Physics 1502Q:
4.1 Gauss’s Law II

Announcements & Reminders

- Pre-lab due **at the beginning of your lab this week**
- Reading Assignment due **Sunday at 11:59 PM**
- Homework due **Monday at 11:59 PM**
- **Office Hours:**
  - Posted on HuskyCT

Preview of this week and next week

Su	M	T	W	Th	F	Sa
6 Reading Assignment Due 11:59 PM	7 HW Due 11:59 PM	8 Gauss's Law II	9	10 Electric Potential Energy Electric Potential I Paper Quiz	11 Lab 4: Electric Potential Pre-lab 4 Due before lab	12
13 Reading Assignment Due 11:59 PM	14 HW Due 11:59 PM	15 Electric Potential II	16	17 Capacitance	18 NO LAB NO PRELAB Exam 1 4 PM	19

Gauss's Laws II

LEARNING GOALS

By the end of this unit, you should be able to:

- Identify the three fundamental types of symmetries that are applicable with Gauss's Law
- Apply Gauss's Law to find the electric fields produced by charges with spherical, cylindrical and planar symmetries
- Identify the properties of electric fields inside conductors

Gauss's Law (Recap)

"closed loop integral" tells you to integrate over the entire surface

$$\Phi_E = \underbrace{\oint \vec{E} \cdot d\vec{A}}_{\text{Net flux through a closed surface}} = \frac{q_{enc}}{\varepsilon_0}$$

total charge enclosed by the surface

Net flux through a closed surface

ε_0 = 8.85 x 10^{-12} C^2 N^{-1} m^{-2}
"permittivity constant"

Gauss's law is always true, but it is only useful for finding the electric Field in situations with certain <u>symmetry</u>.

Gauss's Law Symmetries (Recap)

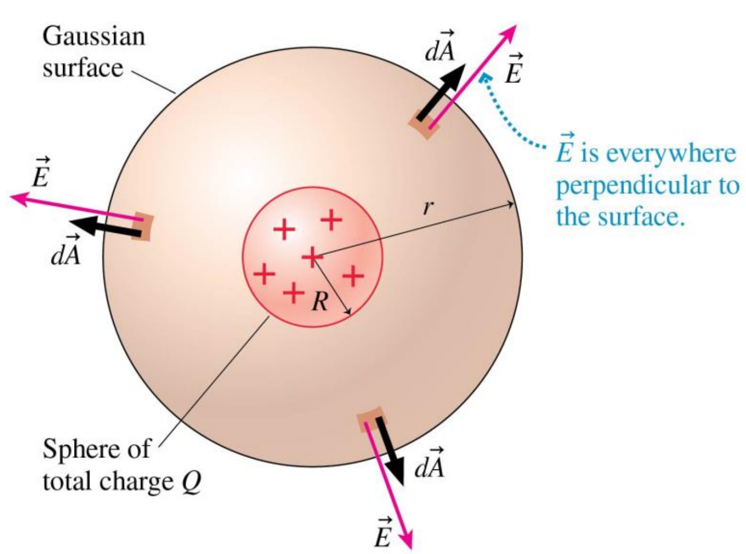
$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Always true for closed surfaces!

In cases with symmetry (and correct Gaussian surface) one can pull E outside and get

$$\boxed{E = \frac{q_{enc}}{A\varepsilon_0}}$$

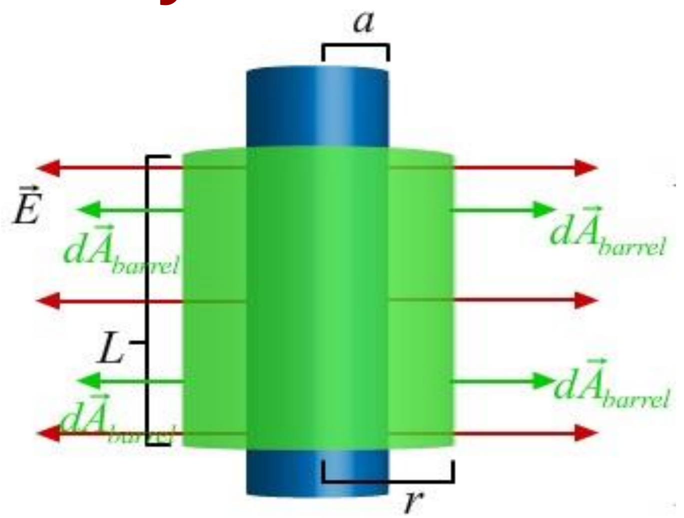
Spherical



$$A = 4\pi r^2$$

Total surface area

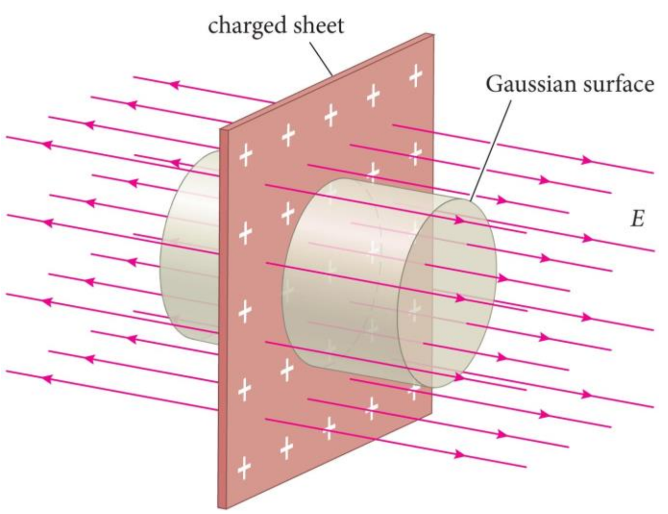
Cylindrical



$$A = 2\pi r L$$

Ignore the top and bottom

Planar



$$A = 2\pi r^2$$

Only the top and bottom

What's so special about these Gaussian surfaces? (Recap)

