# Trajectory of Alternating Direction Method of Multipliers and Adaptive Acceleration

**Clarice Poon** (University of Bath)
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Constrained and composite optimisation problem:

$$\min_{x\in\mathbb{R}^n, y\in\mathbb{R}^m} R(x) + J(y) \quad \text{such that} \quad Ax + By = b \tag{$\mathcal{P}$}$$

under basic assumptions

- $R, J$ are proper, convex, lower semi-continuous functions.
- $A : \mathbb{R}^n \to \mathbb{R}^p$ and $B : \mathbb{R}^m \to \mathbb{R}^p$ are injective linear operators.
- $\mathrm{ri}\,(\mathrm{dom}(R) \cap \mathrm{dom}(J)) \neq \emptyset$ and the set of minimizers is non-empty.

# Alternating Direction Method of Multipliers (ADMM)

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**Question:** *How should one accelerate the convergence of ADMM?*

Given a fixed point sequence $z_{k+1} = \mathcal{F}(z_k)$, accelerate by

$$\begin{aligned} \bar{z}_{k+1} &= z_k + \alpha_k(z_k - z_{k-1}), \quad \alpha_k > 0 \\ z_{k+1} &= \mathcal{F}(\bar{z}_{k+1}) \end{aligned}$$

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Inertial is well-studied for algorithms such as gradient descent and Forward-Backward.
Improves the objective convergence rate from $\mathcal{O}(k^{-1})$ to $\mathcal{O}(k^{-2})$.

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The performance of inertial-ADMM in general is less clear.

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**Based on this trajectory analysis:**

2. We obtain insight into when inertial will work and fail.

3. We develop an acceleration scheme with local acceleration rates.

**Augmented Lagrangian**: For $\gamma > 0$ and Lagrangian multiplier $\psi \in \mathbb{R}^p$

$$\mathcal{L}(x, y, \psi) \stackrel{\text{def.}}{=} R(x) + J(y) + \langle \psi,\, Ax + By - b \rangle + \frac{\gamma}{2}\|Ax + By - b\|_2^2.$$

The ADMM iterations:

$$\begin{aligned}
x_k &= \operatorname{argmin}_{x\in\mathbb{R}^n} R(x) + \frac{\gamma}{2}\|Ax + By_{k-1} - b + \tfrac{1}{\gamma}\psi_{k-1}\|^2, \\
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Then, $z_k = \mathcal{F}(z_{k-1})$ for some fixed point operator $\mathcal{F}$ [†].

[†] Due to the equivalence between ADMM and Douglas-Rachford splitting [Gabay '83].

# The ADMM iterates

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\end{aligned}$$

*We will analyse the behaviour of* $\{z_k\}_k$.

$R$ is partly smooth at $x$ relative to a set $\mathcal{M} \ni x$ if $\partial R(x) \neq \emptyset$ and

**Smoothness:**

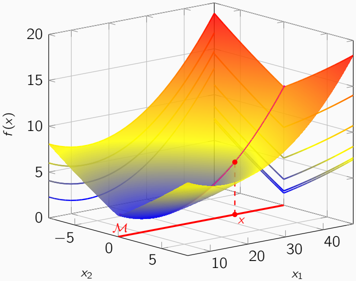
$\mathcal{M}$ is a $C^2$-manifold, $R|_{\mathcal{M}}$ is $C^2$ near $x$.

**Sharpness:**

Tangent space $\mathcal{T}_{\mathcal{M}}(x)$ is $\mathrm{par}\left(\partial R(x)\right)^{\perp}$.

**Continuity:**

$\partial R$ is continuous along $\mathcal{M}$ near $x$.




$\mathrm{par}(C)$: sub-space parallel to $C$, where $C$ is a non-empty convex set.

$\mathrm{PSF}_x(\mathcal{M}_x)$: function that is partly smooth at $x$ relative to $\mathcal{M}_x$.

