Module 3A6 - 3 - Internal Flows and Heat Exchangers

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Internal Flows

In the last chapter we discussed how the momentum and thermal boundary layers grow from a leading edge of a surface. Clearly, if the flow is confined at some point, the boundary layers of the various surfaces exchanging momentum and heat with the fluid will merge, forming the core or *bulk* flow. The simplest possible situation to consider is steady flow through a pipe or channel, as shown in Fig. 1.

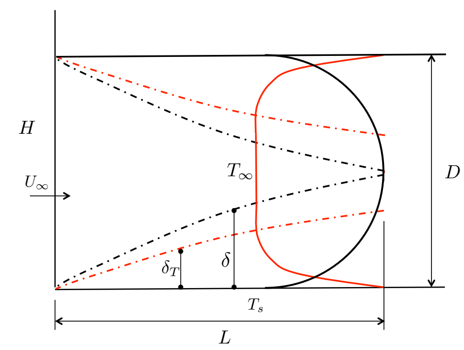


Figure 1: Entrance flow and heat transfer into a pipe.

The mean or *bulk* velocity is defined as:

$$U_b = \frac{1}{\rho A}\int_0^A \rho u \, dA \tag{3.1}$$

and the bulk temperature is defined in the same manner as the mean or bulk velocity:

$$T_b = \frac{1}{\dot{m} c_p}\int_0^A \rho u c_p T \, dA \tag{3.2}$$

The flow is classified as laminar or turbulent if Re based on the bulk mean velocity, U_b, and the diameter, D, of the tube is less or more than the transition value of approximately 2000, respectively. For tubes with circular cross section, D is the geometric diameter of the tube. For non-circular cross section, D is the hydraulic diameter, $D_h = \frac{4A_c}{P}$, where A_c is the cross sectional area of the tube and P is the wetted perimeter. The flow is called fully developed with respect to the momentum or boundary layer if beyond a certain entrance length L_δ the velocity or temperature profiles remain unchanged.

Laminar Internal flows

Let's consider the simplest situation of a laminar, fully-developed flow in a pipe. You have seen the solution to this problem in your IB classes, called a Poiseuille or Hagen-Poiseuille flow. Here is the derivation in a few steps considering the cylindrical element of radius r and length dx as in Fig. 2. Here p is the pressure, u the axial velocity, and we apply momentum conservation,

$$\frac{d\tau}{dr} = r\frac{dp}{dx}$$

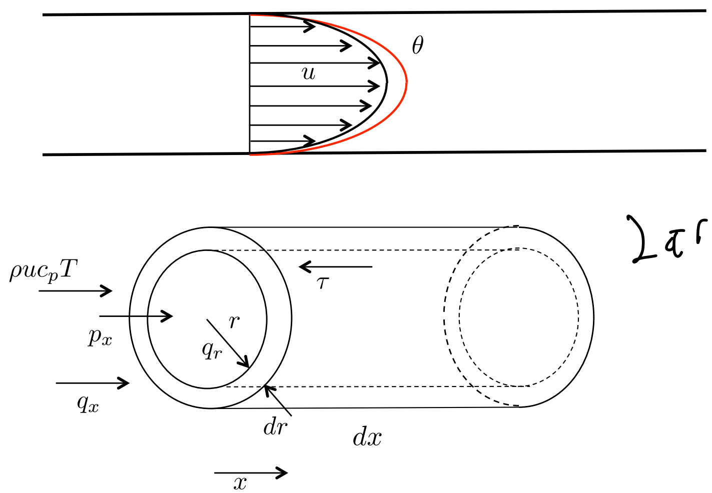


Figure 2: Control volume for internal laminar fully developed flow.

unsteady term

convective term

$$u \cdot \nabla T = \alpha \nabla^2 T$$

assuming constant density and using the assumptions of fully developed steady flow and symmetry so that $u = u(r)$. The radial velocity is zero, which means that the pressure is uniform across r. The balance between pressure and shear forces on a cylindrical shell of radius r, thickness dr and length dx yields:

laminar

$$2\pi r\, dr \frac{dp}{dx} dx = \frac{d}{dr}(2\pi r \tau)\, drdx = \frac{d}{dr}(2\pi r \mu \frac{du}{dr})\, drdx \quad \rightarrow \quad u = \frac{1}{4\mu}\frac{dp}{dx} r^2 + C \tag{3.3}$$

Using boundary conditions $u(R) = 0$, we have

$\frac{dp}{dx}$ is constant

$$u = \frac{1}{4\mu}\frac{dp}{dx}(r^2 - R^2) \tag{3.4}$$

$$\frac{u}{U_0} = (1 - \frac{r^2}{R^2}) \tag{3.5}$$

which is the familiar parabolic profile for laminar flow for U_o set to the central velocity, which can be shown to be $U_o = 2U_b$, where $U_b = \frac{1}{R^2}\int_0^R u 2\pi r\, dr$ is the bulk velocity.