1. E-field is constant at the surface

2. E-field is parallel to surface vector → dot product turns into scalar product

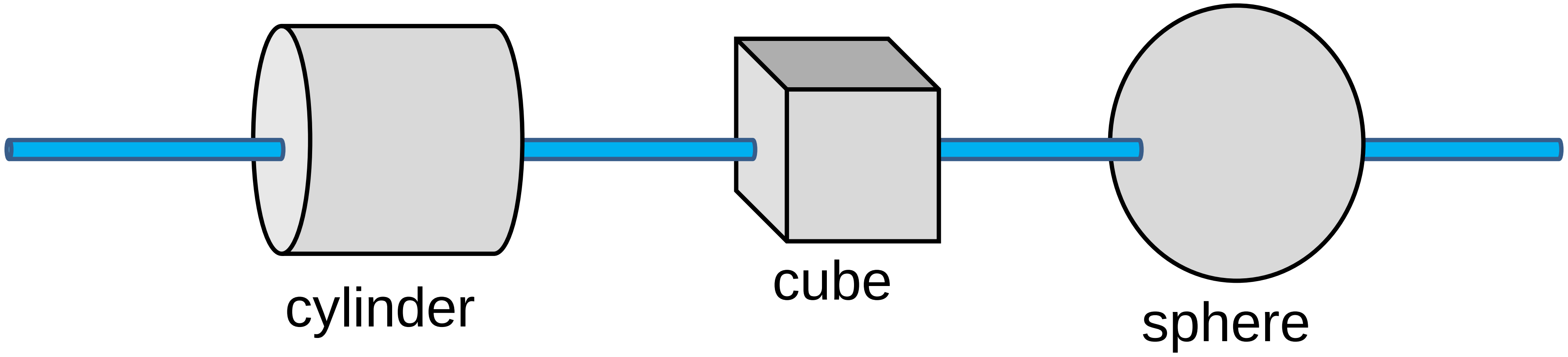
We can rewrite:

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \qquad \text{as} \qquad E = \frac{Q_{enc}}{A\varepsilon_0}$$

These three special cases can be used to calculate E or Q depending on what we know

Question: Gauss's Law

The figure below shows an infinitely long rod with a uniform positive linear charge density λ.

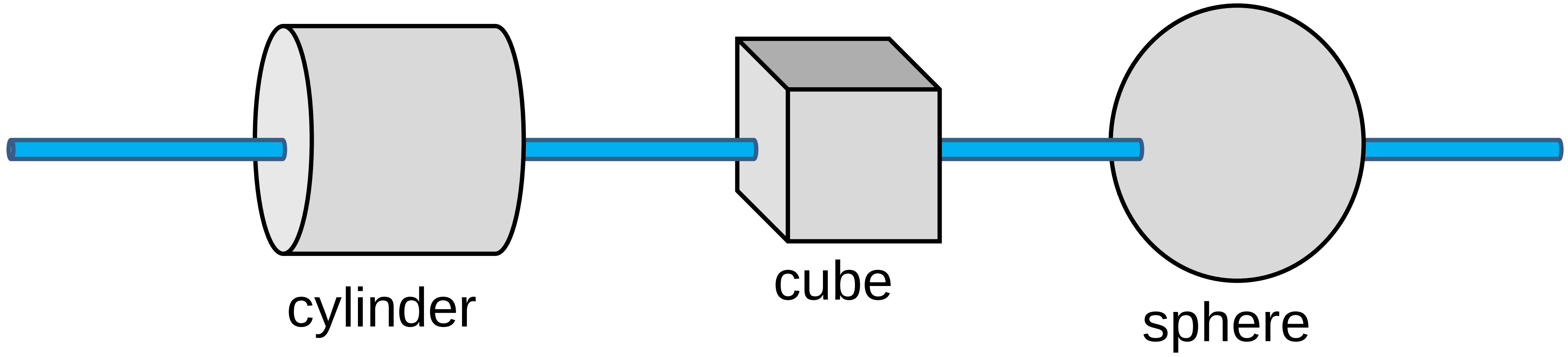


For which surface is it easiest to calculate $\oint \vec{E} \cdot d\vec{A}$?

A) cylinder
B) cube
C) sphere
D) It is equally easy for all three surfaces.

Question: Gauss's Law

The figure below shows an infinitely long rod with a uniform positive linear charge density λ.