**Examples:** $\ell_1, \ell_{1,2}, \ell_\infty$-norm, nuclear norm, total variation.

If $R \in \mathrm{PSF}_{x^\star}(\mathcal{M}^R_{x^\star})$ and $J \in \mathrm{PSF}_{y^\star}(\mathcal{M}^J_{y^\star})$, then under **non-degeneracy conditions** around $x^\star$ and $y^\star$:

**Manifold identification and local linearisation** [Liang, Fadili & Peyré '16]:
There exists $K \in \mathbb{N}$ and a matrix $M_{\text{ADMM}}$ such that for all $k \geqslant K$,

- $x_k \in \mathcal{M}^R_{x^\star}$ and $y_k \in \mathcal{M}^J_{y^\star}$
- $z_k - z^\star = M_{\text{ADMM}}(z_{k-1} - z^\star) + o(\|z_{k-1} - z^\star\|)$

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- $z_k - z^\star = M_{\text{ADMM}}(z_{k-1} - z^\star) + o(\|z_{k-1} - z^\star\|)$

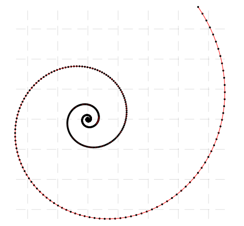
The behaviour of $z_k$ is eventually **regular**.

# Partial smoothness and sequence trajectory

Let $v_k \stackrel{\text{def.}}{=} z_k - z_{k-1}$ and $\theta_k = \angle(v_k, v_{k-1})$.

**Two non-smooth terms**

$R$ and $J$ are locally polyhedral around $x^\star$ and $y^\star$.



**Spiral trajectory:**

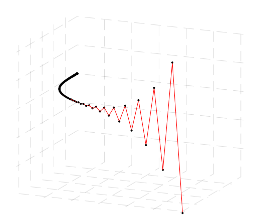
$$\cos(\theta_k) = \cos(\alpha) + \mathcal{O}(\eta^{2k})$$

with $\eta < 1, \alpha > 0$.

$M_{\text{ADMM}}$ has **complex** eigenvalues

**At least one smooth term**

$A$ is a full rank square matrix and $R$ is locally $\mathcal{C}^2$ around $x^\star$.



**Straight line trajectory:**

$\cos(\theta_k) \to 1$ when

$$\gamma > \|(A^\top A)^{-\frac{1}{2}} \nabla^2 R(x^\star)(A^\top A)^{-\frac{1}{2}}\|.$$

$M_{\text{ADMM}}$ has all **real** eigenvalues

# Partial smoothness and sequence trajectory

One inertial-ADMM iteration:

$$
\begin{aligned}
x_k &= \mathrm{argmin}_{x\in\mathbb{R}^n} R(x) + \tfrac{\gamma}{2}\|Ax - \tfrac{1}{\gamma}(\bar{z}_{k-1} - 2\psi_{k-1})\|^2,\\
z_k &= \psi_{k-1} + \gamma A x_k,\\
\bar{z}_k &= z_k + a_k(z_k - z_{k-1}),\\
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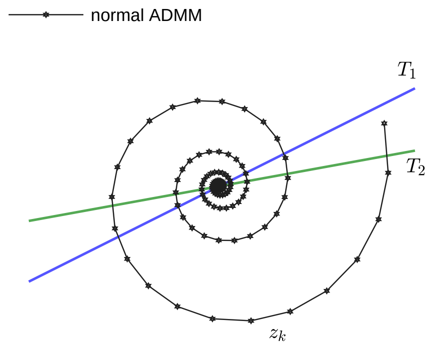
\end{aligned}
$$

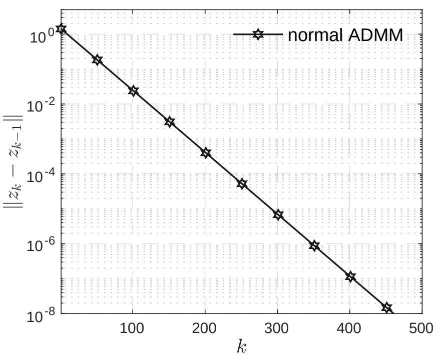
Intuition: inertial-ADMM accelerates if $z_k$ is moving along a straight path...

