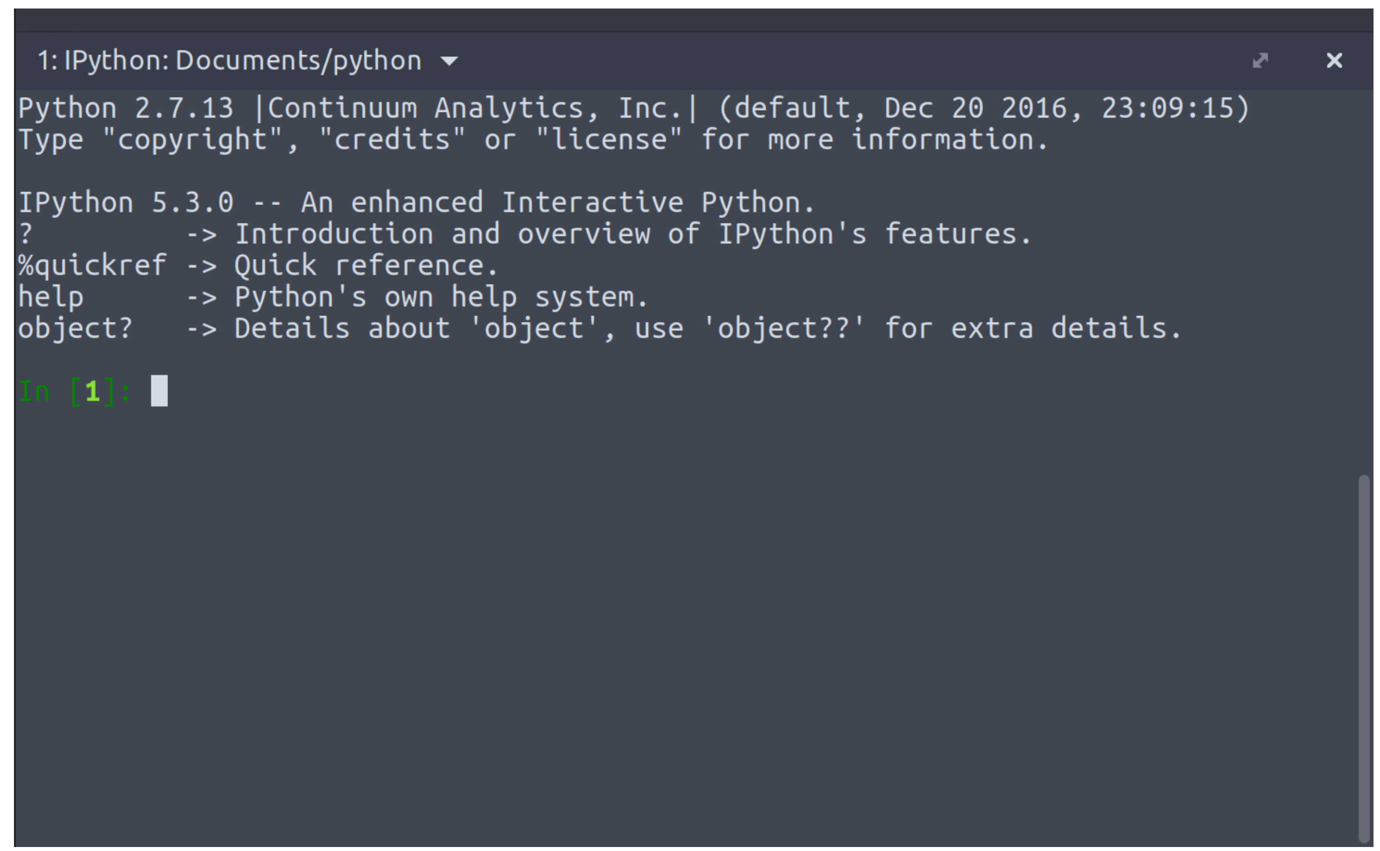
Obtaining Python

- Anaconda is a Python distribution, developed by Continuum Analytics, and specifically designed for scientific computing.
- Comes with its own package manager (conda). Many important packages (the *SciPy stack*) are pre-installed.
- Two versions: Python 2.7 and 3.6. Like the book, we will be using Python 2.7, which is still the industry standard. Most of our code should run on both with minimal adjustments.
- Obtain it from here. I recommend adding it to your PATH upon installation.
- Optional: Install the RISE plugin to allow viewing notebooks as slide shows:

In [1]:
```
#uncomment the next line to install. Note: "!" executes shell commands.
#!conda install -y -c damianavila82 rise
```

IPython Shell

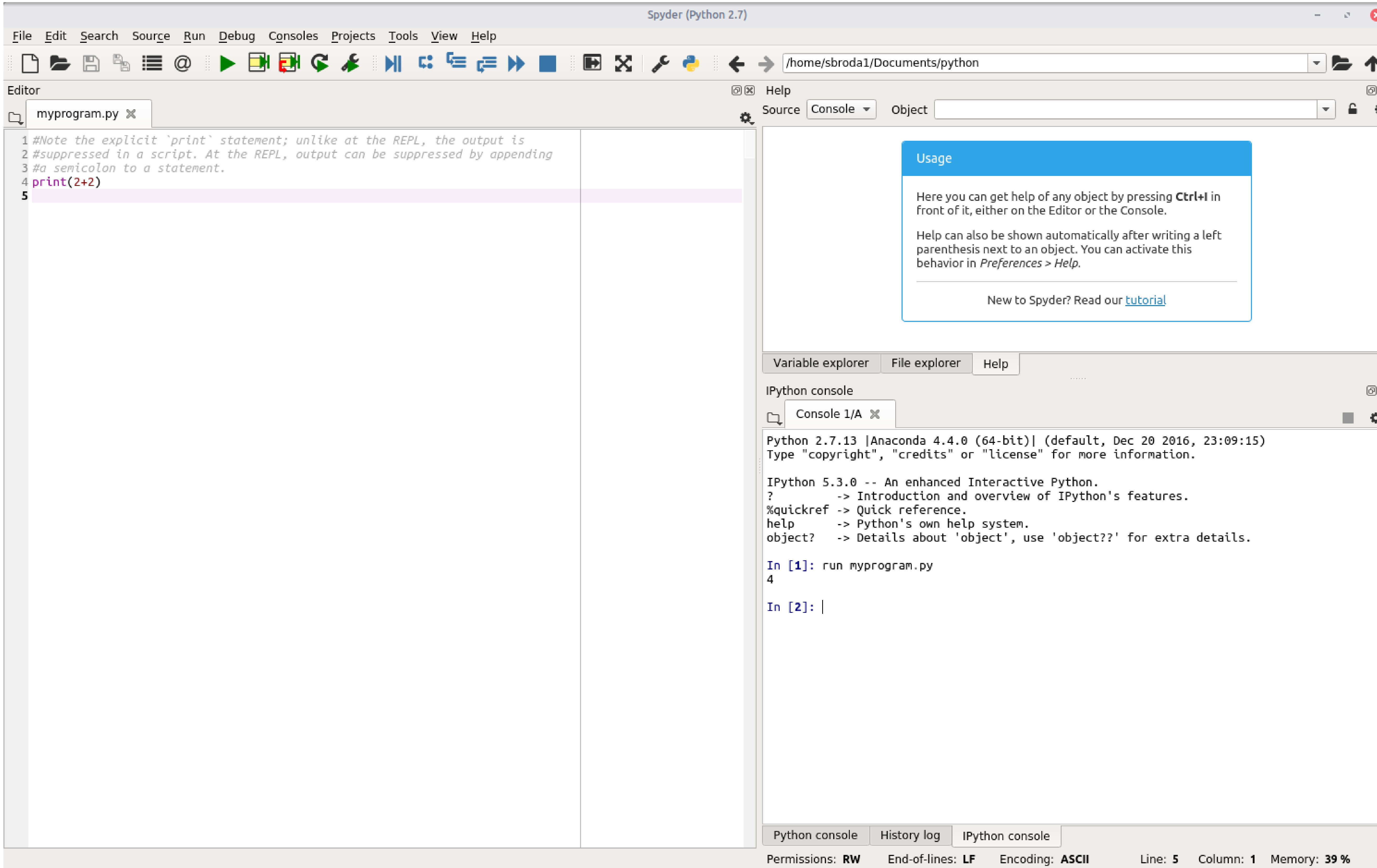
- Python features a *read-eval-print loop* (REPL) which allows you to interact with it.
- The most bare-bones method of interactive use is via the *IPython shell*:



- For now, you can treat it as a fancy calculater. Try entering `2+2`. Use `quit()` or `exit()` to quit, `help()` for Python's interactive help.

