Estimating Philippine Dealing System Treasury (PDST) Reference Rate Yield Curves using a State-Space Representation of the Nelson-Siegel Model

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Interest Rate and Yield Curve Modelling

- Gaining significance and popularity among finance and economics professionals with the development of more complex financial instruments:
  - Forecasting and fitting yield curves
  - Pricing financial derivatives
  - Hedging risks

Objectives of the Study

- To understand how state-space representation works for yield curve modelling;
- To apply the dynamic latent factor approach into the Philippine Dealing System Treasury Reference Rates and see if results yielded adequately represents the yield curve;
- To serve as a foundation to further elevate modelling approaches in the Philippines (policy making, asset and liability management, etc).

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Interest Rate Models

- Stochastic interest rate models
  - Vasicek Model and its variants
  - Cox-Ingersoll-Ross (CIR) Model
- Statistical interest rate models
  - Inclusion of time-varying parameters; dynamic regression models
  - Addresses the constraints of one-factor models
  - ARCH, GARCH models
  - Vector Autoregression

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Development of the Dynamic Nelson-Siegel Model

- Nelson-Siegel (1987) (cf. [6])
  - Parametric approach to capture yield curve shapes
  - In order to attain usual yield curve shapes (humped, monotonic, or s-shaped), forward rates are assumed to be solutions of second-order differential equation
  - If $f_t(\tau)$ is the instantaneous forward rate at time t with maturity τ, then it is assumed that

$$f_t(\tau) = \beta_1 + \beta_2 e^{-\lambda\tau} + \beta_3 \lambda e^{-\lambda\tau} \tag{1}$$

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Development of the Dynamic Nelson-Siegel Model

- It is known that the yield curve is the mean value of the forward rate curve from $t = 0$ to $t = \tau$ (cf. [1], [8]), hence

$$\begin{aligned} y_t(\tau) &= \frac{1}{\tau}\int_0^\tau f_t(u)du \\ &= \beta_1 + \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \end{aligned} \tag{2}$$

- Diebold and Li (2005) (cf. [2]) provide a dynamic interpretation of the model parameters, writing

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) \tag{3}$$

where $\{\beta_{1,t}\}$, $\{\beta_{2,t}\}$, $\{\beta_{3,t}\}$, and λ_t are time series.

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Development of the Dynamic Nelson-Siegel Model

- $\{\beta_{1,t}\}$, $\{\beta_{2,t}\}$, and $\{\beta_{3,t}\}$ can be interpreted as the long-term factor loading (Level factor), the short-term factor loading (Slope factor), and the medium term factor loading (Curvature factor), respectively.
- $\{\lambda_t\}$ can be interpreted as the decay rate of the factors.
- We make the representation that $\beta_{1,t} = L_t$, $\beta_{2,t} = S_t$, and $\beta_{3,t} = C_t$, so the model becomes

$$y_t(\tau) = L_t + S_t \left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \right) + C_t \left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right) \tag{4}$$

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Dynamic Latent Factor Model

- Since the level, slope, and curvature factors are time evolving, we assume that they follow an order-1 vector autoregressive model VAR(1) with the following specification

$$\begin{bmatrix} L_t \\ S_t \\ C_t \end{bmatrix} = \begin{bmatrix} \mu_L \\ \mu_S \\ \mu_C \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} L_{t-1} \\ S_{t-1} \\ C_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_{L,t} \\ \eta_{S,t} \\ \eta_{C,t} \end{bmatrix}. \tag{5}$$

- Let $\mathbf{x}_t$ denote the state vector, $\boldsymbol{\mu}$ be the mean vector of the states, $\mathbf{B}$ be the coefficient matrix, and $\mathbf{w}_t$ be the error vector (assumed to be multivariate Gaussian with mean zero and covariance matrix $\mathbf{Q}$)

$$\mathbf{x}_t = \boldsymbol{\mu} + \mathbf{B}\mathbf{x}_{t-1} + \mathbf{w}_t, \qquad \mathbf{w}_t \sim MVN(0, \mathbf{Q}). \tag{6}$$

This will be known as the **state equation**.