Find $z \in T_1 \cap T_2$. Solve using ADMM

$$\min_{x,y} \iota_{T_1}(x) + \iota_{T_2}(y) \quad \text{such that} \quad x - y = 0.$$

Consider $z_k \overset{\text{def.}}{=} \psi_{k-1} + \gamma x_k$. **Standard ADMM:**



# Failure of inertial-ADMM

Find $z \in T_1 \cap T_2$. Solve using ADMM

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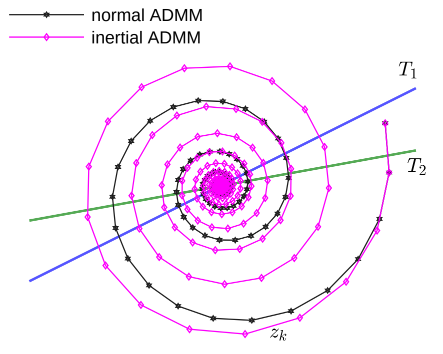
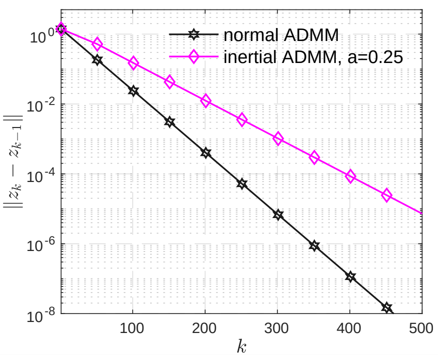
Consider $z_k \stackrel{\text{def.}}{=} \psi_{k-1} + \gamma x_k$. **Inertial-ADMM with $a = 0.25$:**

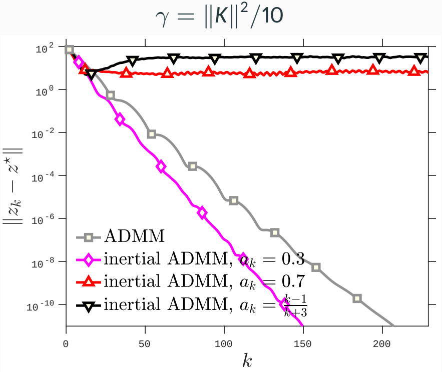


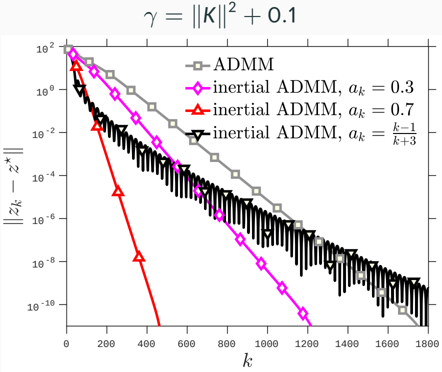



# Failure of inertial-ADMM

## LASSO example:

$$\min_{x,y\in\mathbb{R}^n} \mu\|x\|_1 + \tfrac{1}{2}\|Ky - f\|_2^2 \quad \text{such that} \quad x - y = 0.$$




## Eventual trajectory:

- Straight line when $\gamma > \|K\|^2$
- $M_{\text{ADMM}}$ may have complex leading eigenvalue if $\gamma \leqslant \|K\|^2$.

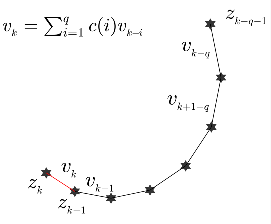
**Goal:** Given past points $\{z_{k-j}\}_{j=0}^{q}$, predict $z_{k+1}$.

**Idea:** Define $v_j \stackrel{\text{def.}}{=} z_j - z_{j-1}$,

**S.1)** Fit the past directions $v_{k-1}, \ldots, v_{k-q}$ to the latest direction $v_k$:

$$c_k \stackrel{\text{def.}}{=} \operatorname{argmin}_{c \in \mathbb{R}^q} \|V_{k-1}c - v_k\|^2,$$

where $V_{k-1} = [v_{k-1}, \ldots, v_{k-q}] \in \mathbb{R}^{n \times q}$.



