# FUNDAMENTALS

## BASIC PROGRAMMING MODEL

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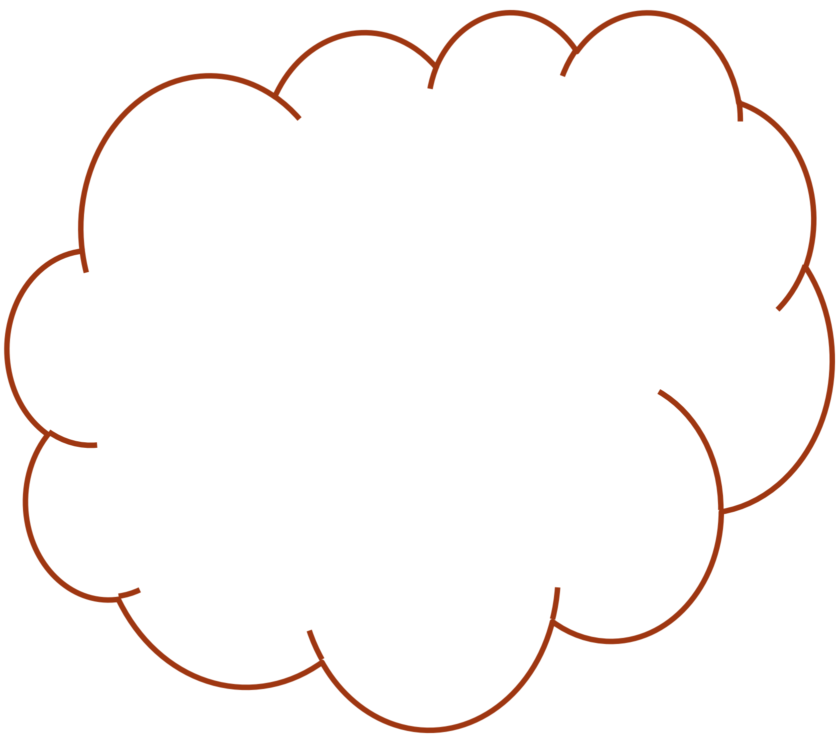
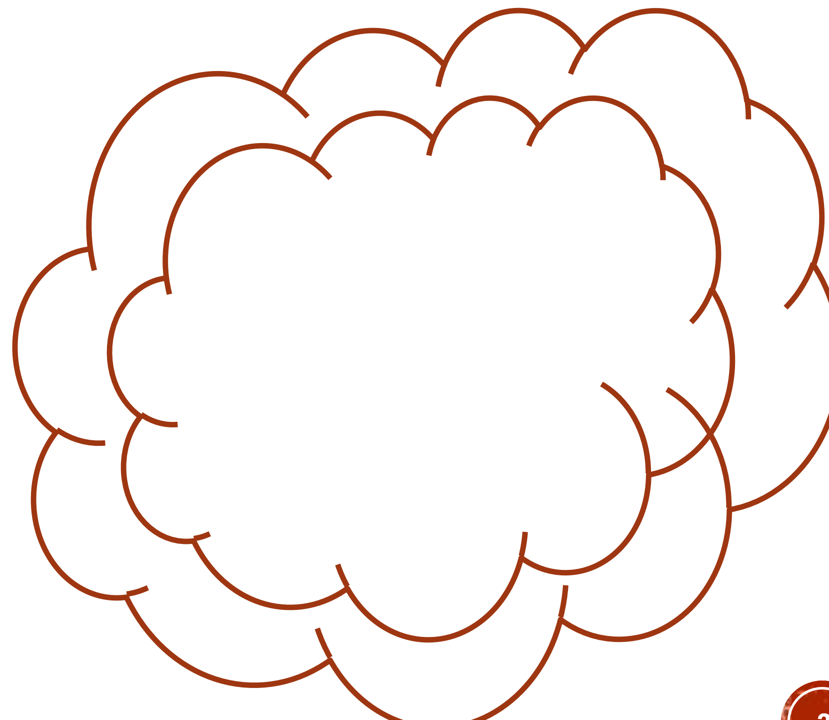
# RECURSION