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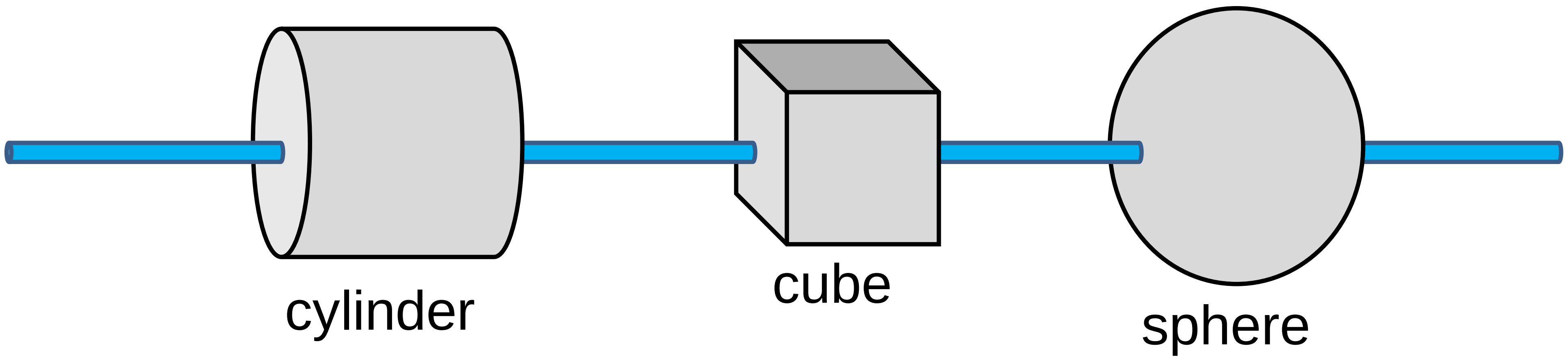

Three Gaussian surfaces are shown above.
For which of the Gaussian surfaces is Gauss's law true?

A) Cylinder
B) Cube
C) Sphere
D) All of them

Question: Gauss's Law

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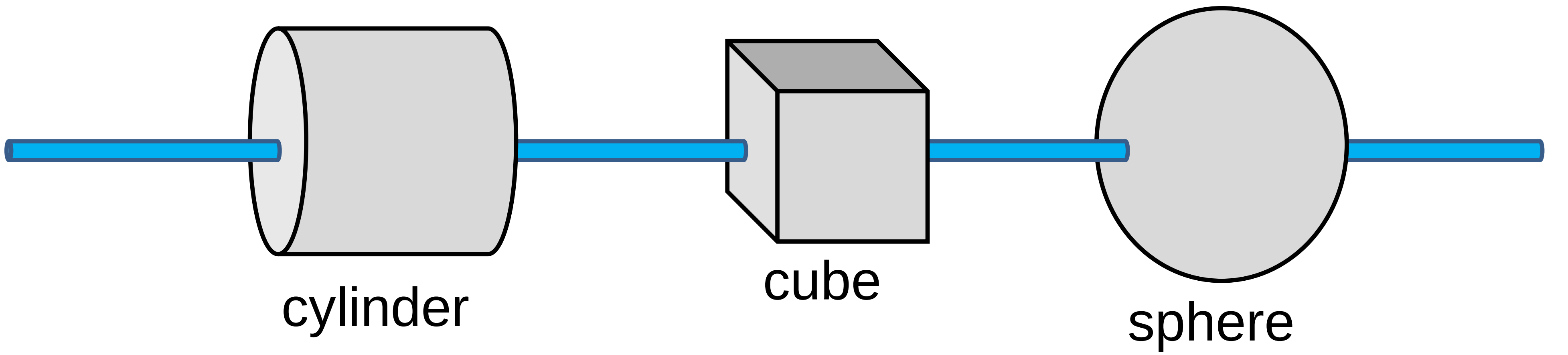


Three Gaussian surfaces are shown above.
For which of the Gaussian surfaces is Gauss's law true?

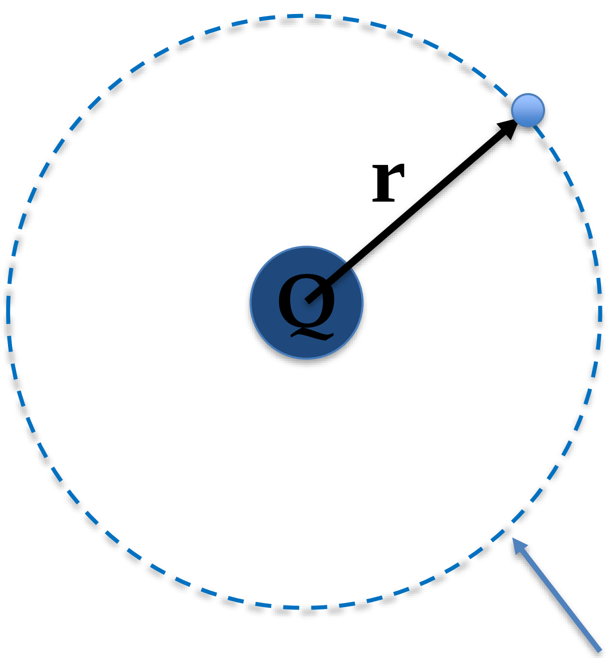
A) Cylinder
B) Cube
C) Sphere
D) All of them

Gauss's Law is always true, regardless of the surface!

But some surfaces are easier to consider than others…

Gauss's Law Example: Spherical Case (Recap)

Find the electric field at a radius r away from a sphere of charge Q (NOT a point charge).



