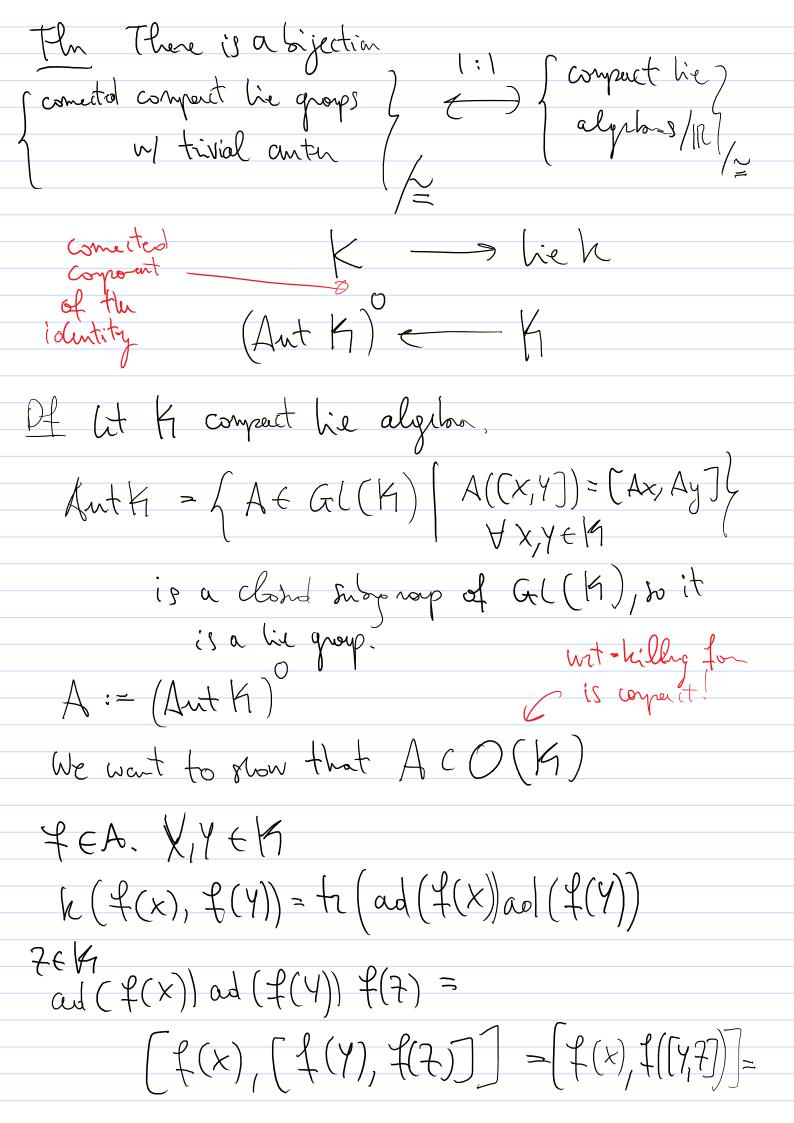
COMPACT UZ ALGEBRAS & LICLING FORMS. Det ht g be fin. din he algebra over a field /h. The hilling form on g is a bilinear for oblind vie  $h(x, y) = t_1(ad(x)ad(y)) \forall x, y \in g$  $ad(x): q \rightarrow g$ Y -> (x,Y] Since tr(AB) = tr(BD) => h is synetric. EXAMPLE 8/2(IR) e,h, f  $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} / \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$  $ad(e) = \begin{pmatrix} 0 - 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{in Seveis } \{e, h, f\}$  $h(e,e) = tr(ad(e)^2) = 0$ so it is not obtainte

Del A lie algebra /R is compact if the hilling form
h is myretive definite.

Del The anter of a lie algebra of z(q)={X+g \ (x,y)=0} g=lie G thm  $\frac{2(q) = \text{lie}(\text{Im AdG})}{\text{Im AdG}} \leq \frac{2(G)}{\text{Im AdG}}$ Ad:  $G \rightarrow G(g) = \frac{1}{1}$ if  $G \in G$  is corrected.  $\frac{1}{1} = \frac{1}{1} =$ trop (et & se a compact lie group w/ finite 7(G), then g = lie & is a corport lie algebra Exacts on g via Ad Grapaet = ) J Grinvariant scalar product on g AdGCO(q)X ∈ Lie G =)  $\forall t \in \mathbb{R}$   $e^{xt} \in G =$ )

=)  $\forall t \in \mathbb{R}$   $e^{xt} \in G =$ )  $\Rightarrow \forall t \in \mathbb{R}$   $e^{xt} \in G =$ )  $\Rightarrow \forall t \in \mathbb{R}$   $e^{xt} \in G =$ )  $\Rightarrow \forall t \in \mathbb{R}$   $e^{xt} \in G =$ )  $\Rightarrow \forall t \in \mathbb{R}$   $e^{xt} \in G =$ )  $\Rightarrow \forall t \in \mathbb{R}$   $e^{xt} \in G =$ )  $\Rightarrow \forall t \in G =$   $\Rightarrow \forall t \in G =$ )  $\Rightarrow \forall t \in G =$   $\Rightarrow \forall t \in G =$ 