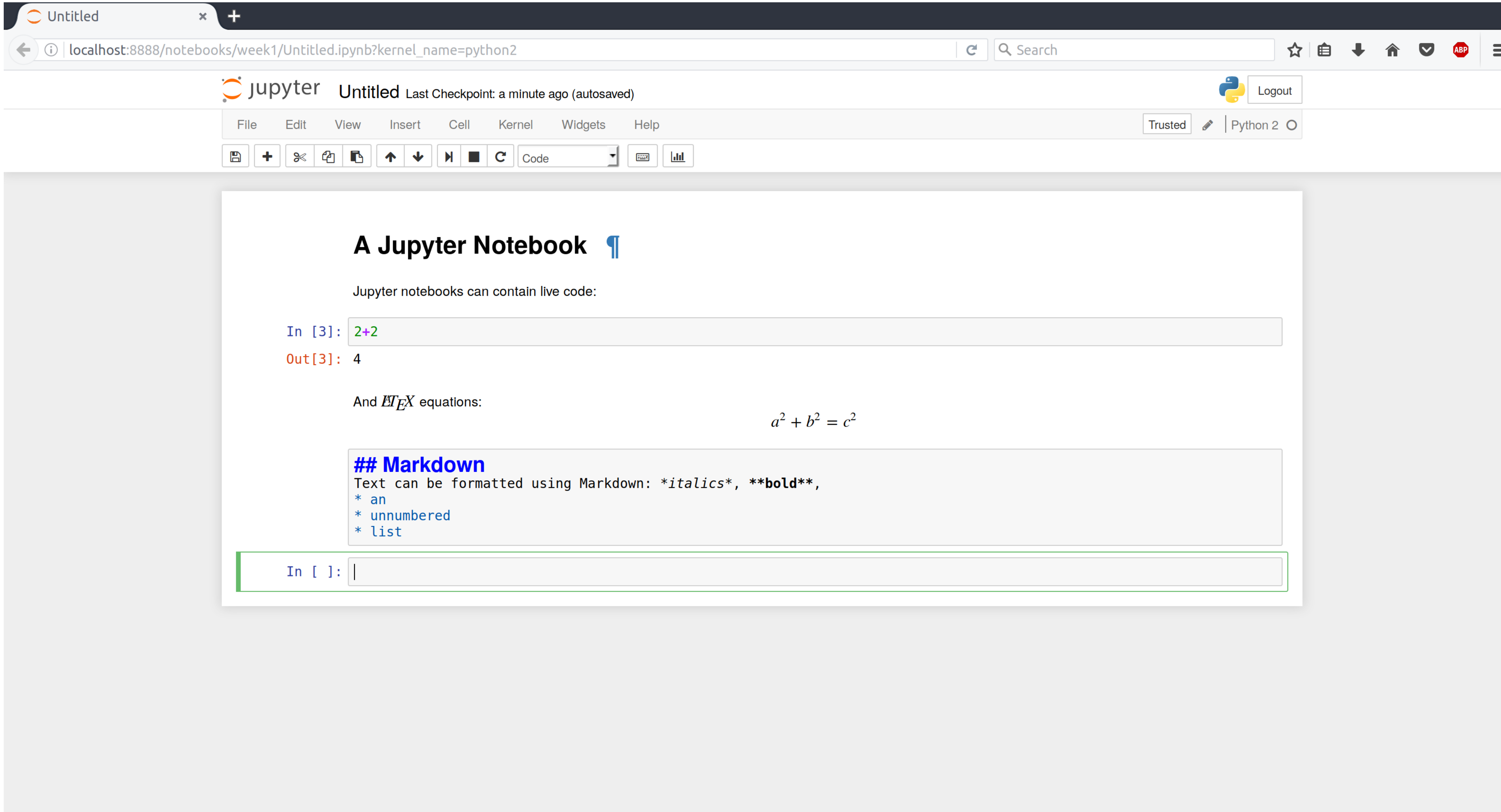
Writing Python Programs

- Apart from using it interactively, we can also write Python *programs* so we can rerun the code later.
- A Python program (called a *script* or a *module*) is just a text file, typically with the file extension `.py`.
- It contains Python commands and comments (introduced by the # character)
- To execute a program, do `run filename.py` in IPython (you may need to navigate to the right directory by using the `cd` command).
- While it is possible to code Python using just the REPL and a text editor, many people prefer to use an *integrated development environment* (IDE).
- Anaconda comes with an IDE called *Spyder* (Scientific PYthon Development EnviRonment), which integrates an editor, an IPython shell, and other useful tools.


Spyder (Python 2.7)
File Edit Search Source Run Debug Consoles Projects Tools View Help
/home/sbroda1/Documents/python
Editor
myprogram.py
#Note the explicit `print` statement; unlike at the REPL, the output is
#suppressed in a script. At the REPL, output can be suppressed by appending
#a semicolon to a statement.
print(2+2)
Help
Source Console Object
Usage
Here you can get help of any object by pressing Ctrl+I in front of it, either on the Editor or the Console.
Help can also be shown automatically after writing a left parenthesis next to an object. You can activate this behavior in Preferences > Help.
New to Spyder? Read our tutorial
Variable explorer File explorer Help
IPython console
Console 1/A
Python 2.7.13 |Anaconda 4.4.0 (64-bit)| (default, Dec 20 2016, 23:09:15)
Type "copyright", "credits" or "license" for more information.
IPython 5.3.0 -- An enhanced Interactive Python.
? -> Introduction and overview of IPython's features.
%quickref -> Quick reference.
help -> Python's own help system.
object? -> Details about 'object', use 'object??' for extra details.
In [1]: run myprogram.py
4
In [2]:
Python console History log IPython console
Permissions: RW End-of-lines: LF Encoding: ASCII Line: 5 Column: 1 Memory: 39 %

