8 Hedging & MC-methods

~~Simplify Assumptions~~

Δ-hedging Principles (Seller's P.O.V)

1) Sell call options collect premium P per option.

Premium collected ~~we~~ reflects difference $S_0 - K$ and

Current Spot

time left to expiration t.

But ~~we~~ seller could lose $S_t - K$ at time t if $S_t > K$.

we can then purchase at time 0 a number of shares of the stock.

Seller loses profit at time t if the stock goes up in value.

But if stock was owned at time 0, we still sell stock for profit. (Just not market rate profit).
This helps negate ~~a~~ future loses.

2) Δ-hedging, at time of selling calls, purchase per contract

Δ_{C_0} share of stock.

A time of expiration per option sold:

total profit of

$$\text{Premium} - C_t + \overbrace{(S_t - S_0)}\Delta_{C_0}$$

If $S_t > S_0$

the value positive

So it counters $-C_t \leq 0$

Remark: To consider expected profits at time 0 w.r.t. risk-free interest:

Total Profit: (at expiration

$$\text{Premium} - C_t + (S_t - e^{rt} S_0)\Delta_{C_0}.$$

Multiple Hedges:

$0 < t_1 < t = \text{Expiration}.$

- Seller of call option collects premium P

- Δ-hedges @ time 0 by buying Δ_{C_0} shares of stock per contract.

- <u>Time t_1:</u> Seller adjusts stock portfolio to own $\Delta_{C_{t_1}}$ shares of stock and come back to Δ-neutral position.

Stock Profit/loss at time t_1 is

$$\left(S_{t_1} - e^{t_1 r} S_0\right) \cdot \Delta_{C_0}$$

Discounted back to time 0:

Stock Profit is

$$e^{-t_1 r}\left(S_{t_1} - e^{t_1 r} S_0\right) \Delta_{C_0}$$

- <u>At expiration:</u>

Stock Profit/Loss from time t_1 to time t:

(discounted to time 0)

$$e^{-rt}\left(S_t - e^{r(t-t_1)} S_{t_1}\right)\Delta_{C_{t_1}}$$

Total Profit discounted to time 0 is

Sold call option

Stock Profit @ t_1

$$\text{Premium} - e^{-rt} Max(S_t - K, 0)$$

$$+ e^{-t_1 r}\left(S_{t_1} - e^{t_1 r} S_0\right)\Delta_{C_0}$$

$$+ e^{-tr}\left(S_t - e^{r(t-t_1)} S_{t_1}\right)\Delta_{C_{t_1}}$$

Stock profit @ t.

<u>More generally</u>:

Seller can Δ-neutralize portfolio at time interval

$$t_0 = 0 < t_1 < t_2 < \cdots < t_n = t$$

Seller's P&L discounted to time 0 is

- Premium $- e^{-rt} C_t$

$$+ \sum_{i=1}^{n} e^{-rt_i}\left(S_{t_i} - e^{r(t_i - t_{i-1})} S_{t_{i-1}}\right) \Delta_{C_{t_{i-1}}}$$