Spherical Gaussian surface

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = \oint E dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = EA = \frac{q_{enc}}{\varepsilon_0}$$

For a spherical surface, we know that $A = 4\pi r^2$,

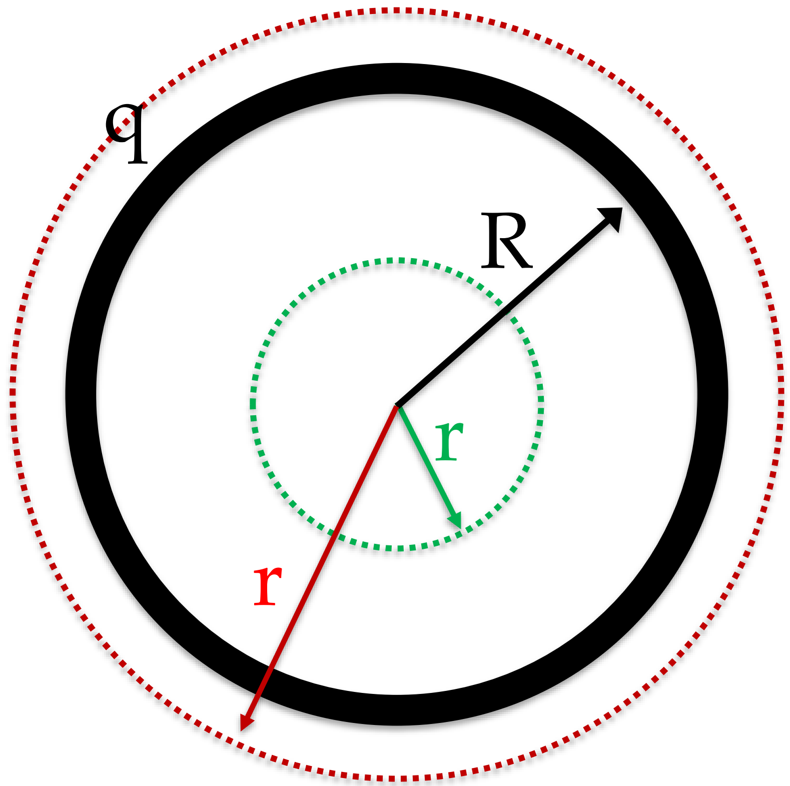
$$q_{enc} = Q \longrightarrow E\,4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{kQ}{r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

The equation is the same as for a point charge, even though this is NOT a point charge!!

Gauss's Law Example: Hollow Sphere (Recap)

A thin, uniformly charged, spherical shell of radius R with total charge q. Find the electric field inside and outside the shell.



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

For inside the shell, $r < R$ draw a Gaussian surface, and apply Gauss's Law to the surface.

$$q_{enc} = 0 \qquad \oint \vec{E} \cdot d\vec{A} = EA = 0$$

$$\boxed{E = 0}$$

For outside the shell, $r > R$ draw a Gaussian surface, and apply Gauss's Law to the surface.

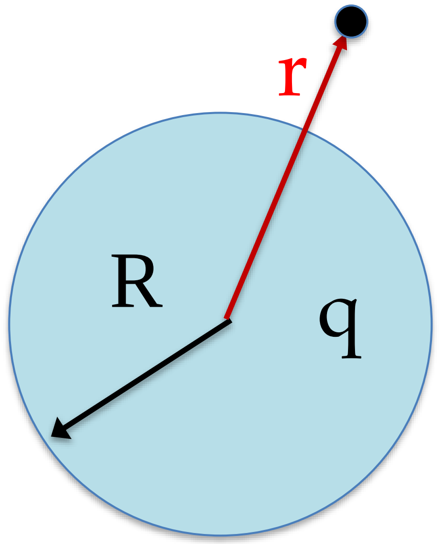
$$\oint \vec{E} \cdot d\vec{A} = EA = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.

Spherical Symmetry (Recap)

Any charged object that has spherical symmetry will create an electric field <u>outside</u> the object, with the form

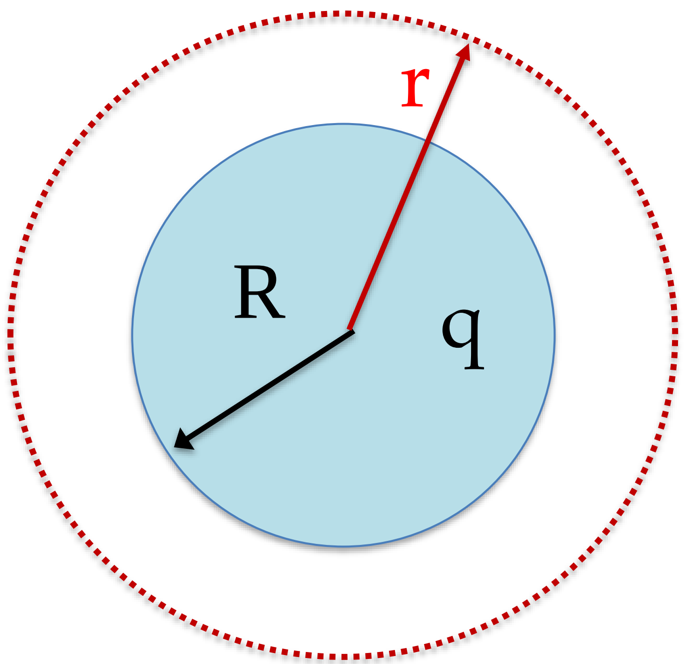


$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{enc}}{r^2}$$

where r is measured from the center.