Jupyter Notebooks

- Another option is the *Jupyter notebook* (JUlia PYThon (e) R, formerly known as IPython notebook).
- It's a web app that allows you to create documents (`*.ipynb`) that contain text (formatted in [Markdown]), live code, and equations (formatted in $\LaTeX$).
- In fact these very slides are based on Jupyter notebooks. You can find them on my [Github page].
- The slides are also available on my [Microsoft Azure page], where you can also see them as a slide show and/or run them (after cloning them; requires a free Microsoft account).


Untitled
localhost:8888/notebooks/week1/Untitled.ipynb?kernel_name=python2
Search
jupyter
Untitled
Last Checkpoint: a minute ago (autosaved)
Logout
File
Edit
View
Insert
Cell
Kernel
Widgets
Help
Trusted
Python 2
Code
A Jupyter Notebook ¶
Jupyter notebooks can contain live code:
In [3]: 2+2
Out[3]: 4
And LaTeX equations:
$a^2 + b^2 = c^2$
Markdown
Text can be formatted using Markdown: *italics*, **bold**,
* an
* unnumbered
* list
In [ ]:

- A notebook consists of cells, each of which is either designated as Markdown (for text and equations), or as code.
- You should take a moment to familiarize yourself with the keyboard shortcuts. E.g., `enter` enters edit mode, `esc` enters command mode, `ctrl-enter` evaluates a cell, `shift-enter` evaluates a cell and selects the one below.
- Useful references:
    - Jupyter documentation;
    - Markdown cheat sheet;
    - Latex math cheat sheet.

Python Basics

Variables

- A variable is a named memory location. It is assigned using "=" (technically, "=" binds the name on the LHS to the result of the expression on the RHS).

In [2]:
```
a = 2
a = a+1  #bind the name a to the result of the expression a+1
print(a)  #show the result
```

```
3
```

In [3]:
```
a += 1  #shorthand for a=a+1
print(a)
```

```
4
```

- Variable names can be made up from letters, numbers, and the underscore. They may not start with a number. Python is case-sensitive: A is not the same as a.