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Dynamic Latent Factor Model

The **observation equation** can then be written in matrix form as

$$\begin{bmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_k) \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{1-e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} & \dfrac{1-e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} - e^{-\lambda_t \tau_1} \\ 1 & \dfrac{1-e^{-\lambda_t \tau_2}}{\lambda_t \tau_2} & \dfrac{1-e^{-\lambda_t \tau_2}}{\lambda_t \tau_2} - e^{-\lambda_t \tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \dfrac{1-e^{-\lambda_t \tau_{12}}}{\lambda_t \tau_k} & \dfrac{1-e^{-\lambda_t \tau_k}}{\lambda_t \tau_k} - e^{-\lambda_t \tau_k} \end{bmatrix} \begin{bmatrix} L_t \\ S_t \\ C_t \end{bmatrix} + \begin{bmatrix} e_{\tau_1,t} \\ e_{\tau_2,t} \\ \vdots \\ e_{\tau_k,t} \end{bmatrix}. \tag{7}$$

Let $\mathbf{y}_t$ denote the observation vector (vector of yield rates), $\mathbf{Z}$ be the coefficient matrix containing the factor loadings, and the observation error vector by $\mathbf{v}_t$. The above equation can be written as

$$\mathbf{y}_t = \mathbf{Z}\mathbf{x}_t + \mathbf{v}_t \qquad \mathbf{v}_t \sim MVN(0, \mathbf{R}). \tag{8}$$

State-Space Representation

Given the prior discussion, the problem of estimating the yield curve is thus equivalent to solving for the parameters of the **state-space model**

$$\begin{aligned} \mathbf{y}_t &= \mathbf{Z}\mathbf{x}_t + \mathbf{v_t} \qquad \text{(observation equation)} \\ \mathbf{x}_t &= \boldsymbol{\mu} + \mathbf{B}\mathbf{x}_{t-1} + \mathbf{w}_t \qquad \text{(state equation)}, \end{aligned} \tag{9}$$

where $\mathbf{v}_t \sim MVN(0, \mathbf{R})$ and $\mathbf{w}_t \sim MVN(0, \mathbf{Q})$.

Methodology

- **Data:** Philippine Dealing System Treasury Rates
  - Assumed to be a suitable proxy for zero rates
  - Coverage: March 19, 2007 to April 30, 2014 (daily rates, 1746 data points)
  - Maturities: 12 maturities (from 1-month to 25-year rates); τ is expressed in months.
- **Statistical Program:** `R` using the `MARSS` package (cf. [4])

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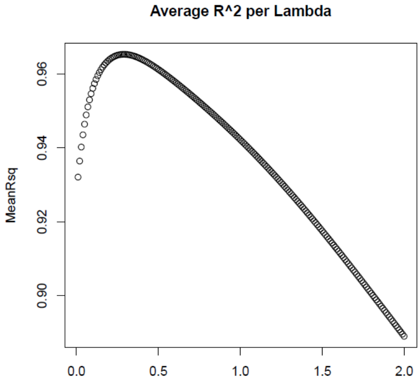
Grid Search for λ

- We assume that there is one value of λ that governs the decay rate, hence $\{\lambda_t\}$ will be treated as a constant λ w.r.t. time t.
- A **grid-search** will be performed for the decay factor:
  - For each λ in the interval $[0.01, 2.00]$ in increments of 0.01, form the matrix $\mathbf{Z}$.
  - For each λ above, regress the yield rates $y_t(\tau)$, against $\mathbf{Z}$ for each t from March 19, 2007 to April 30, 2014.
  - Compute for the R^2 in each of the regressions in the previous step and compute for the average R^2.
  - The value of λ that will be used in the state-space model will be the λ for which the average R^2 is the highest.

Grid Search for λ

- The optimal value of λ is 0.29. Below is a graph summarizing the average R^2 per value of λ in $[0.01, 2.00]$



- State-space system is now reduced into linear regression problem and VAR parameter estimation.