Gauss's Law Example: Solid Sphere

A solid uniform sphere of total charge q, R is the sphere's radius, and R is the radial distance from the center of the sphere. Find the electric field everywhere.



For $r > R$,

$$\oint \vec{E} \cdot d\vec{A} = \oint E dA \cos 0^\circ = \oint E dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = EA = \frac{q_{enc}}{\varepsilon_0}$$

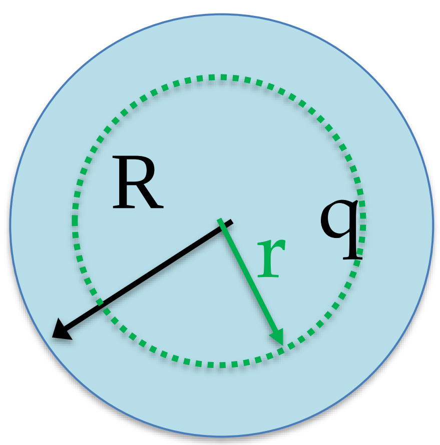
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{enc}}{r^2}$$

$$q_{enc} = \mathrm{q}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

Gauss's Law Example: Solid Sphere

A solid <u>uniform</u> sphere of total charge q, R is the sphere's radius, and R is the radial distance from the center of the sphere. Find the electric field everywhere.

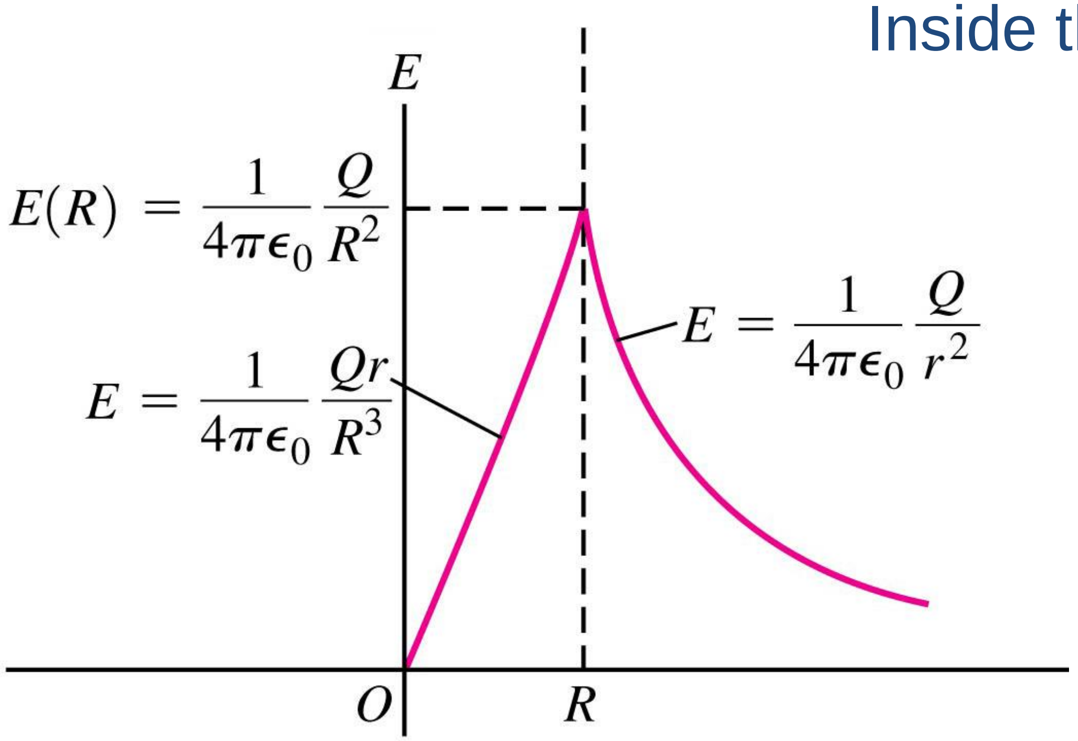


For $r < R$, only part of the charge is enclosed.

$$\oint \vec{E} \cdot d\vec{A} = \oint E dA \cos 0^\circ = \oint E dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = EA = \frac{q_{enc}}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{enc}}{r^2}$$