Built-in Types

Attributes and Methods

- Any Python object has a *type*.
- One can use the `type` function to show the type of an object:

```
In [4]: type(a)  #Functions take one or more inputs (in parentheses) and return an output.
```

```
Out[4]: int
```

- Objects can have attributes and methods associated with them:

```
In [5]: a.real  #an attribute (internal variable stored inside an object)
```

```
Out[5]: 4
```

```
In [6]: a.bit_length()  #a method (function that operates on objects of a particular type)
```

```
Out[6]: 3
```

Numeric Types

- Computers distinguish between integers and floating point numbers.
- Python integers can be arbitrary large (will use as many bits as necessary).
- Python floats are between $\pm 1.8 \cdot 10^{308}$, but are stored with just 64 bits of precision.
- Hence, not all real numbers can be represented, and floating point arithmetic is not exact:

In [7]:
```
a = 1.0; type(a)  #Note that variables can change type: a was an int before
```
Out[7]:
```
float
```

In [8]:
```
a-0.9
```
Out[8]:
```
0.09999999999999998
```

Arithmetic

- The basic arithmetic operations are `+`, `-`, `*`, `/`, and `**` for exponentiation:

In [9]:
```
2*(3-1)**2
```
Out[9]:
```
8
```

- If any of the operands is a `float`, then Python will convert the others to float, too:

In [10]:
```
2*(3-1.0)**2
```
Out[10]:
```
8.0
```

- Note that / performs floor division in Python 2.7 (not 3.6) when both arguments are `ints`:

In [11]:
```
c=3
c/2
```

Out[11]: 1

- We need to convert one argument to float to get the usual division:

In [12]:
```
c/2.0
```

Out[12]: 1.5

In [13]:
```
float(c)/2
```

Out[13]: 1.5

Booleans

- A `bool` can take one of two values: `True` or `False`.
- They are returned by *relational operators*: <, <=, >, >=, == (equality), != (inequality), and can be combined using the *logical operators* `and`, `or`, and `not`.

In [14]:
```
1 <= 2 < 4
```
Out[14]:
```
True
```

In [15]:
```
1 < 2 and 2 < 1
```
Out[15]:
```
False
```

In [16]:
```
not(1 < 2)
```
Out[16]:
```
False
```

Sequence Types: Containers with Integer Indexing

Strings

- Strings hold text. They are constructed using either single or double quotes:

In [17]:
```
s1 = "Python"; s2 = ' is easy.'; s1+s2  #Concatenation
```
Out[17]:
```
'Python is easy.'
```

- Strings can be indexed into:

In [18]:
```
s1[0]  #Note zero-based indexing
```
Out[18]:
```
'P'
```

In [19]:
```
s1[-1]  #Negative indexes count from the right:
```
Out[19]:
```
'n'
```

- We can also pick out several elements ("*slicing*"). This works for all *sequence types* (lists, NumPy arrays, ...).

In [20]:
```
s1[0:2]  #Elements 0 and 1; left endpoint is included, right endpoint excluded.
```
Out[20]:
```
'Py'
```

In [21]:
```
s1[0:6:2] #start:stop:step
```
Out[21]:
```
'Pto'
```

In [22]:
```
s1[::-1] #start and stop can be ommitted; default to 0 and len(str)
```
Out[22]:
```
'nohtyP'
```

- Strings are *immutable*:

In [23]:
```
#Wrapping this in a try block so the error doesn't break `Run all` in Jupyter.
try:
    s1[0] = "C"  #This errors.
except TypeError as e:
    print(e)
```
```
'str' object does not support item assignment
```

- Python has many useful methods for strings:

In [24]:
```
print(', '.join(filter(lambda m: callable(getattr(s1, m)) and not m.startswith("_"), dir(s1))))
```

```
capitalize, center, count, decode, encode, endswith, expandtabs, find, format, index, isalnum, isalpha, isdig
it, islower, isspace, istitle, isupper, join, ljust, lower, lstrip, partition, replace, rfind, rindex, rjust,
rpartition, rsplit, rstrip, split, splitlines, startswith, strip, swapcase, title, translate, upper, zfill
```