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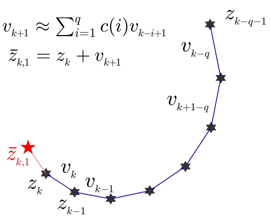

# Adaptive acceleration for ADMM (A$^3$DMM)

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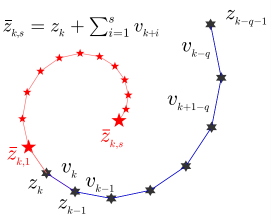
Repeat $s$ times to predict $z_{k+s}$.



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Repeat $s$ times to predict $z_{k+s}$.

**Define:** $H(c_k) \stackrel{\text{def.}}{=} \left[ \begin{array}{c|c} c_k & \begin{array}{c} \mathrm{Id}_{q-1} \\ \hline 0_{1,q-1} \end{array} \end{array} \right]$ and $\mathcal{E}_{s,q,k} = V_k \left( \sum_{j=1}^{s} H(c_k)^j \right)_{(:,1)}$.

The ***s*-step extrapolation** is $\bar{z}_{k,s} = z_k + \mathcal{E}_{s,q,k}$.

**Initial:** Let $s \geqslant 1$, $q \geqslant 1$, $\bar{q} = q + 1$. Let $\bar{z}_0 = z_0 \in \mathbb{R}^p$ and $V_0 = 0_{p \times q}$.

**Repeat:** For $k \geqslant 1$

$$
\begin{aligned}
y_k &= \mathrm{argmin}_{y \in \mathbb{R}^m} J(y) + \tfrac{\gamma}{2} \| By + \tfrac{1}{\gamma}(\bar{z}_{k-1} - \gamma b) \|^2 \\
\psi_k &= \bar{z}_{k-1} + \gamma(By_k - b) \\
x_k &= \mathrm{argmin}_{x \in \mathbb{R}^n} R(x) + \tfrac{\gamma}{2} \| Ax - \tfrac{1}{\gamma}(\bar{z}_{k-1} - 2\psi_k) \|^2 \\
z_k &= \psi_k + \gamma A x_k \\
v_k &= z_k - z_{k-1} \quad \text{and} \quad V_k = [v_k, V_k(:, 1:q-1)]
\end{aligned}
$$

If $\mathrm{mod}(k, \bar{q}) = 0$: Compute coefficients $c_k$ and let $C_k \stackrel{\text{def.}}{=} H(c_k)$

If $\rho(C_k) < 1$: $\quad \bar{z}_k = z_k + a_k \mathcal{E}_{s,q,k}$; $\quad$ else: $\quad \bar{z}_k = z_k$.

If $\mathrm{mod}(k, \bar{q}) \neq 0$: $\bar{z}_k = z_k$.

# Remarks

Global convergence is guaranteed for appropriate choice of $a_k$.

Local acceleration depends on $\varepsilon_k \stackrel{\text{def.}}{=} \min_c \|V_{k-1}c - v_k\|$.

- If $M_{\text{ADMM}}$ is diagonalisable, then $\varepsilon_k = \mathcal{O}(|\lambda_{\bar{q}}|^k)$ where $\lambda_{\bar{q}}$ is the $\bar{q}^{th}$ largest eigenvalue.
- Guaranteed local acceleration for $q = 2$ if $R$ and $J$ are polyhedral.

Related to vector extrapolation techniques from the 1960's.

[Aitken '27, Wynn '62, Andersen '65...]