Inside the sphere there is a <u>uniform</u> volume charge density ρ,

$$q = \rho V$$

$$q_{enc} = \rho V_{enc}$$

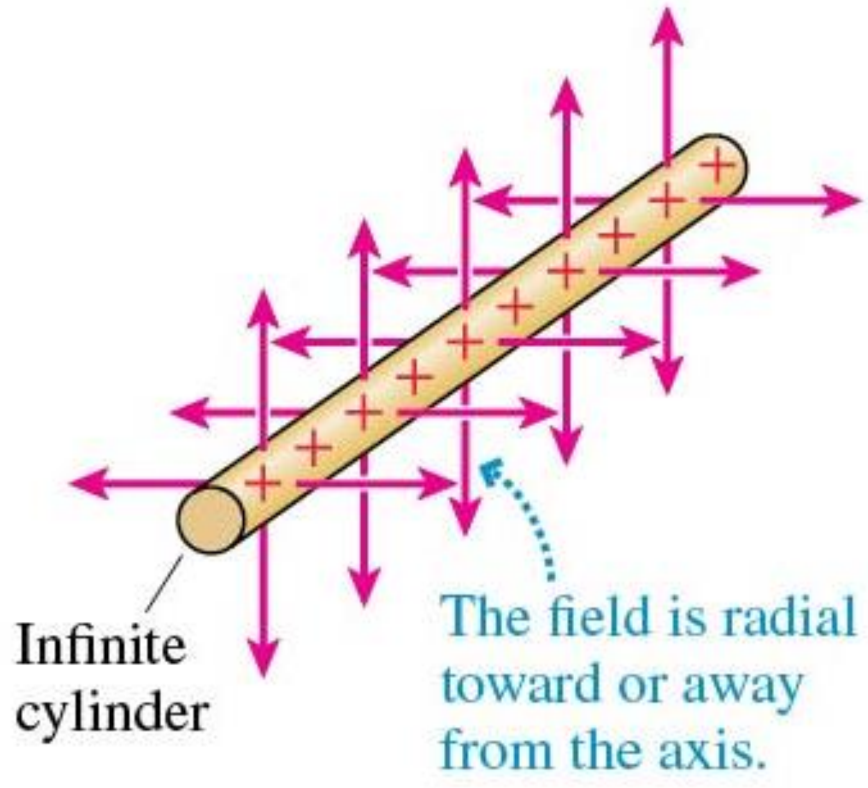
$$\Rightarrow q_{enc} = q\frac{V_{enc}}{V} = q\frac{r^3}{R^3}$$

$$\boxed{E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r}$$

Cylindrical Symmetry

- Cylindrical symmetry involves symmetry with respect to:
  - Translation parallel to the axis.
  - Rotation about the axis.
  - Reflection in any plane containing or perpendicular to the axis.

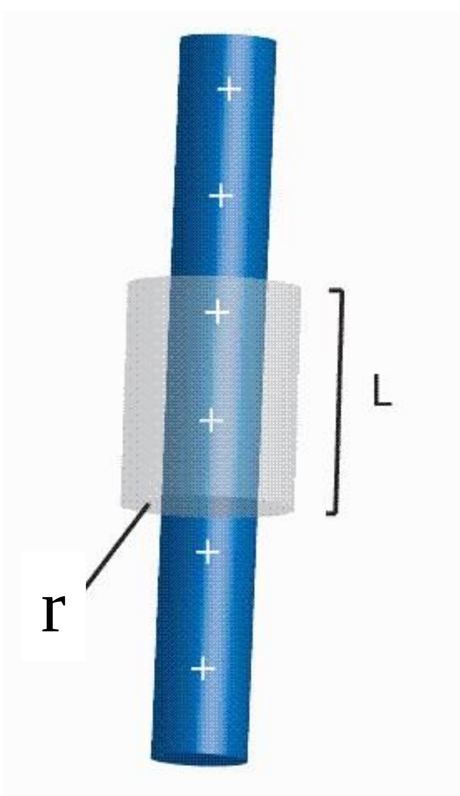
Cylindrical symmetry



Coaxial cylinders

Gauss's Law - Cylindrical Symmetry

An infinitely long charged rod has uniform charge density of λ. For the gray Gaussian surface (radius *r*, length *L*), find the electric flux flowing through it and the electric field at the surface of the Gaussian surface



$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$$= \oint \vec{E} \cdot d\vec{A} = \oint E dA \cos 0^\circ$$

$$\Rightarrow \Phi_e = E \oint dA = EA = \frac{\lambda L}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\lambda L}{\varepsilon_0 A} \qquad A = 2\pi r L$$

$$\boxed{E = \frac{\lambda}{2\pi r \epsilon_0}}$$

Question: Electric Field Flux & Gauss's Law

An infinitely long charged rod has a uniform charge density λ and passes through a gray cylinder. The cylinder in case 2 has twice the radius and half the length compared to the cylinder in case 1.