In [25]:
```
help(s1.upper)
```

```
Help on built-in function upper:

upper(...)
    S.upper() -> string

    Return a copy of the string S converted to uppercase.
```

In [26]:
```
(s1+s2).replace('easy','hard').upper()
```

Out[26]:
```
'PYTHON IS HARD.'
```

Lists

- Lists are indexable collections of arbitrary (though usually homogeneous) things:

In [27]:
```
list1 = [1, 2., 'hi']; print(list1)
```

```
[1, 2.0, 'hi']
```

- The function `len` returns the length of a list (or any other sequence):

In [28]:
```
len(list1)
```

Out[28]:
```
3
```

- Like strings, they support indexing, but unlike strings, they are *mutable*:

In [29]:
```
list1[2] = 42; print(list1)
```

```
[1, 2.0, 42]
```

- Note the following:

In [30]:
```python
list2 = list1  #Bind the name list2 to the object list1. This does not create a copy:
list2[0] = 13
print(list1)  #list2 and list1 are the _same_ object!
```

```
[13, 2.0, 42]
```

In [31]:
```python
list3 = list1[:]  #This DOES create a copy.
list3 == list1  #Tests if all elements are equal.
```

Out[31]:
```
True
```

In [32]:
```python
list3 is list1  #Tests if list3 and list1 refer to the same object.
```

Out[32]:
```
False
```

In [33]:
```python
list2 is list1
```

Out[33]:
```
True
```

- Lists of integers can be constructed using the `range` function:

```
In [34]: range(1, 11, 2)  #start, stop, [,step]
Out[34]: [1, 3, 5, 7, 9]
```

```
In [35]: range(5)  #start and step can be ommited.
Out[35]: [0, 1, 2, 3, 4]
```

- *List comprehensions* allow creating lists programmatically:

```
In [36]: [x**2 for x in range(1, 10) if x > 3 and x < 7]
Out[36]: [16, 25, 36]
```

- The `for` and `if` statements will be discussed in more detail later.

- Methods for lists:

In [37]:
```
print(', '.join(filter(lambda m: callable(getattr(list1, m)) and not m.startswith("_"), dir(list1))))
```

```
append, count, extend, index, insert, pop, remove, reverse, sort
```

In [38]:
```
list1.append(13);  #append 13 to the list `l1`
print(list1)
```

```
[13, 2.0, 42, 13]
```

In [39]:
```
list1.remove(13)  #remove first occurence of 13 from l1
print(list1)
```

```
[2.0, 42, 13]
```

- Note: Table 4-2 in the book incorrectly states that `remove[i]` removes the element at index `i`. For that, use

In [40]:
```
del(list1[0]); print(list1)
```

```
[42, 13]
```

- `del` can also be used to delete variables (technically, to unbind the variable name).

xranges

- An `xrange` is similar to a list created with `range`, but it is more memory efficient because the list elements are created on demand (*lazily*).

In [41]:
```
xrange(1, 10, 2)[3]
```
Out[41]:
```
7
```

Tuples

- A `tuple` is an immutable sequence. It is created with round brackets:

In [42]:
```
(1, 2., 'hi')
```
Out[42]:
```
(1, 2.0, 'hi')
```

Other built-in datatypes

- Other built-in datatypes include `sets` (unordered collections) and `dicts` (collections of key-value pairs). See Hilpisch (2014), pp. 92-94.

Control Flow

- Control flow refers to the order in which commands are executed within a program.
- Often we would like to alter the linear way in which commands are executed. Examples:
  1. *Conditional branch*: Code that is only evaluated if some condition is true.
  2. *Loop*: Code that is evaluated more than once.

Conditional Branch: The `if-else` statement

In [43]:
```
x = 3  #Uncomment the next line for interactive use.
#x = int(raw_input("Enter a number between 0 and 9: "))  #`raw_input` returns a string. `int` converts to inte
if x < 0:
    print("You have entered a negative number.")
elif x > 9:
    print("You have entered a number greater than 9.")
else:
    print("Thank you. You entered %s." %x)  #String interpolation.
```

```
Thank you. You entered 3.
```

- Notes:
    1. Code blocks are introduced by colons and *have* to be indented.
    2. The `if` block is executed if and only if the first condition is true
    3. The optional `elif` (short for 'else if') block is executed if and only if the first condition is false and the second one is true. There could be more than one.
    4. The optional `else` block is executed if and only if none of the others was.

