Problems

Persons arrive at a taxi stand with room for W taxis according to a Poisson process with rate λ. A person boards a taxi upon arrival if one is available and otherwise waits in a line. Taxis arrive at the stand according to a Poisson process with rate μ. An arriving taxi that finds the stand full departs immediately; otherwise, it picks up a customer if at least one is waiting, or else joins the queue of waiting taxis. Use an $M/M/1$ queue formulation to obtain the steady-state distribution of the person's queue. What is the steady-state probability distribution of the taxi queue size when $W = 5$ and λ and μ are equal to 1 and 2 per minute, respectively?

Solution

Consider a Markov chain with state

$$n = \text{Number of people waiting} + \text{number of empty taxi positions}$$

Then the state goes from n to $n+1$ each time a person arrives and goes from n to $n-1$ (if $n \geq 1$) when a taxi arrives. Thus the system behaves like an $M/M/1$ queue with arrival rate 1 per minute and departure rate 2 per minute. Therefore the occupancy distribution is

$$p_n = (1-\rho)\rho^n$$

where $\rho = 1/2$. State n, for $0 \leq n \leq 4$ corresponds to 5, 4, 3, 2, 1 taxis waiting while $n > 4$ corresponds to no taxi waiting. Therefore

$$P(5 \text{ taxis waiting}) = 1/2$$

$$P(4 \text{ taxis waiting}) = 1/4$$

$$P(3 \text{ taxis waiting}) = 1/8$$

$$P(2 \text{ taxis waiting}) = 1/16$$

$$P(1 \text{ taxi waiting}) = 1/32$$

and $P(\text{no taxi waiting})$ is obtained by subtracting the sum of the probabilities above from unity. This gives $P(\text{no taxi waiting}) = 1/32$.