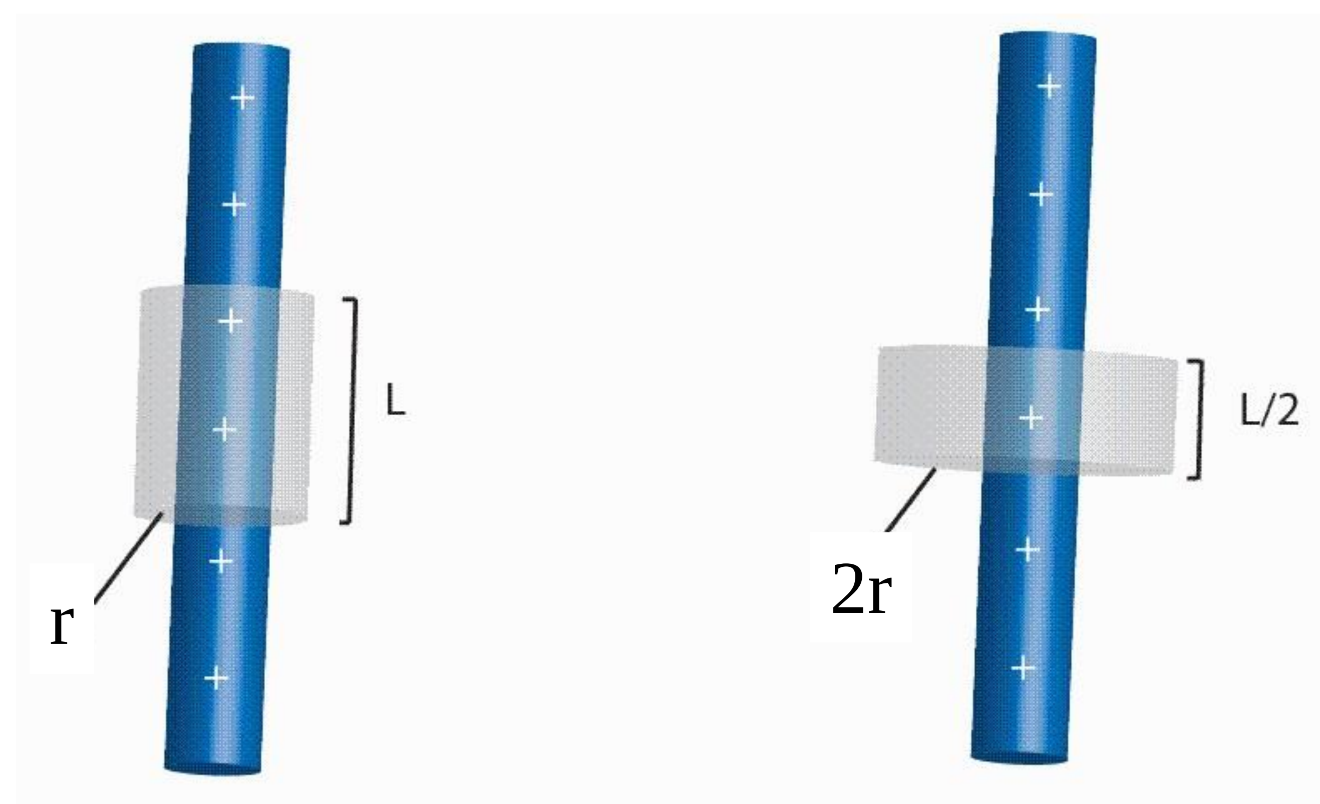
How does the flux of the first case compare with the second case?

(A) $\Phi_1 = 2\Phi_2$

(B) $\Phi_1 = \Phi_2$

(C) $\Phi_1 = ½\ \Phi_2$

(D) $\Phi_1 = 0$



Case 1 Case 2

Question: Electric Field Flux & Gauss's Law Answer

An infinitely long charged rod has a uniform charge density λ and passes through a gray cylinder. <u>The cylinder in case 2 has twice the radius and half the length compared to the cylinder in case 1.</u>

How does the flux of the first case compare with the second case?