`While` loops

- Similar to `if`, but jumps back to the `while` statement after the `while` block has finished.
- The `else` block is executed when the condition becomes false (not if the loop is exited through a `break` statement; see next).

In [44]:
```
x = 1  #Set this to -1 to run.
while x < 0 or x > 9:
    x = int(raw_input("Enter a number between 0 and 9: "))
    if x < 0:
        print("You have entered a negative number.")
    elif x > 9:
        print("You have entered a number greater than 9.")
else:
    print("Thank you. You entered %s." %x)
```

```
Thank you. You entered 1.
```

- Alternative implementation:

In [45]:
```
while False:  #Change to True to run.
    x = int(raw_input("Enter a number between 0 and 9: "))
    if x < 0:
        print("You have entered a negative number.")
        continue  #Skip remainder of loop body and go back to `while`.
    if x > 9:
        print("You have entered a number greater than 9.")
        continue
    print("Thank you. You entered %s." %x)
    break  #Exit innermost enclosing loop.
```

`For` Loops

- A `for` loop iterates over the elements of a sequence (e.g., a list):

In [46]:
```
for letter in "Python":
    print(letter)
```

```
P
y
t
h
o
n
```

- `letter` is called the loop variable. Every time the loop body is executed, it will in turn assume the value of each element of the sequence.

- `For` loops are typically used to execute a block of code a pre-specified number of times; `range` and `xrange` are often used in that case:

In [47]:
```
squares = []
for i in xrange(10):
    squares.append(i**2)
print(squares)
```

```
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
```

- Question: What does the following compute?

In [48]:
```
n = 7
f = 1
for i in xrange(n):
    f *= i+1
```

Modules

- Python's functionality is organized in *modules*.
- Some of these are part of Python's *standard library* (e.g., `math`). Others are part of *packages*, many of which come preinstalled with Anaconda (e.g., `numpy`).
- Modules need to be imported in order to make them available:

In [49]:
```python
import math
math.factorial(7)
```

Out[49]:
```
5040
```

- You can use *tab completion* to discover which functions are defined by `math`: after importing, enter `math.` and press the `Tab` key. Alternatively, use dir(math):

In [50]:
```python
print(', '.join(filter(lambda m: not m.startswith("_"), dir(math))))  #just so the output fits on the slide
```

```
acos, acosh, asin, asinh, atan, atan2, atanh, ceil, copysign, cos, cosh, degrees, e, erf, erfc, exp, expm1, f
abs, factorial, floor, fmod, frexp, fsum, gamma, hypot, isinf, isnan, ldexp, lgamma, log, log10, log1p, modf,
pi, pow, radians, sin, sinh, sqrt, tan, tanh, trunc
```

- Note that importing the module does not bring the functions into the *global namespace*: they need to be called as `module.function()`.
- It is possible to bring a function into the global namespace; for this, use

In [51]:
```
from math import factorial
factorial(7)
```
Out[51]:
```
5040
```

- It is even possible to import all functions from a module into the global namespace using `from math import *`, but this is frowned upon; it pollutes the namespace, which may lead to name collisions.
- *Packages* can contain several modules. They are imported the same way:

In [52]:
```
import numpy
numpy.random.rand()
```
Out[52]:
```
0.08736320389265984
```

- Optionally, you can specify a shorthand name for the imported package/module:

In [53]:
```
import numpy as np
np.sqrt(2.0)  #Note that this is not the same function as math.sqrt
```

Out[53]:
```
1.4142135623730951
```

- Conventions have evolved for the shorthands of some packages (e.g., `np` for `numpy`). Following them improves code readability.
- For the same reason, it is good practice to put your `import` statements at the beginning of your document (which I didn't do here).