## Remarks

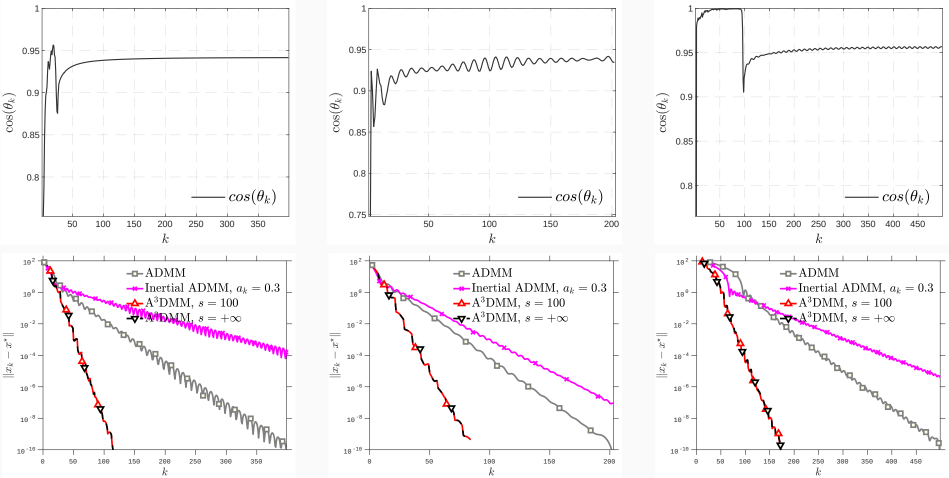
Implementation:

- Typically set $q \leqslant 10$.
- Extra memory cost of $p \times (q+1)$ (storing $V_k$).
- Extra computation cost of $q^2 p$ every $(q+2)$ iterations.
- One could also extrapolate $\{x_k, y_k\}$ simultaneously. But this would require extra storage of past directions.

# Experiment: 2 non-smooth terms

Basis pursuit type problem with $\Omega \stackrel{\text{def.}}{=} \{x \in \mathbb{R}^n \; ; \; Kx = f\}$:

$$\min_{x,y\in\mathbb{R}^n} R(x) + \iota_\Omega(y) \quad \text{such that} \quad x - y = 0.$$




$\ell_1$-norm $\qquad$ $\ell_{1,2}$-norm $\qquad$ Nuclear norm

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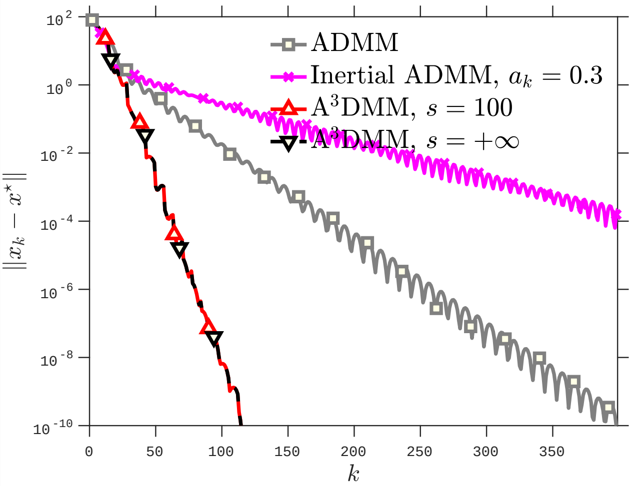


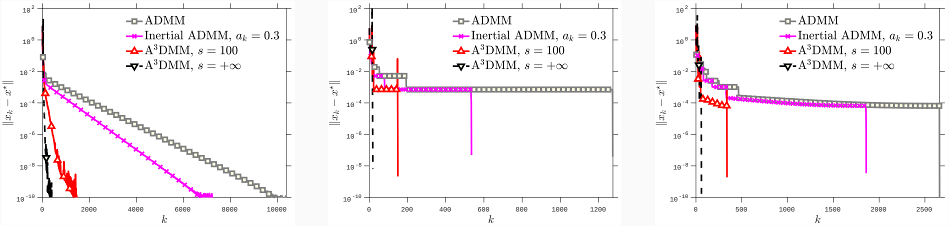

$\ell_1$-norm

Inertial ADMM is **slower** than ADMM as eventual trajectory is a spiral.