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State-Space Parameter Estimation

- The matrix $\mathbf{B}$ (state equation coefficient matrix) and the error vector correlation matrices $\mathbf{R}$ and $\mathbf{Q}$ are estimated using the expectation-maximization (EM) algorithm embedded in the `MARSS` package.
- Estimates for $\{L_t\}$, $\{S_t\}$, and $\mathbf{C_t}$ are filtered and smoothed during the estimation procedure (using Kalman filtering and smoothing).

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Coefficient Matrix of State Equation

Table: Coefficient matrix and mean vector of the state equation. Standard errors are in parentheses.

	L_{t-1}	S_{t-1}	C_{t-1}	μ
L_t	0.9974	-0.0018	0.0087	0.0290
	(0.0006)	(0.0012)	(0.0030)	(0.0039)
S_t	0.0007	0.9769	0.0253	-0.0442
	(0.0003)	(0.0037)	(0.0056)	(0.0091)
C_t	0.0015	0.0008	0.9898	-0.0687
	(0.0005)	(0.0004)	(0.0012)	(0.0136)

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Correlation of State Errors

Table: Correlation matrix of the state error variables. Standard errors are in parentheses.

	L_t	S_t	C_t
L_t	0.0129	-	-
	(0.0430)	-	-
S_t	-0.0059	0.0242	-
	(0.0020)	(0.0035)	-
C_t	-0.0270	0.0066	0.2143
	(0.0034)	(0.0029)	(0.0106)

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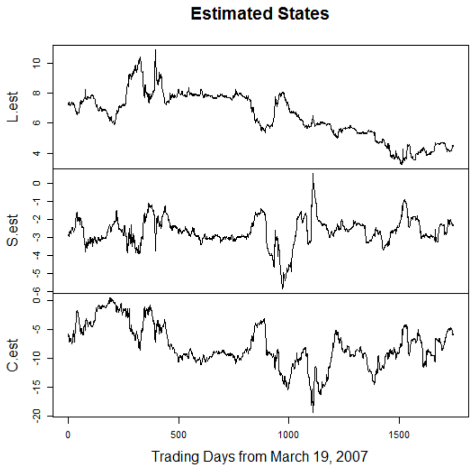
Estimated Latent Factors

Figure: Time series plots of estimated (filtered and smoothed) level, slope, and curvature latent factors.



Empirical Proxies for Latent Factors

In [3], empirical proxies for the state variables are defined as follows

$$\begin{aligned} L_{\text{proxy},t} &= \frac{1}{3}[y_t(3) + y_t(24) + y_t(120)] \\ S_{\text{proxy},t} &= y_t(120) - y_t(3) \\ C_{\text{proxy},t} &= 2y_t(24) - [y_t(3) + y_t(120)] \end{aligned} \tag{10}$$

Below is the correlation matrix for the estimated state variables and their respective proxies.

	L_t (Proxy)	S_t (Proxy)	C_t (Proxy)
L_t	0.9375	-	-
	(<2.2 e-16)	-	-
S_t	-	0.7476	-
	-	(<2.2 e-16)	-
C_t	-	-	0.4378
	-	-	(<2.2 e-16)

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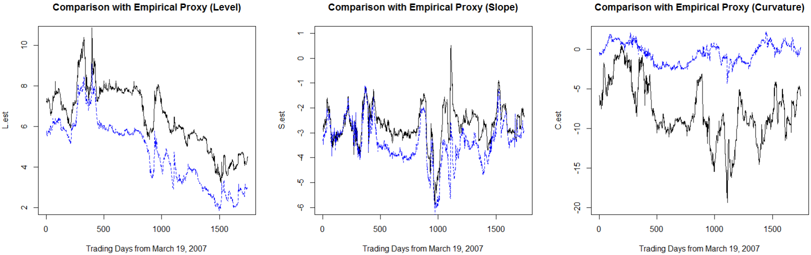
Empirical Proxies for Latent Factors

Figure: Time series plots comparing estimated states (solid black lines) and their empirical proxies (dashed blue lines).