Now let us consider the heat transfer across the same cylindrical shell, assuming fully developed thermal conditions (so that velocity only depends on r). The energy balance for an annular element of area $(2\pi r)\,dx$ is:

$$
\begin{aligned}
\frac{\partial}{\partial r}(2\pi r q_r)\,drdx + \frac{\partial}{\partial x}(2\pi r q_x)\,drdx + \frac{\partial}{\partial x}(\rho u c_p T)(2\pi r)\,drdx &= 0 \\
-\lambda \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) - \lambda r \frac{\partial^2 T}{\partial x^2} + \rho u c_p r \frac{\partial T}{\partial x} &= 0 \\
-\alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) - \frac{\partial^2 T}{\partial x^2}\right) + u\frac{\partial T}{\partial x} &= 0
\end{aligned}
\tag{3.6}
$$

which is simply the energy equation in cylindrical coordinates.

Let us now consider a particular case in which $\frac{\partial T}{\partial x}$ is constant, or more generally, the case in which $\frac{\partial \theta}{\partial x}$ is constant, with $\theta = \frac{T(r,x) - T_s(x)}{T_b(x) - T_s(x)}$. This is called fully thermally developed flow. Now we can neglect the second term in the equation, so that:

$$\frac{1}{ur}\left(\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right) = \frac{1}{\alpha}\frac{\partial T}{\partial x} \tag{3.7}$$

$$h\, 2\pi r\,(T_w - T_b) = \rho c_p T_b$$

where we have been mindful to keep the term $u(r)$ on the left hand side.

The equation is now integrable to yield:

$$
\begin{aligned}
T &= \frac{1}{\alpha}\frac{\partial T}{\partial x}u_0\left(\frac{r^2}{4} - \frac{r^4}{16R^2}\right) + C_1 \ln r + C_2 \\
T - T_c &= \frac{1}{\alpha}\frac{u_0 R^2}{4}\frac{\partial T}{\partial x}\left[\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4\right]
\end{aligned}
\tag{3.8}
$$

where T_c is the temperature at the centre of the tube. The final constants T_c and $\frac{\partial T}{\partial x}$, are obtained by realising that the bulk temperature and heat flux at the wall are set by the boundary conditions:

$$T_b = \frac{1}{\dot{m}c_p}\int_0^R \rho u c_p T 2\pi r\,dr = T_c + \frac{7}{96}\frac{u_0 R^2}{\alpha}\frac{\partial T}{\partial x} \tag{3.9}$$

$$T_s = T_c + \frac{3}{16}\frac{u_0 R^2}{\alpha}\frac{\partial T}{\partial x} \tag{3.10}$$

$$q_s = hP(T_s - T_b) = \lambda P\left(\frac{\partial T}{\partial R}\right)_{r=R} = \lambda 2\pi R \frac{u_0 R}{4\alpha}\frac{\partial T}{\partial x} \tag{3.11}$$

We conclude that, if $\dfrac{\partial T}{\partial x}$ is constant:

- The heat transfer to or from the wall is constant, and the temperature difference between wall and centre remains constant – this means that the surface and centre temperature increase at the same rate.
- The heat flux at the wall is constant, and the heat transfer coefficient (and the Nusselt number) remains constant.

Substituting the value of the bulk temperature difference obtained from the gradient at the wall, we have:

$$h = \frac{24}{11}\frac{\lambda}{R} \tag{3.12}$$

and the Nusselt number based on the diameter is:

$$Nu = \frac{hD}{\lambda} = 4.364 \tag{3.13}$$

Similar expressions can be derived for constant temperature flows, although not in such a straightforward manner.

Question 3-1 Show that for constant heat flux the bulk temperature increases linearly.

Solution The bulk temperature is given as:

$$T_b = \frac{1}{\dot{m}c_p}\int_0^R \rho u c_p T 2\pi r\, dr = T_c + \frac{7}{96}\frac{u_0 R^2}{\alpha}\frac{\partial T}{\partial x}$$

the second term is constant, since we assumed that the temperature gradient is constant. Therefore, the change in T_b follows that of T_c. From Eq. (3.9),

$$\frac{\partial (T - T_c)}{\partial x} = \frac{\partial T}{\partial x} - \frac{dT_c}{dx} = \frac{1}{\alpha}\frac{u_0 R^2}{4}\frac{\partial^2 T}{\partial x^2}\left[\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4\right] = 0$$

Therefore, $\dfrac{dT_c}{dx} = \dfrac{\partial T}{\partial x} = const$, and the bulk and centreline temperatures increase linearly with x.

Turbulent Internal Flows

Internal flows become turbulent when $Re_D = U_b D/\nu > 2000$, where U_b is the bulk mean velocity, D is the diameter of the tube and ν is the kinematic viscosity of the fluid. In turbulent flow, the momentum and heat transfer include both a laminar and a turbulent contribution.