Functions

Defining Functions

- User-defined functions are declared using the `def` keyword:

In [54]:
```
def mypower(x, y):  #zero or more arguments, here two
    """Compute x^y."""
    return x**y
mypower(2, 3)  #positional arguments
```

Out[54]:
```
8
```

- The *docstring* is shown by the help function:

In [55]:
```
help(mypower)
```

```
Help on function mypower in module __main__:

mypower(x, y)
    Compute x^y.
```

Several Outputs

- Functions can have more than one output argument:

In [56]:
```
def plusminus(a, b):
    return a+b, a-b
c, d = plusminus(1, 2); c, d
```

Out[56]:
```
(3, -1)
```

Keyword Arguments

- Instead of *positional arguments*, we can also pass *keyword arguments*:

In [57]:
```
mypower(y=2, x=3)
```
Out[57]:
```
9
```

- Functions can specify *default arguments*:

In [58]:
```
def mypower(x, y=2):  #default arguments have to appear at the end
    """Compute x^y."""
    return x**y
mypower(3)
```
Out[58]:
```
9
```

In [59]:
```
mypower(3, 3)
```
Out[59]:
```
27
```

Variable Scope

- Variables defined in functions are local (not visible in the calling scope):

In [60]:
```
def f():
    z = 1
f()
```

In [61]:
```
try:
    print(z)  #x is local to function f!
except NameError as e:
    print(e)
```

```
name 'z' is not defined
```

Calling Convention

- Python uses a *calling convention* known as *call by object reference*.
- This means that any modifications a function makes to its (mutable) arguments are visible to the caller (i.e., outside the function):

In [62]:
```
x = [1]  #Recall that lists are mutable.
def f(y):
    y[0] = 2  #Note: no return statement. Equivalent to `return None`.
f(x); print(x)  #Note that x has been modified in the calling scope.
```

```
[2]
```

Nested Functions

- Functions can be defined inside other functions. They will only be visible to the enclosing function.
- Nested functions can see variables defined in the enclosing function.

In [63]:
```
def mypower(x, y):
    def helper():  #No need to pass in x and y:
        return x**y  #The nested function can see them!
    a = helper()
    return a
mypower(2, 3)
```

Out[63]: 8

Advanced Material on Functions

Splatting and Slurping

- Splatting: passing the elements of a sequence into a function as positional arguments, one by one.

In [64]:
```
def mypower(x, y):
    return x**y
args = [2, 3]  #a list or a tuple
mypower(*args)  #Splat (unpack) args into mypower as positional arguments.
```
Out[64]: 8

- We can splat keyword arguments too, but we need to use a `dict` (key-value store):

In [65]:
```
kwargs={'y': 3, 'x': 2}  #a dict
mypower(**kwargs)  #splat keyword arguments
```
Out[65]: 8

- Slurping allows us to create *vararg* functions: functions that can be called with any number of positional and/or keyword arguments.

In [66]:
```
def myfunc(*myargs, **mykwargs):
    for (i, a) in enumerate(myargs): print("The %sth positional argument was %s." %(i, a))
    for a in mykwargs: print("Got keyword argument %s=%s." %(a,mykwargs[a]))
myfunc(0, 1, x=2, y=3)
```

```
The 0th positional argument was 0.
The 1th positional argument was 1.
Got keyword argument y=3.
Got keyword argument x=2.
```

- The asterisk means "collect all (remaining) positional arguments into a tuple".
- The double asterisk means "collect all (remaining) keyword arguments into a dict".

Closures

- Functions are *first class objects* in Python.
- This implies, inter alia, that functions can return other functions.
- Such functions are called *closures*, because they close around (capture) the local variables of the enclosing function.

In [67]:
```
def makemultiplier(factor):
    """Return a function that multiplies its argument by `factor`."""
    def multiplier(x):
        return x*factor
    return multiplier
timesfive = makemultiplier(5)
type(timesfive)
```

Out[67]:
```
function
```

In [68]:
```
timesfive(3)
```

Out[68]:
```
15
```

Anonymous Functions

- Anonymous functions (or *lambdas*) are functions without a name (duh...) and whose function body is a single expression.
- They are often useful for functions that are needed only once (e.g., to return from a function, or to pass to a function).
- E.g., the previous example could be written

In [69]:
```
def makemultiplier(factor):
    """Return a function that multiplies its argument by `factor`."""
    return lambda x: x*factor
timesfive=makemultiplier(5)
timesfive(3)
```

Out[69]:
```
15
```