Problems

3.1 A communication line capable of transmitting at a rate of 50,000 bps can accommodate 10 sessions, each generating Poisson traffic at a rate of 2.5 pkts/sec, using time-division multiplexing (TDM). In each session, packet lengths are distributed such that 10% of the packets are 100 bits long and the rest are 1,500 bits long.

(a) Determine the average number of packets in queue, average delay per packet and the average number of packets in the system.
(b) Repeat part (a) for the case when short packets are given non-preemptive priority over the long packets.

Solution (《Data Networks》书上题目3.36):

(a)

For each session, the arrival rates, average transmission times and utilization factors for the short packets (class 1), and the long packets (class 2) are

λ_1＝0.25 packets/second, $1/\mu_1$=0.02 seconds, ρ_1=0.005

λ_2＝2.25 packets/second, $1/\mu_2$=0.3 seconds, ρ_2=0.675

The corresponding second moments of transmission time are

$E[X_1^2] = 0.0004$, $E[X_2^2] = 0.09$

The total arrival rate for each session is $\lambda = 2.5$ packets/second. The overall 1st and 2nd moments of the transmission time, and overall utilization factors are given by

$1/\mu = 1/\mu_1 \times 10\% + 1/\mu_2 \times 90\% = 0.272$

$E[X^2] = 0.1 \times 0.0004 + 0.9 \times 0.09 = 0.081$

$\rho = 2.5 \times 0.272 = 0.68$

We obtain the average time in queue W via the P - K formula

$W = \frac{\lambda E[X^2]}{2\times(1-\rho)} = 0.3164.$

The average time in the system is $T = \frac{1}{\mu} + W = 0.588$. The average number in queue and in the system are $N_Q = \lambda W * 10 = 7.91$, and $N = \lambda T * 10 = 14.7$.

(b)

$$W_1 = \frac{\lambda_1 E[X_1^2] + \lambda_2 E[X_2^2]}{2 \times (1-\rho_1)} = 0.1018$$

$$W_2 = \frac{\lambda_1 E[X_1^2] + \lambda_2 E[X_2^2]}{2 \times (1-\rho_1) \times (1-\rho_1-\rho_2)} = 0.318$$

$T_1 = 1/\mu_1 + W_1 = 0.1218$

$T_2 = 1/\mu_2 + W_2 = 0.618$

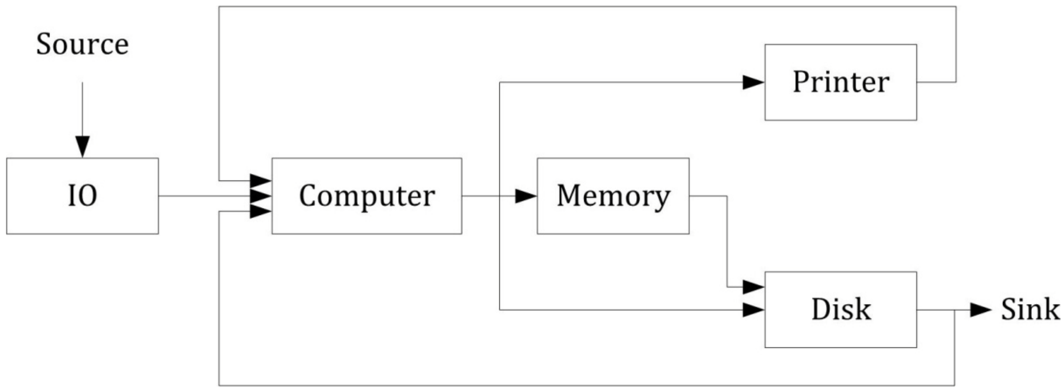
$N_{Q1} = \lambda_1 W_1 = 0.025$

$N_{Q2} = \lambda_2 W_2 = 0.716$

$N_1 = \lambda_1 T_1 = 0.0305$

$N_2 = \lambda_2 T_2 = 1.39$

3.2 Consider the queueing network given below, which consists of N = 5 single server first-come first-serve nodes.



States: Source-0, Computer-1, Printer-2, Memory-3, Disk-4, IO-5, Sink-6

The service times of the jobs at each node are exponentially distributed with respective means:

$$\frac{1}{\mu_1}=0.02 \text{ second}, \ \frac{1}{\mu_2}=0.1 \text{ second}, \ \frac{1}{\mu_3}=0.02 \text{ second},$$

$$\frac{1}{\mu_4}=0.04 \text{ second}, \ \frac{1}{\mu_5}=0.02 \text{ second}.$$

The jobs enter the network from the source with the interarrival time exponentially distributed with the parameter:

$$\lambda = 10 \text{ jobs/second}.$$

Furthermore, the routing probabilities are given as follows:

$$p_{12} = 0.2, \ p_{13} = 0.4, \ p_{14} = 0.4, \ p_{34} = p_{21} = p_{51} = 1,$$

$$p_{41} = 0.5, \ p_{46} = 0.5.$$

(a) Express the number in states 1, 2, 3, 4, and 5 as $(k_1k_2k_3k_4k_5)$. What is the steady-state probability of state $(k_1k_2k_3k_4k_5) = (1,2,1,4,1)$?

(b) What is the total number of jobs in the network?

Solution:

(a)

$\lambda_1 = p_{21}\lambda_2 + p_{41}\lambda_4 + p_{51}\lambda_5$

$\lambda_2 = p_{12}\lambda_1$

$\lambda_3 = p_{13}\lambda_1$

$\lambda_4 = p_{14}\lambda_1 + p_{34}\lambda_3$

$\lambda_5 = \lambda$

$\therefore$ $\lambda_1=25$, $\lambda_2=5$, $\lambda_3=10$, $\lambda_4=20$, $\lambda_5=10$

$P[(k_1 k_2 k_3 k_4 k_5) = (1,2,1,4,1)] = 0.0000655$

(b) 6.5