We invoke the same picture as in the case of the flat plate turbulent boundary layers. However, here in place of the outer flow we have the bulk flow, and the flow is fully developed. A full development of the theory to obtain the friction coefficient, which is largely based on experiments, is available in fluid mechanics textbooks and the 3A1 notes. Here we present only the essential elements. Recalling the analysis in section , the mean momentum balance for a differential tube $dx \times r$, assuming that the *turbulent eddy diffusivity* ν_t in the tube is constant[1] yields:

Constant momentum flux in turbulent flows near walls

turbulent velocity

laminar viscosity

$$\frac{r}{2}\frac{d\overline{p}}{dx} = \rho\left[(\nu + \nu_t)\frac{d\overline{u}}{dr}\right] = \tau, \qquad \Longrightarrow \quad \frac{\tau}{\tau_w} = \frac{r}{R}, \tag{3.14}$$

where τ_w and R are the shear stress at the wall and the tube radius respectively, and we have applied the boundary condition $\frac{d\overline{u}}{dr} = 0;$ at $r = 0$.

In order to obtain a value for the C_f or friction factor $f = 4C_f$, we need to know the mean velocity profile near the wall. The development of the theory follows a similar path of approximations as in the case of boundary layers, but with a few additional approximations to account for the boundary condition of the integral reaching the bulk velocity. Ultimately, the results are summarised as a friction factor f as a function of Reynolds number and surface roughness in a series of charts or equations, called the *Moody diagram* (Fig. 3. The equations for f are typically implicit. For smooth pipes, the most commonly used version is:

in turbulent area of boundary layer follow law of wall (logarithm based dependence)

$$\frac{1}{\sqrt{f}} = A\log_{10}(Re\sqrt{f} - B) \tag{3.15}$$

where accepted values for A and B are 1.93 and 0.54, respectively.

A simpler expression is valid up to $Re_D < 10^5$:

$$f = 4C_f = 0.316\, Re_D^{-1/4} \tag{3.16}$$

By using any of the above relation for $f = 4C_f$, the Nusselt number can be obtained via the Reynolds–Colburn analogy:

$$St\, Pr^{2/3} = \frac{Nu}{Re}Pr^{-1/3} = \frac{C_f}{2} = \frac{f}{8}$$

[1] Recall that in contrast, in the case of the developing boundary layer the *turbulent flux* τ was taken as constant.

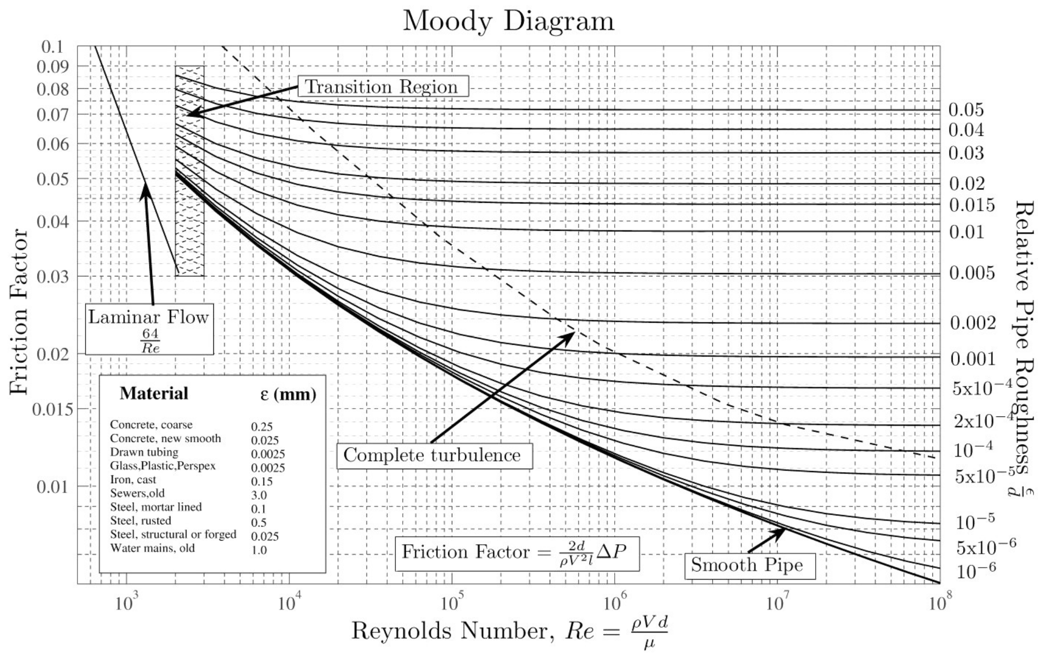


Figure 3: Friction factor $f = 4C_f$ for duct flow [Credits Wikimedia Commons].

Using the equation in Eq. (3.16) we have:

$$\frac{Nu}{Re} Pr^{-1/3} = \frac{f}{8} = 0.0395 \, Re_D^{-1/4} \tag{3.17}$$

$$Nu = 0.0395 \, Re_D^{3/4} Pr^{1/3} \tag{3.18}$$

There are many such correlations available in the heat transfer literature. A commonly used correlation for pipe flows is the *Dittus–Boelter correlation*:

$$\boxed{Nu \approx 0.023 Re_D^{4/5} Pr^n; \quad n = 0.4 \ \text{for heating}, \quad n = 0.3 \ \text{for cooling the fluid} \qquad (3.19)}$$

The above expressions can be used for flows in ducts of other cross section by simply replacing D by the hydraulic diameter, D_h. In general, different cross sections will require different constants, which are typically obtained experimentally. The present equations are sufficient for an engineering estimation before more computationally intensive calculations are attempted.