# Experiment: LASSO

Consider the LASSO problem

$$\min_{x,y\in\mathbb{R}^n} R(x) + \frac{1}{2}\|Ky - f\|^2 \quad \text{such that} \quad x - y = 0.$$

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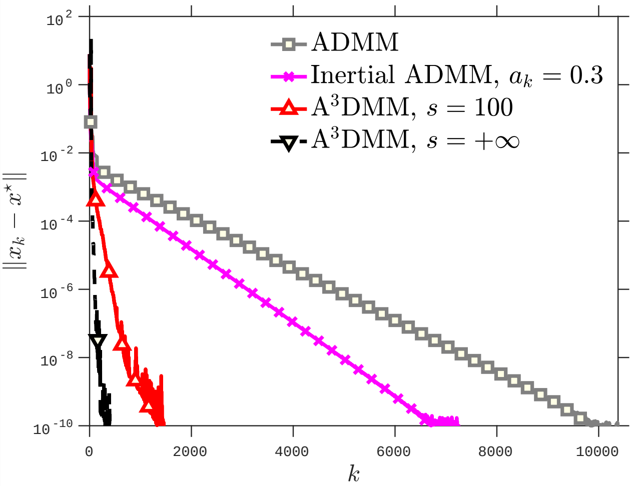


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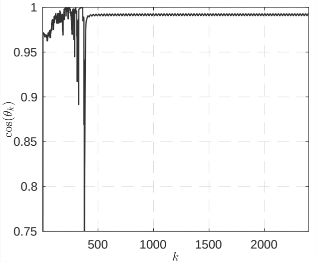
Inertial ADMM does accelerate, but A$^3$DMM is significantly faster.

## Experiment: Total variation based image inpainting

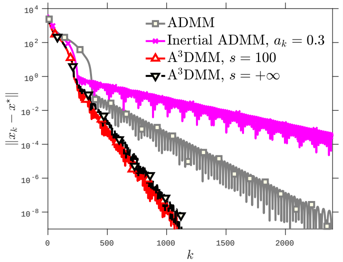
Let $\Omega \stackrel{\text{def.}}{=} \{x \in \mathbb{R}^{n\times n} \;;\; P_{\mathcal{D}}(x) = f\}$, $P_{\mathcal{D}}$ randomly sets 50% pixels to zero and consider

$$\min_{x\in\mathbb{R}^{n\times n}} \ \|y\|_1 + \iota_\Omega(x) \quad \text{such that} \quad \nabla x - y = 0.$$

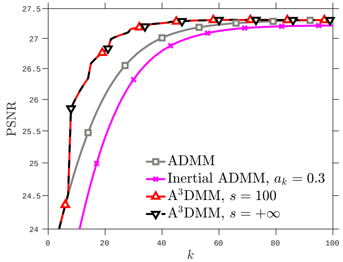
Angle $\{\theta_k\}$



$\|x_k - x^\star\|$




PSNR




- Both functions are polyhedral, trajectory is a spiral.
- Inertial ADMM is **slower** than ADMM.

# Experiment: Total variation based image inpainting

Original image

ADMM, PSNR = 26.6935

Inertial ADMM, PSNR = 26.3203

Corrupted image

A$^3$DMM $s = 100$, PSNR = 27.1668

A$^3$DMM $s = +\infty$, PSNR = 27.1667

## Summary of contributions

**Trajectory of ADMM** For sequence $\{z_k\}_{k\in\mathbb{N}}$

- When both $R$ and $J$ are locally polyhedral around the fixed point, $\{z_k\}_{k\in\mathbb{N}}$ eventually moves along a **spiral**.
- When at least one of $R$ or $J$ is smooth, the trajectory of $\{z_k\}_{k\in\mathbb{N}}$ depends on $\gamma$ and can be either a spiral or a **straight line**.

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**An adaptive acceleration for ADMM**

- The different trajectory behaviour of ADMM can lead to the **failure** of the inertial technique.
- We propose an acceleration strategy based on the idea of following the sequence trajectory.

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**Poster: East Exhibition Hall B+C #115!**