# THE RECURSION CONCEPT

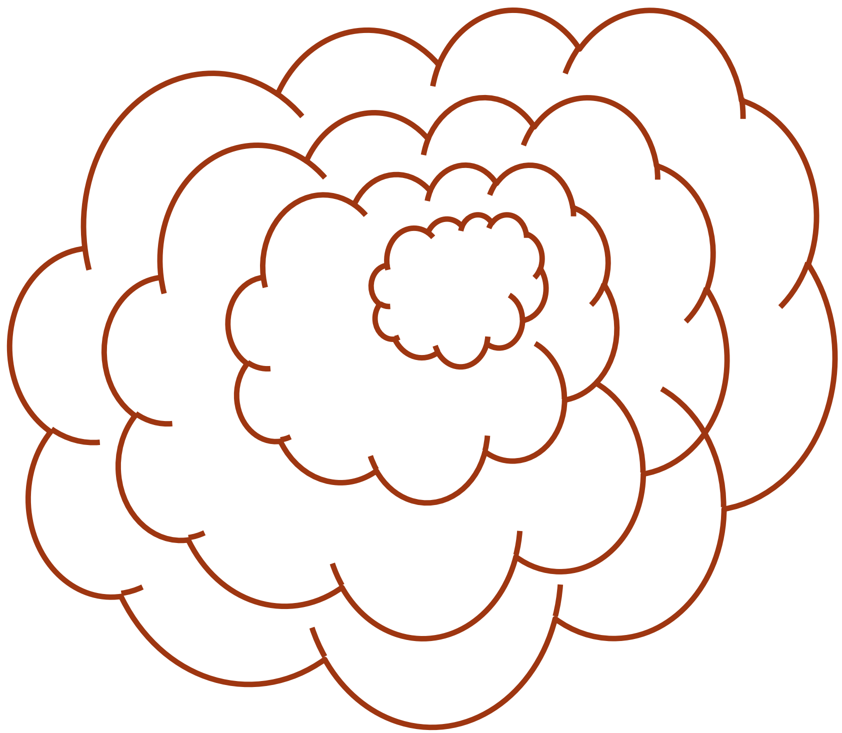
Suppose I want to draw a puffy cloud. Well, I can use PowerPoint's handy cloud tool. Only, it's not really puffy. So, I copy the first cloud, shrink it, and move it into the original cloud. That's okay, but that inner cloud doesn't look puffy. Maybe I can repeat the copy…





# THE RECURSION CONCEPT

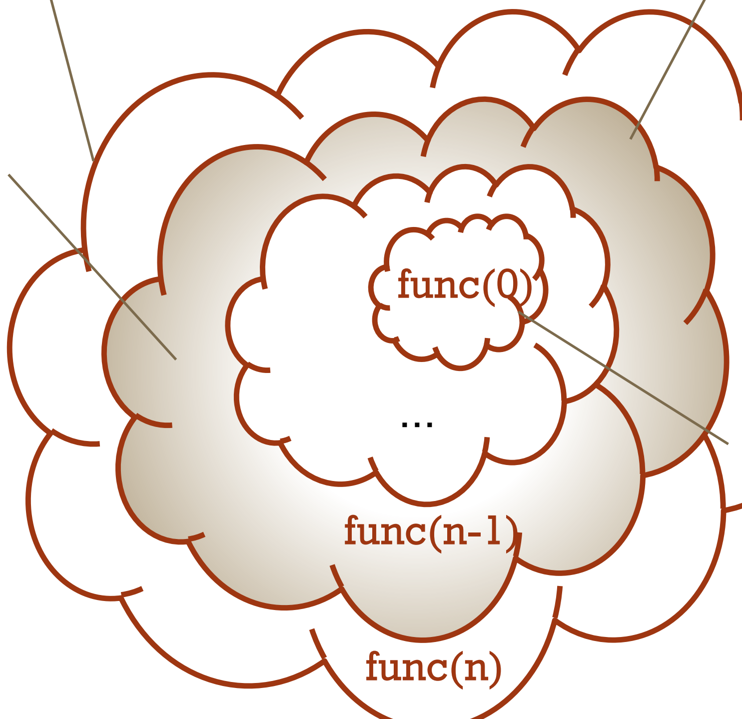
… and so I do. And again. And again. Until I've ended up shrinking the smaller cloud so much, I really don't need another one. So, I built a cloud out of a cloud! This is recursion.