From Fig. 3 and the various correlations, we can see that the Nusselt number increases with a positive power of Re. Therefore, in general, the heat transfer is increased by higher velocities.

Consider that for turbulent flows, the friction factor decreases as $Re^{-1/4}$, so that the pressure drop should increase as $U^{7/4}$. The heat transfer rate should increase with $U^{3/4}$. Therefore, one should be careful about the benefit of increasing velocities regarding the pressure drop penalty.

Influence of surface roughness

The presence of wall roughness increases C_f (see Fig. 3) and thus Nu is expected to increase. Experiments suggests that Nu for a rough pipe is

$$\frac{Nu}{Nu_o} = \left(\frac{C_f}{C_{f_o}}\right)^n, \quad \text{with} \quad n = 0.68Pr^{0.215}, \tag{3.20}$$

where Nu_o is the Nusselt number for a smooth pipe.

The above result suggests that the heat transfer rate can be enhanced by artificially roughening the surface. However, experimental studies showed that Nu does not increases further with roughness when $\frac{C_f}{C_{f_o}} > 4$. Thus, one needs to pay sufficient attention in introducing the artificial roughness otherwise the pumping power requirement will increase, for marginal or no gain in the heat transfer rate. Typically, the size of the roughness element should be within the viscous sub-layer to gain benefit on heat transfer augmentation.

Heat exchangers are devices used to transfer heat from one fluid stream to another. So-called radiators used in automobiles and in air–conditioners as well as for space heating are examples of simple heat exchangers. Complex heat exchangers are essential in power plants using fossil fuels or nuclear power, food processing, refining oil, and chemical processing industries. In this section we introduce the fundamental elements of heat exchanger calculation and design. The ultimate design of these devices is an involved optimisation involving the best arrangement for maximising heat exchange whilst minimising cost, flow pumping requirements, weight and space usage.

There are many types of heat exchangers, according to the flow arrangement and construction. In a direct transfer heat exchanger, the hot fluid mixes directly with the cold fluid, producing a mean temperature. This is a common arrangement in power plants, where warm water from the condensation in gas turbines is pumped to mix with the incoming cold water. In indirect heat exchangers, which will be considered here, each fluid is kept separate from the other by a heat-conducting wall, typically a metal. Here we will only consider the most common steady heat exchangers. Finally, the geometric arrangement of heat exchangers can vary significantly, with the fluids coming in co-flow, counterflow, cross flow, with an infinite variety of the number of passes of tubes and channels through the heat exchangers, the use of fins and turbulence enhancing features.

In general, since increased velocities lead to higher Reynolds numbers, and the Nusselt number is proportional to Re^n, it is advantageous to have smaller pipes or ducts through which each stream is forced. The limit is provided by the pressure drop, which also increases with velocity: at some point, it becomes difficult to provide sufficient pressure to the fluid to flow through

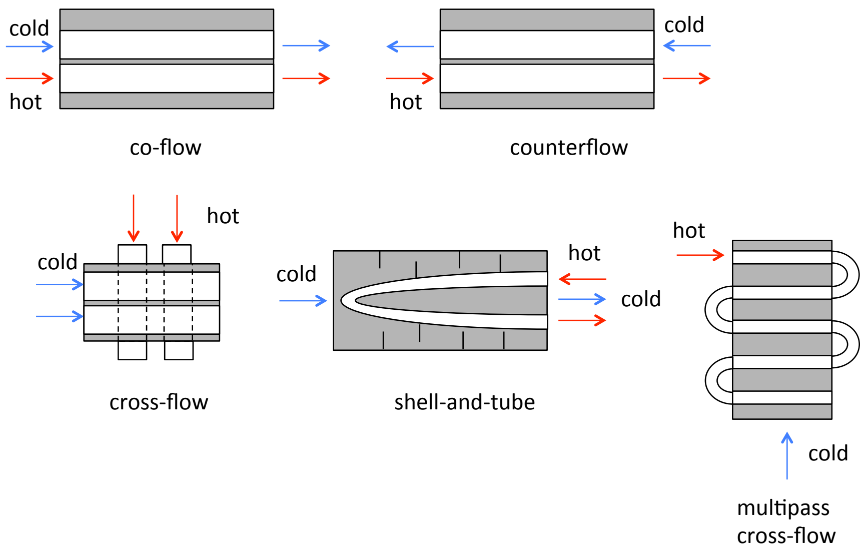


Figure 4: Typical heat exchange configurations.

small passages.