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Variance of Observation Errors

Maturity	Variance	Std. Error	Upper CI Limit	Lower CI Limit
1 month	0.0024	0.0038	0.0022	0.0026
3 months	0.2873	0.0091	0.2692	0.3074
6 months	0.4117	0.0109	0.3857	0.4404
1 year	0.0016	0.0071	0.0015	0.0017
2 years	0.3695	0.0103	0.3462	0.3953
3 years	0.4428	0.0112	0.4148	0.4737
4 years	0.2529	0.0084	0.2369	0.2706
5 years	0.1664	0.0069	0.1559	0.1780
7 years	0.0015	0.0032	0.0014	0.0016
10 years	0.1546	0.0066	0.1448	0.1654
20 years	2.6339	0.0274	2.4675	2.8178
25 years	3.7359	0.0326	3.4999	3.9967

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In-Sample Validation

	19-Mar-07			30-Apr-14		
Maturity	**Predicted**	**Actual**	**Squared Difference**	**Predicted**	**Actual**	**Squared Difference**
1 month	4.0425	4.0482	0.0000	1.7681	1.7721	0.0000
3 months	3.8711	4.1232	0.0635	1.4716	1.4646	0.0000
6 months	4.1394	4.5839	0.1976	1.6358	2.1708	0.2862
1 year	4.9848	4.9482	0.0013	2.3992	2.3979	0.0000
2 years	5.9710	5.5286	0.1957	3.3446	3.0521	0.0856
3 years	6.3753	5.6771	0.4875	3.7367	3.3604	0.1416
4 years	6.5805	5.9286	0.4249	3.9358	3.5667	0.1363
5 years	6.7037	6.1321	0.3267	4.0554	4.0188	0.0013
7 years	6.8445	6.8911	0.0022	4.1921	4.1267	0.0043
10 years	6.9501	7.4339	0.2341	4.2946	4.4063	0.0125
20 years	7.0733	8.7518	2.8173	4.4142	5.3521	0.8796
25 years	7.0979	9.0964	3.9938	4.4381	5.6896	1.5662
		MSE	0.7287		**MSE**	0.2595

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Out-Sample Validation

	2-May-14			15-May-14		
Maturity	**Predicted**	**Actual**	**Squared Difference**	**Predicted**	**Actual**	**Squared Difference**
1 month	1.7714	1.7646	0.0000	1.7784	1.6667	0.0125
3 months	1.4868	1.4667	0.0004	1.4665	1.3271	0.0194
6 months	1.6496	2.1583	0.2588	1.6222	2.0413	0.1756
1 year	2.3947	2.3958	0.0000	2.3856	2.2792	0.0113
2 years	3.3157	3.0437	0.0740	3.3370	2.9646	0.1387
3 years	3.6974	3.3458	0.1236	3.7320	3.3500	0.1459
4 years	3.8913	3.5750	0.1001	3.9326	3.5771	0.1264
5 years	4.0078	4.0125	0.0000	4.0531	3.9708	0.0068
7 years	4.1409	4.1604	0.0004	4.1908	4.0729	0.0139
10 years	4.2407	4.4042	0.0267	4.2941	4.3104	0.0003
20 years	4.3571	5.3458	0.9775	4.4146	5.2979	0.7802
25 years	4.3804	5.6458	1.6012	4.4387	5.4854	1.0956
		MSE	0.2636		**MSE**	0.2105

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Conclusion

- The Nelson-Siegel model for yield curves and its dynamic latent factor variants have adequately characterized the dynamics of the PDST-F reference rate.
- Limitations of the model include its inability to predict rates with longer maturities, having estimated large measurement error variances for such rates.
- Improvements may be made in terms of parameter estimation, the use of non-linear least squares to characterize the dynamics of the decay latent factor, and statistical goodness-of-fit tests in model validation.
- The Nelson-Siegel model may be used as a more accurate alternative to stochastic interest rate models, since as a statistical model, the Nelson-Siegel model is already calibrated through the use of data.

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Conclusion

- Other multivariate models can be considered for the state factors (e.g. multivariate GARCH, copulas, etc) to better characterize the correlation structure of the state variables.
- Further research in the same field may analyze the effect of latent factors, as well as several macroeconomic indicators such as inflation rate and economic capacity, as in [2] and [5].

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