# RECURSION OVERVIEW

A full size problem (n) contains a sub-problem (n-1).

Each time the problem is analyzed, some part of the solution is computed and is combined with the solution for the sub-problem, to form the solution for the whole.



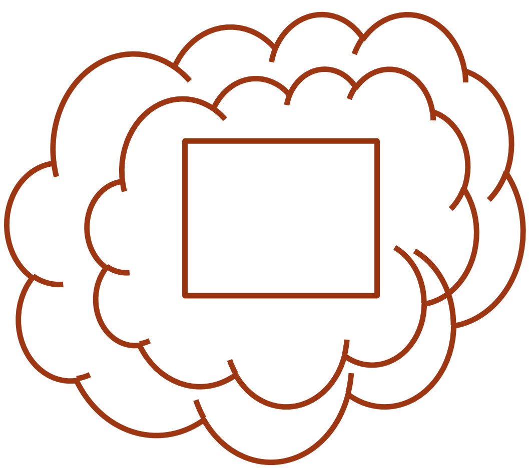

The sub-problem is the same type of problem as the main problem but is easier to solve. It is “smaller” according to a size metric.

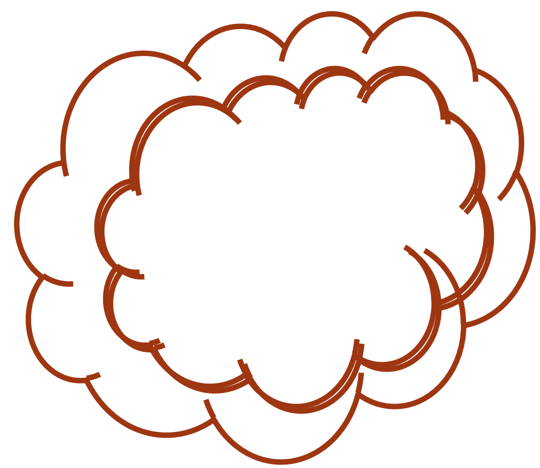
The sub-problems continue until they are small enough to be trivial.

# THE KEYS TO RECURSION

For all valid inputs, a recursive method must make the problem smaller during each call, and make ***progress*** towards some base case (***termination***).

The type of problem should not change, nor the sub-problem be the same as the original problem.





# RECURSIVE METHODS

- Recursive structure is an important concept in computer science. By definition, a method is recursive if it calls itself within the method.
- There is a special type of recursive structure, which calls itself only once and in the last statement. We refer to this as the tail-recursion. (Tail recursion is very similar to a do-while loop.)
- Why do we need recursion?

# THE FANTASTIC FOUR APPROACH

## SCAFFOLDING THE DESIGN OF RECURSIVE METHODS

1. Formulate the size-n problem.
2. Find the stopping condition and the corresponding return value.
3. Formulate the size-m problem and find m. In many cases, m = n - 1;
4. Construct the solution of size-n problem from size-m problem.

Let's think about implementing **factorial** in terms of these steps.

(Chen and Tsai. *Introduction to Programming Languages*, Kendall Hunt Publishing.)

# 1. FORMULATE THE SIZE-N PROBLEM

## THE FANTASTIC FOUR ABSTRACT OF WRITING RECURSIVE FUNCTIONS

- Like a loop, recursion is necessary only if you want to solve a problem that needs to repeat the same operations.
- We assume the number of iterations is n. In most cases, n is obvious. For example, if we want to compute factorial n!, the size n is already given.
- Formulating the size-n problem, in some cases, is merely choosing a function name and using n as the parameter of the function. Thus, the size-n problem for factorial problem is

  `int factorial(int n)`

- The return value of the size-n is what the function is supposed to compute, or the value we are looking for. Obviously, in this step, we do not need to design the solution for size-n problem.