Let us consider the heat exchange between two streams, one hot and one cold. Here we start by considering the co-flow situation, with mass flow rates $\dot{m}$, bulk temperatures T and heat exchange coefficient h for each stream, and perimeter P for each interface, with c or h indicating the cold or hot streams, respectively. All the heat is assumed to be transferred between the two fluids, with no losses to the environment. The material separating the two streams has conductivity k and thickness δ. We start by considering the overall resistance to heat transfer between the hot and cold side within region dx in Fig.5 :

$$dq = h_h(T_h - T_{s,h})P_h\,dx = \frac{\lambda}{\delta}(T_{s,h} - T_{s,c})\bar{P}\,dx = h_c(T_{s,c} - T_c)P_c\,dx \tag{3.21}$$

$$(T_h - T_c) = \frac{dq}{dx}\frac{1}{U\mathcal{P}} \tag{3.22}$$

where

$$\frac{1}{U\mathcal{P}} = \frac{1}{h_h P_h} + \frac{\delta}{k\bar{P}} + \frac{1}{h_c P_c} \tag{3.23}$$

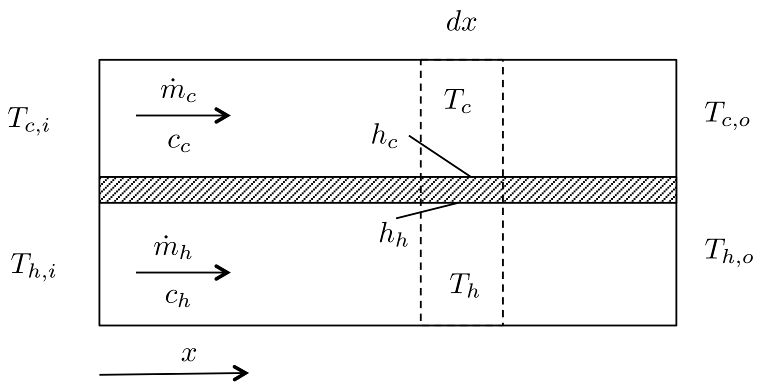


Figure 5: Heat exchanger diagram for co-flow configuration.

where U is the global heat transfer coefficient and $\mathcal{P}$ the mean perimeter, $\bar{P}$ as the average perimeter between hot and cold interfaces, and the subscript s indicates the surface[2].

The energy flux along dx can be written for the cold and hot fluids as:

$$\overbrace{\dot{m}_c c_c}^{C_c} \frac{dT_c}{dx} = -\overbrace{\dot{m}_h c_h}^{C_h} \frac{dT_h}{dx} = U\mathcal{P}(T_h - T_c) \tag{3.24}$$

$$C_h \frac{dT_h}{dx} = -U\mathcal{P}(T_h - T_c) \qquad C_c \frac{dT_c}{dx} = U\mathcal{P}(T_h - T_c) \tag{3.25}$$

$$\frac{d(T_h - T_c)}{dx} = -U\mathcal{P}(T_h - T_c)\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \tag{3.26}$$

Integrating this equation we have:

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \frac{\Delta T_o}{\Delta T_i} = \exp\left[-U\mathcal{P}L\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\right] \tag{3.27}$$

where L is the length of the heat exchanger. From the overall energy equation for the heat

[2] In general, there could be additional resistances due to rusting or fouling of surfaces, as well as lowering of resistances due to surface finning. These can be incorporated in the overall transfer coefficient

exchanger, we have:

$$Q = C_c(T_{c,o} - T_{c,i}) = C_h(T_{h,i} - T_{h,o}) \tag{3.28}$$

$$C_c = \frac{Q}{T_{c,o} - T_{c,i}} \qquad C_h = \frac{Q}{T_{h,i} - T_{h,o}} \tag{3.29}$$

Therefore, we can write an equation for the overall heat transfer, Q, as:

$$U\mathcal{P}L\left(\frac{1}{C_h} + \frac{1}{C_c}\right) = \frac{UA}{Q}(T_{h,i} - T_{h,o} + T_{c,o} - T_{c,i}) = -\ln\left(\frac{\Delta T_o}{\Delta T_i}\right) \tag{3.30}$$

from which we obtain :

$$\boxed{Q = UA\,\underbrace{\frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)}}_{\text{LMTD}} = U\ A\ \text{LMTD}} \tag{3.31}$$

where

$$\boxed{\text{LMTD} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)} = \frac{\Delta T_i - \Delta T_o}{\ln(\Delta T_i/\Delta T_o)}} \tag{3.32}$$

The **log mean temperature difference**, LMTD, can therefore be directly related to the heat transfer rate. Of course, in general we know the inlet temperatures of each stream, but not the final temperatures. This will be addressed in later sections. For now, we observe that the analysis for the counterflow heat exchanger would be the same, except for the fact that the outgoing hot and cold temperatures, and the temperature differences and inlet and outlet are different, as illustrated in Fig. 6.

In the case of multi pass, cross-flow and other heat exchangers (Fig. 4), the analysis is clearly more complex, as the relationship between the temperatures of the two gases can become a complex function of the location of the tube or tubes and so on. For standard geometric configurations, it is possible to generate correction factors that multiply the LMTD to correct for the enhanced heat transfer. As always, such correlations are available from standard handbooks.

