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IWYM
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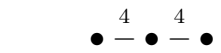
Coxeter Groups

Question 1. Describe the Coxeter complex of $I_2(m)$ for any $m \geq 2$ and for $m = \infty$.

Question 2. Describe the Coxeter complex of $\tilde{B}_2$, whose Coxeter graph is



Strongly suggested tip: play around with https://www.jgibson.id.au/lievis/affine_weyl/.

Question 3. Let $W = S_4$, the symmetric group on $\{1, 2, 3, 4\}$. Then W has the structure of a Coxeter group with $S = \{s_1, s_2, s_3\}$ where s_i denotes the transposition $(i, i + 1)$. Show that $w = s_1 s_2 s_1 s_3 s_2 s_1$ is a reduced expression. Show that for any $s \in S$ we hae $\ell(ws) = \ell(w) - 1$ and find a simple reflection that can be removed from the reduced expression of $\ell(w)$ to give a reduced expression for ws.

Question 4. Let $\{s, t, u\}$ be the simple reflections inside the Coxeter group of type A_3. Show that the subgroup generated by (su) and t is a Coxeter group of type $B_2 = I_2(4)$, with simple reflections $\{su, t\}$, by checking the braid relation.