CCM218 Notes

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1 Probability

1.1 Foundations

In 1933, Andrey Kolmogorov published a book which is know as one of the first formal approaches in Probability Theory. In his book, Foundations of Probability Theory, Kolmogorov introduces probability with three axiomes.

1.2 Fundamental Definitions

- Let Ω be the set of all elementary events. We call Ω a sample space of the experiment.
- Let Σ be the subsets of Ω that form a σ-algebra, satisfying:
    1. Σ contains at least one subset of Ω
    2. Let A be a set of elements. If $A \in \Sigma$, so $\Omega / A \in \Sigma$
    3. If A_1,A_2,..., $A_n \in \Sigma$, so $A_1 \cup A_2 \cup ... \cup A_n \in \Sigma$.
- The elements of Σ are denoted random events
- If A,$B \in \Sigma$, so:
    1. $A \cup B \in \Sigma$
    2. $A \cap B \in \Sigma$
- $\Omega \in \Sigma$
- $\varnothing \in \Sigma$
- $\langle\ \Omega,\Sigma\ \rangle$ is defined as the mensurable space.

1.3 Probability

For $A \in \Sigma$, we now introduce the Probability of A as a non-negative real number.

The probability P Domain and Image are represented by: P:$\Sigma \mapsto [0,1]$.

$So,$ $\forall$A$\in$ Σ, $\exists$ $P(A) \geq 0$, satisfying:

- P(Ω) = 1.
- If A,B $\in \Sigma$ and $A \cap B = \varnothing$, so P(A) + P(B) = P$(A \cup B)$

We also define $\langle$ Ω, Σ, P $\rangle$ as the probability space.

Theorem 1. Let $A \in \Sigma$, we have that P(A) + P(A^c) = 1.

Proof. Since A^c is the logical negation of A, we have that $A^c = \Omega/A$, the sum P(A) + P(A^c) = P(A) + P(Ω/A) = P(Ω) = 1.

Theorem 2. P$(\varnothing)$ = 0, the probability of the empty set.

Proof. Let $A \in \Sigma$, we have that P$(A \cup \varnothing)$ = P(A) + P$(\varnothing)$ = P(A)
$\Rightarrow P(\varnothing)$ = P(A) - P(A) = 0.
(Note that $A \cup \varnothing = A$)

Theorem 3. If A_1,A_2,..., $A_n \in \Sigma$, and $A_i \cap A_j = \varnothing$, $\forall$ i, j between 1 and n, then:

$$\text{P}(\bigcup_{j=1}^{n} A_j) = \textstyle\sum_{j=1}^{n} \text{A}_j$$

Proof. Considering n = 2, we have simply reduce to the case in the definition of probability. Using induction, we can proof that this equality holds $\forall$ n, where n is a non-negative number.

Obs: Using induction, if we assume that the equation is true for N, we notice that $A_N \cap A_{N+1} = \varnothing$, and the proof is almost complete.