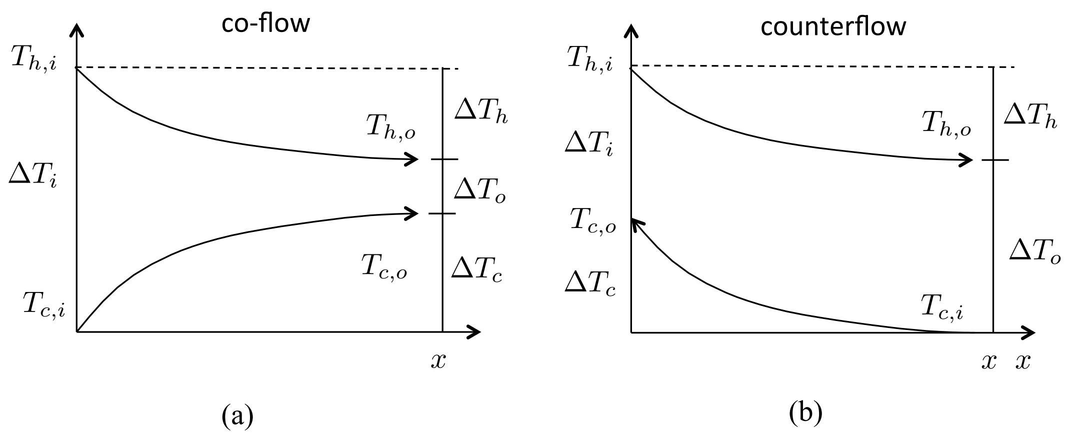


Figure 6: Variation of temperature along the length of the heat exchanger in (a) co-flow and (b) counterflow arrangements.

Question 3-2 Obtain an expression for the temperature of the hot and cold streams in co-flow as a function of x.

Solution

From the integration of Eq. (3.26) up to x we have:

$$\frac{T_h - T_c}{T_{h,i} - T_{c,i}} = \exp\left[-U\mathcal{P}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)x\right]$$

Substituting into Eq. (3.26), we have:

$$\frac{dT_h}{dx} = -\frac{U\mathcal{P}}{C_h}(T_{h,i} - T_{c,i})\exp\left[-U\mathcal{P}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)x\right]$$

$$T_{h,i} - T_h = (T_{h,i} - T_{c,i})\frac{1}{C_h}\frac{1}{\dfrac{1}{C_h} + \dfrac{1}{C_c}}\exp\left[-U\mathcal{P}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)x\right]$$

and using a similar substitution for T_c,

$$T_c - T_{c,i} = (T_{h,i} - T_{c,i})\frac{1}{C_c}\frac{1}{\dfrac{1}{C_h} + \dfrac{1}{C_c}}\exp\left[-U\mathcal{P}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)x\right]$$

ε–NTU method

In general, we do now know what the outlet temperatures of the heat exchanger will be, yet we may need to design the total area and configuration of a heat exchanger. We start by considering the maximum amount of heat that can be exchanged. This will be the product of the smaller of the heat capacities times the corresponding maximum temperature rise or decrease[3]:

infinitely long) Heat exchanger

$$Q_{\text{max}} = C_{\text{min}}(T_{h,i} - T_{c,i}) \qquad C_{\text{min}} = \min(C_c, C_h) \tag{3.33}$$

It is useful to define an *effectiveness* for the heat exchanger as the *ratio of the actual to the maximum possible heat transfer rate*, which of course can vary from 0 to 1:

$$\varepsilon = \frac{Q}{Q_{\text{max}}} = \frac{C_c(T_{c,o} - T_{c,i})}{C_{\text{min}}(T_{h,i} - T_{c,i})} = \frac{C_h(T_{h,i} - T_{h,o})}{C_{\text{min}}(T_{h,i} - T_{c,i})} \tag{3.34}$$

Whereas the effectiveness can be calculated for simple cases, the overall effectiveness of a heat exchanger depends on the geometry of the heat exchanger, the number of passes and design details. In general, if we are trying to specify the area and configuration of a heat exchanger, we should start from considering how much heat we would like to transfer, working out the minimum effectiveness of the process, and from there the UA of the heat exchanger.

Consider for the moment a co-flow heat exchanger. The energy balance demands that:

$$C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i}) \qquad \rightarrow \qquad \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}} = \frac{C_c}{C_h} \tag{3.35}$$

Returning to the analysis of a heat exchanger, and following the same rationale as above in determining the limits of temperature difference, Eq. (3.27) can be written as:

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \exp\left[-\frac{UA}{C_{min}}\left(1 + \frac{C_{min}}{C_{max}}\right)\right] = \exp[-\text{NTU}(1 + R_c)] \tag{3.36}$$

where the *Number of Transfer Units* (NTU) and *ratio of heat capacities*, R_c, is defined implicitly as:

$$\text{with} \qquad \boxed{\text{NTU} = \frac{UA}{C_{\text{min}}},} \qquad \text{and} \qquad R_c = \frac{C_{\text{min}}}{C_{\text{max}}}.$$

From the overall energy balance in Eq. (3.35)

$$T_{h,o} = T_{h,i} - R_c(T_{c,o} - T_{c,i})$$

[3] You can see this by taking for example $C_c < C_h$: the incoming fluid could only be heated to the inlet temperature of the hot stream. Conversely, if $C_h < C_c$, the hot stream cannot give up more heat than the heat capacity of the hot stream times the maximum temperature drop possible, which is that of the incoming hot stream minus the that of the incoming cold stream.