# 2. FIND THE STOPPING CONDITION AND RETURN VALUE

## THE FANTASTIC FOUR ABSTRACT OF WRITING RECURSIVE FUNCTIONS

- Like a while-loop, every recursive function starts with checking the stopping condition.
- If the stopping condition is true, we return the corresponding value and exit the function.
- Otherwise, we enter the body of the recursive function.
- In some cases, identifying the stopping condition and corresponding value is trivial. For example, the stopping condition of factorial(n) is $n = 0$ and the corresponding value is 1

# 3. FORMULATE THE SIZE-M PROBLEM

## THE FANTASTIC FOUR APPROACH OF WRITING RECURSIVE FUNCTIONS

- The size-m problem is simply the size-n problem with n replaced by m, where $m < n$ is determined by how much we can reduce the size in one step.
- If we can only reduce the problem size by one, m is n-1. For example: factorial(n-1).
- Do not try to define a solution, or the return value in this step! All we need to do here is assume the size-m problem will return a value and use the value, e.g., n*factorial(n-1), to get us to the stopping condition.

- The problem may be resized in many different ways:
  - For the mergesort, $m = \frac{1}{2} n$.
  - For the maze homework, $m = \frac{1}{4} n$.

# 4. CONSTRUCT THE SOLUTION OF SIZE-N PROBLEM

## THE FANTASTIC FOUR APPROACH OF WRITING RECURSIVE FUNCTIONS

- In this step, we will use the assumed solution or the return value for size-m or size-(n-1) problem to construct the solution of the size-n problem.
- This step is application-specific. In the case of factorial, the solution of the size-n problem is

  n*factorial(n-1);

- Sometimes, we need to use the return values of multiple size-m problems, where $0 \leq m < n$ to construct the solution of size-n problem.

# PUTTING FACTORIAL TOGETHER

```
int factorial(int n) {
        if(n == 0)
                return 1;
        else
                return n*factorial(n-1);
}



// Mathematical Definition:
// 1! = 1                        (or 0! = 1)
// N! = N * (N-1)!
```

# MESSING WITH FACTORIAL

# COMPUTING THE NTH FIBONNACI NUMBER

1. Formulate the size-n problem.

   long fib(int n)

2. Find the stopping condition and the corresponding return value.

   if(n == 0) return 0;

   if(n == 1) return 1;

3. Formulate the size-m problem and find m. In many cases, m = n - 1;

   fib(n-1)

   fib(n-2)

4. Construct the solution of size-n problem from size-m problem.

   fib(n-1) + fib(n-2)

# THE NTH FIBONNACI NUMBER

```
public static long fib(int n) {

    if(n == 0)
        return 0;

    if(n == 1)
        return 1;

    return fib(n-1)+ fib(n-2);
}
```

At this point, recursion probably looks easy. And not that useful, but…

**As a general rule, recursion is very useful whenever you are faced with a problem you do not know how to solve.**

Due to the nature of our world, many problems have a naturally recursive structure, which a recursion algorithm is able to leverage.

# PRINTING A LIST

1. Formulate the size-n problem.
   `void displayList(LinearNode node)`

2. Find the stopping condition and the corresponding return value.
   `if(node == null) and return;`

3. Formulate the size-m problem and find m. In many cases, m = n - 1;
   `displayList(node.getNext());`

4. Construct the solution of size-n problem from size-m problem.
   `System.out.println(node); displayList(node.getNext());`

# PRINTING A LIST

```
//iterative
public static void
displayList(LinearNode node) {
   LinearNode iter = node;
   while(iter != null) {
      System.out.println(iter);
      iter = iter.getNext();
   }
}
```

```
//recursive
public static void displayList(LinearNode
node) {
   if(node != null) {
      System.out.println(node);
      displayList(node.getNext());
   }
}
```

# RECURSIVE DATA STRUCTURES

Here's a thought: we've made programs recursive, so might we make data recursive too?

```
public class Thing {
    public Thing thing2;
    private int alongfortheride;

    public Thing(Thing t2, int i) {
        thing2 = t2;
        alongfortheride = i;
    }
}
```

This is the fundamental idea behind the "linked list" data structure.

- This class is self-referential – another form of recursion.
- This makes a list a recursive data structure.
- Made possible by references.