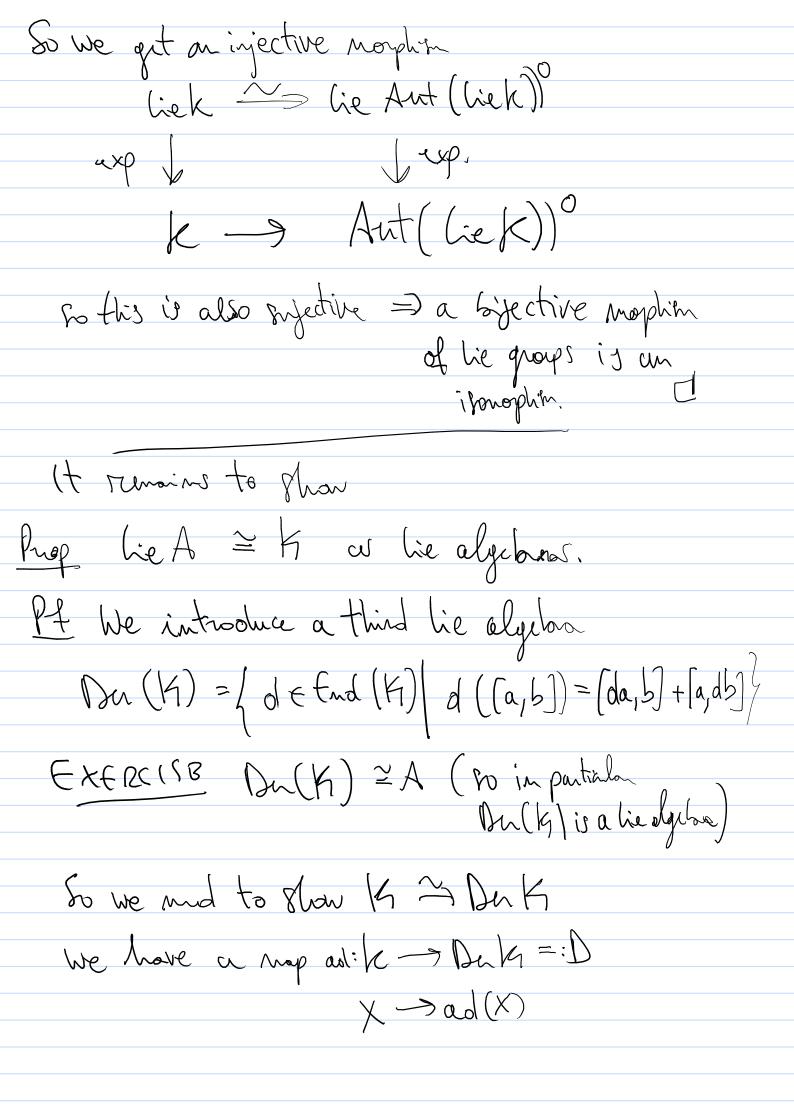


$$= f([X[Y,T]]) = f(adxad(Y)7)$$

=) 
$$ad(f(x))$$
  $ad f(y) = f(ad x ad y)f^{-1}$ 

$$\Rightarrow$$
  $h(\xi(x), \xi(y)) = h(x, y)$ 

What is Z(A)?
A corrected =) Z(A) = km (Ad)
ASJUMPTION (we show it later)
(ie A 2 K as lie alyton and as representations of A
$(re(Ad) = ler(A \rightarrow Gl(K)) = \{ld\} = ) + (A) = le$
We mid to show that the two mops are inverse to each other.
h corpact lie alglo. lie (Aut K)) = K
K corpect lie grap., corrected wy trivial cuter
Need to show:
K= Aut (lieK)
Ad: K -> GL(lieK))
h cometed => len (Ad) = 7(A) = e
=) Ad injective.
gen, X,Y = liek, (g xg-), g yg-1]=g(x,Y)g-).



ad(X) is a derivation YY,ZEK [X,[Y,7]] = -[Y,(7,X]) - [7,[X,Y]][Y, ad(X)] - [7, ad(X)Y] == (ad(x) Y, 7) + (Y, ad(x) 7). K computet = ad is injective ( = 0 = (x,x)) (= 0 = x = 0, D:=Den K, let ko be the hilling form on D. We want to slow ad (K) = D. ad (K) is compact he algebra and it is an ideal of D.

 $y, x \in K, d \in Den L$  (d, ad x)(Y) > d((X,Y)) - (X,dY) = (dX,Y) + (X,dY) - (X,dY) = ad(dX)(Y)

ad kBut if we take  $S \in I, \forall x \in K$  0 = [S, adx] = ad Sx = Sx = 0 = S = 0 = T = 0

 $\rightarrow$   $D \approx ad(K) \approx K$ 

In K ~ Duk = lie A is isonophin of A-reprentations.

Yach, xek, yek

ad(ax)(y) = [ax,y]

 $a(ad x)a^{-1}(y) = a(x,a^{-1}y) = [ax,y]$