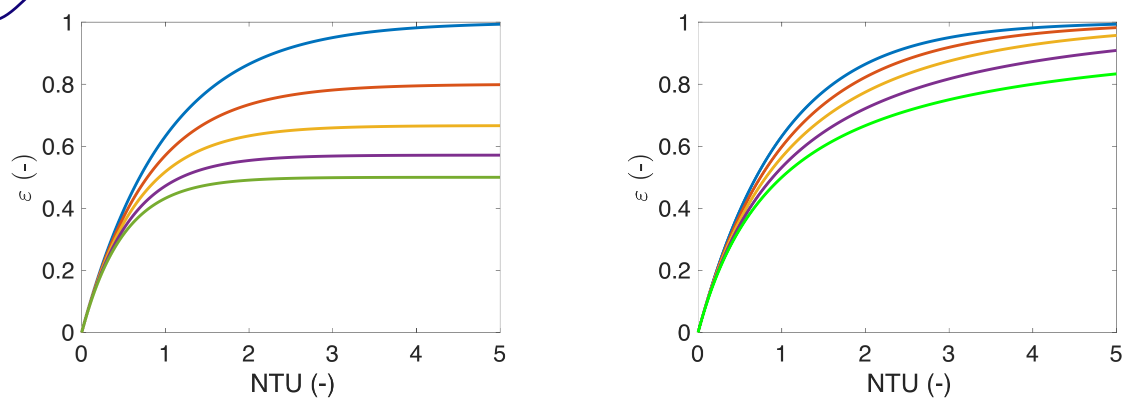
substituting this into Eq. (3.36) and rearranging, we have

$$\varepsilon = \frac{1 - e^{-\mathrm{NTU}(1+R_c)}}{1 + R_c}, \quad \text{or} \quad \mathrm{NTU} = \frac{1}{1+R_c} \ln\left(\frac{1}{1-(1+R_c)\varepsilon}\right) \tag{3.37}$$

A similar analysis for the counterflow arrangement gives:

$$\varepsilon = \frac{1 - e^{-\mathrm{NTU}(1-R_c)}}{1 - R_c e^{-\mathrm{NTU}(1-R_c)}}, \quad \text{or} \quad \mathrm{NTU} = \frac{1}{1-R_c} \ln\left(\frac{1-\varepsilon R_c}{1-\varepsilon}\right) \tag{3.38}$$

Figure 7 shows the effectiveness of single pass co-flow and counterflow heat exchangers.



(a) (b)

Figure 7: Heat exchanger effectiveness for increasing R_c for (a) co-flow and (b) counterflow arrangements.

In the above analysis, it is implicitly assumed that there is no phase change of the fluids. In boiling and condensation processes, the fluid temperature remains essentially constant or the fluid acts as if it has infinite specific heat capcity. In these cases $R_c \longrightarrow 0$ and the above ε-NTU relations become simply

$$\varepsilon = 1 - e^{-\mathrm{NTU}}, \quad \text{with } C_{\min} = \dot{m}_c c_c \text{ for a condenser, } C_{\min} = \dot{m}_h c_h \text{ for a boiler.} \tag{3.39}$$

Therefore, for a given total amount of heat exchange required, and given mass flow rates of cold and hot fluid, one can determine the required heat transfer area A (sizing of the heat exchanger). This can be obtained from NTU for a given condition (ε and R_c are known). The overall heat transfer coefficient can be obtained from the correlations noted in the previous chapter, and often from experiments if the calculations are challenging.

Heat exchanger performance

A number of methods can be used to enhance heat transfer in these devices: for each surface, turbulence-enhancing features can be added to create additional heat transfer, finned surfaces can be used to extend the heat exchange surfaces, and in many cases, multiple passes of tubes to increase the area for heat transfer within a compact volume.

In shell-and-tube heat exchangers, often used for liquid, bundles of tubes are routed through a counter flowing shell containing the other fluid. Automotive heat exchangers use compact highly finned surfaces in contact with the cooling air coming through the front of the car. Air-to-air heat exchangers, such as recuperators for power plants, can be very large, as heat exchange between gases can be very slow and require vast amounts of area. The student is referred to the textbooks for further references.

Question 3-3 Consider a single pass heat exchanger cooling exhaust gases using available cooling water, which remains at 290 K. The exhaust gas mass flow rate is 0.1 kg/s, the heat capacity 1 kJ/kg K, the initial temperature is 400 K and the desired temperature drop is 50 K. Determine the necessary UA. **Solution:**
Since the water temperature does not change significantly, we have $R_c = C_{min}/C_{max} = 0$

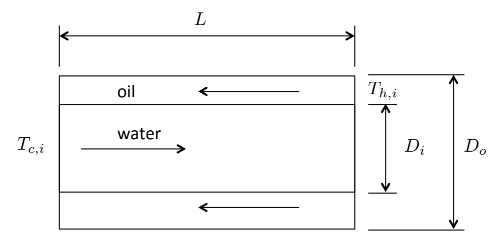
$$\varepsilon = \frac{\dot{Q}}{C_{min}(T_{h,i} - T_{c,i})} = \frac{(1\ \mathrm{kg/s})(1000\ \mathrm{J/kg})(50K)}{(1\ \mathrm{kg/s})(1000\ \mathrm{J/kg})(400-290)\mathrm{K}} = 0.45$$

$$\mathrm{NTU} = -\ln(1-\varepsilon) = 0.60$$

$$\mathrm{UA} = \mathrm{NTU}\ C_{min} = (0.60)(1\ \mathrm{kg/s})(1000\ \mathrm{J/kg}) = 600\ \mathrm{W/K}$$

Question 3-4

For the counter-flow, tubular heat exchanger arrangement shown, oil and water mass flow rates are equal to 0.1 kg/s. The oil comes in hot, and is cooled by water. The properties of the fluids are given below.



Property	water	oil
ρ (kg/m^3)	1000	800
c_p (J/kg K)	4200	1900
ν (m^2/s)	7.0×10^{-7}	1.0×10^{-5}
λ (W/m K)	0.64	0.134
Pr	4.7	140
T_i	30	100

(a) Calculate the heat transfer rate, $\dot{Q}$ and $T_{c,o}$ if $T_{h,i} = 100°$ C

(b) Determine the UA required to achieve $T_{h,o} = 60$ °C using

(i) LMTD method;

(ii) ε–NTU method.

Solution:
(a) Energy balance on the hot oil side:

$$\dot{Q} = \dot{m}_h c_h (T_{h,i} - T_{h,o}) = 0.1 \times 1900 \times (100 - 60) = 7.6 \ \text{kW}$$

on the cold water side:

$$\dot{Q} = \dot{m}_c c_c (T_{c,o} - T_{c,i}) = 0.1 \times 4200 \times (T_{c,o} - 30) = 7.6 \ \text{kW} \implies T_{c,o} = 48.1\ ^\circ\text{C}$$

(b) (i) Using the LMTD method: $\dot{Q} = UA\text{LMTD}$

$$\text{LMTD} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)} = \frac{(60-30) - (100-48.1)}{\ln(30/51.9)} \approx 40\ ^\circ\text{C}, \qquad \implies \quad UA = \frac{\dot{Q}}{\text{LMTD}} = \frac{7600\ \text{W}}{40\ \text{K}} = 190\ \text{W/K}$$

(ii) Using the ε–NTU method:
$\dot{m}_h c_h = 190$ J/kg K $= C_{\text{min}}$; $\dot{m}_c c_c = 420$ J/kg K $= C_{\text{max}}$

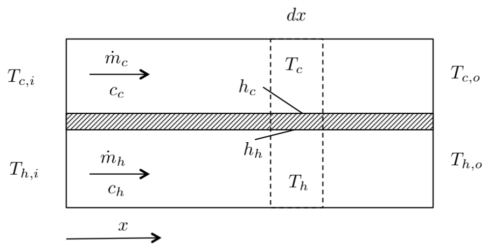
$$\implies R_c = \frac{C_{\text{min}}}{C_{\text{max}}} = 0.452$$

$$\dot{Q}_{\text{max}} = C_{\text{min}}(T_{h,i} - T_{c,i}) = 13.3\ \text{kW} \quad \implies \quad \varepsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} = 0.571$$

NTU relationship in Eq. (3.38) $\rightarrow$ $$\text{NTU} = \frac{1}{1-0.452} \ln\left(\frac{1-(0.571)(0.452)}{1-0.571}\right) = 0.999 = \frac{UA}{C_{\text{min}}}$$

$$UA = 190\ \text{W/K}$$

Epic heat exchanger



where U is the global heat transfer coefficient and $\mathcal{P}$ the mean perimeter, $\bar{\mathcal{P}}$ as the average perimeter between hot and cold interfaces, and the subscript s indicates the surface[2].

The energy flux along dx can be written for the cold and hot fluids as:

$$\overbrace{\dot{m}_c c_c}^{C_c}\frac{dT_c}{dx} = -\overbrace{\dot{m}_h c_h}^{C_h}\frac{dT_h}{dx} = U\mathcal{P}(T_h - T_c) \quad (3.$$

$$C_h\frac{dT_h}{dx} = -U\mathcal{P}(T_h - T_c) \qquad C_c\frac{dT_c}{dx} = U\mathcal{P}(T_h - T_c) \quad (3.$$

$$\frac{d(T_h - T_c)}{dx} = -U\mathcal{P}(T_h - T_c)\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \quad (3.$$

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \exp\left[-\frac{UA}{C_{min}}\left(1 + \frac{C_{min}}{C_{max}}\right)\right] = \exp[-\mathrm{NTU}(1 + R_c)] \quad (3.36)$$

where the *Number of Transfer Units* (NTU) and *ratio of heat capacities*, R_c, is defined implicitly as:

$$\text{with} \quad \boxed{\mathrm{NTU} = \frac{UA}{C_{\min}},} \quad \text{and} \quad R_c = \frac{C_{\min}}{C_{\max}}.$$

From the overall energy balance in Eq. (3.35)

$$T_{h,o} = T_{h,i} - R_c(T_{c,o} - T_{c,i})$$