$\boxed{\text{(A) } \Phi_1 = 2\Phi_2}$

(B) $\Phi_1 = \Phi_2$

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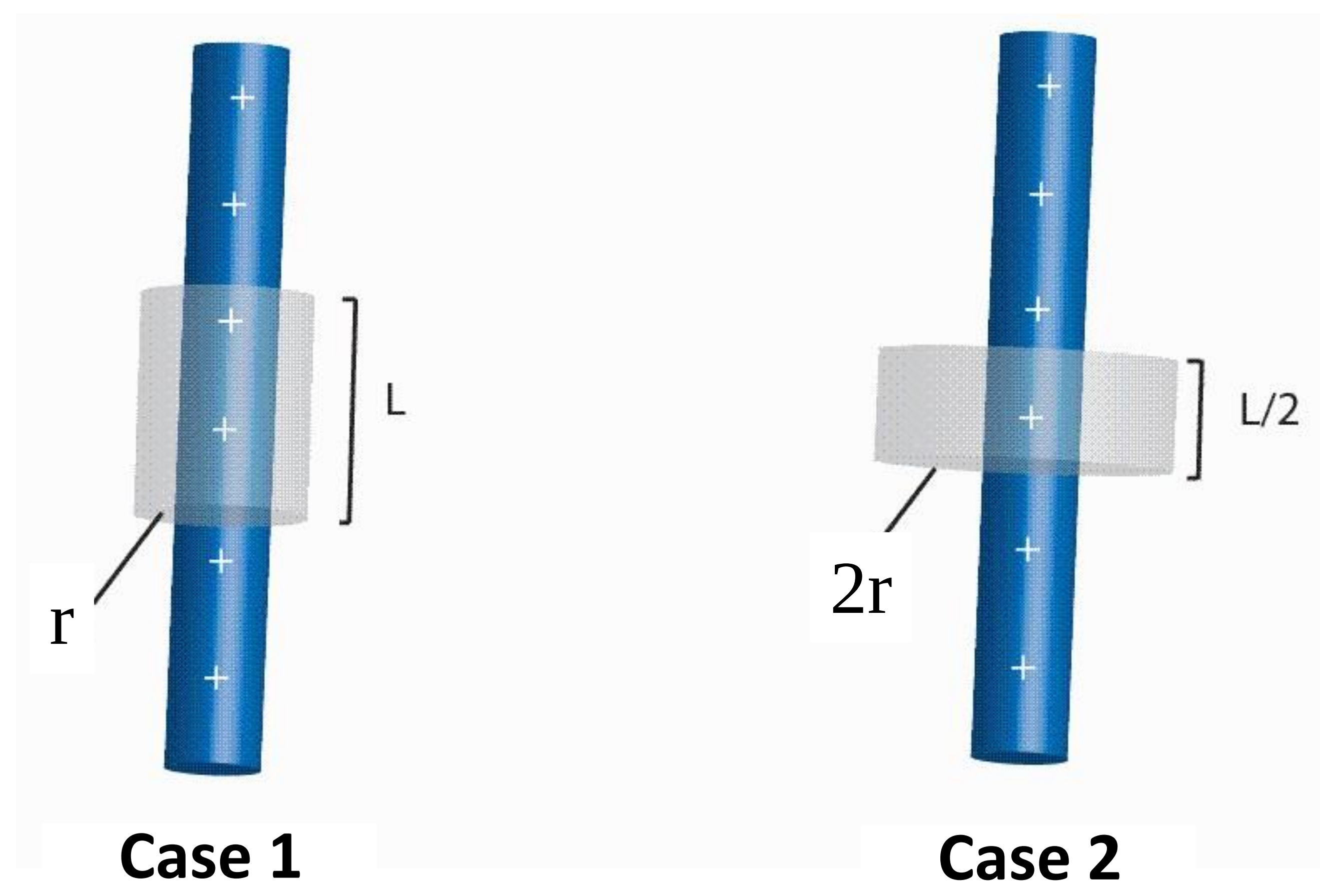
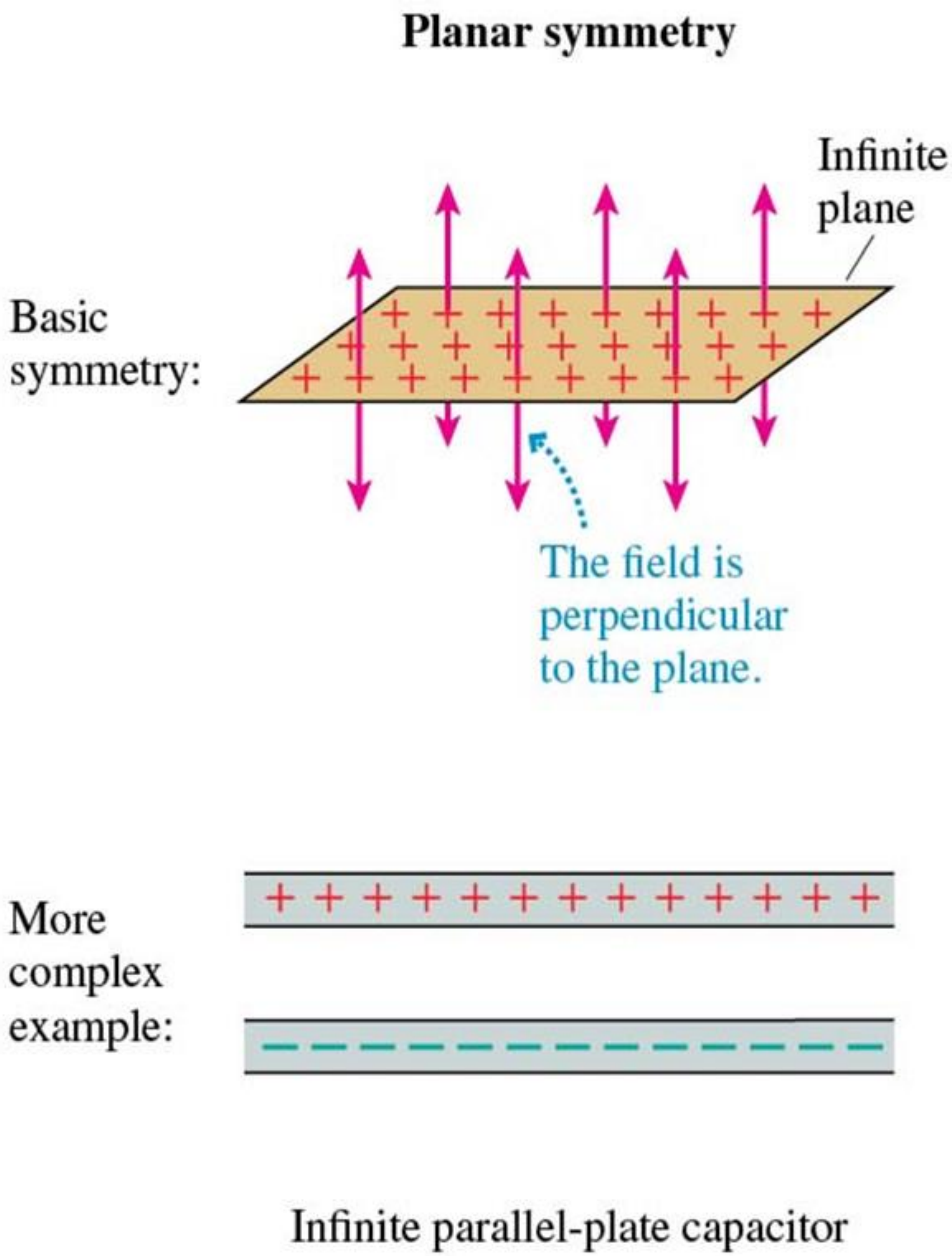


The first case encloses twice as much charge as the second, therefore it has more flux.

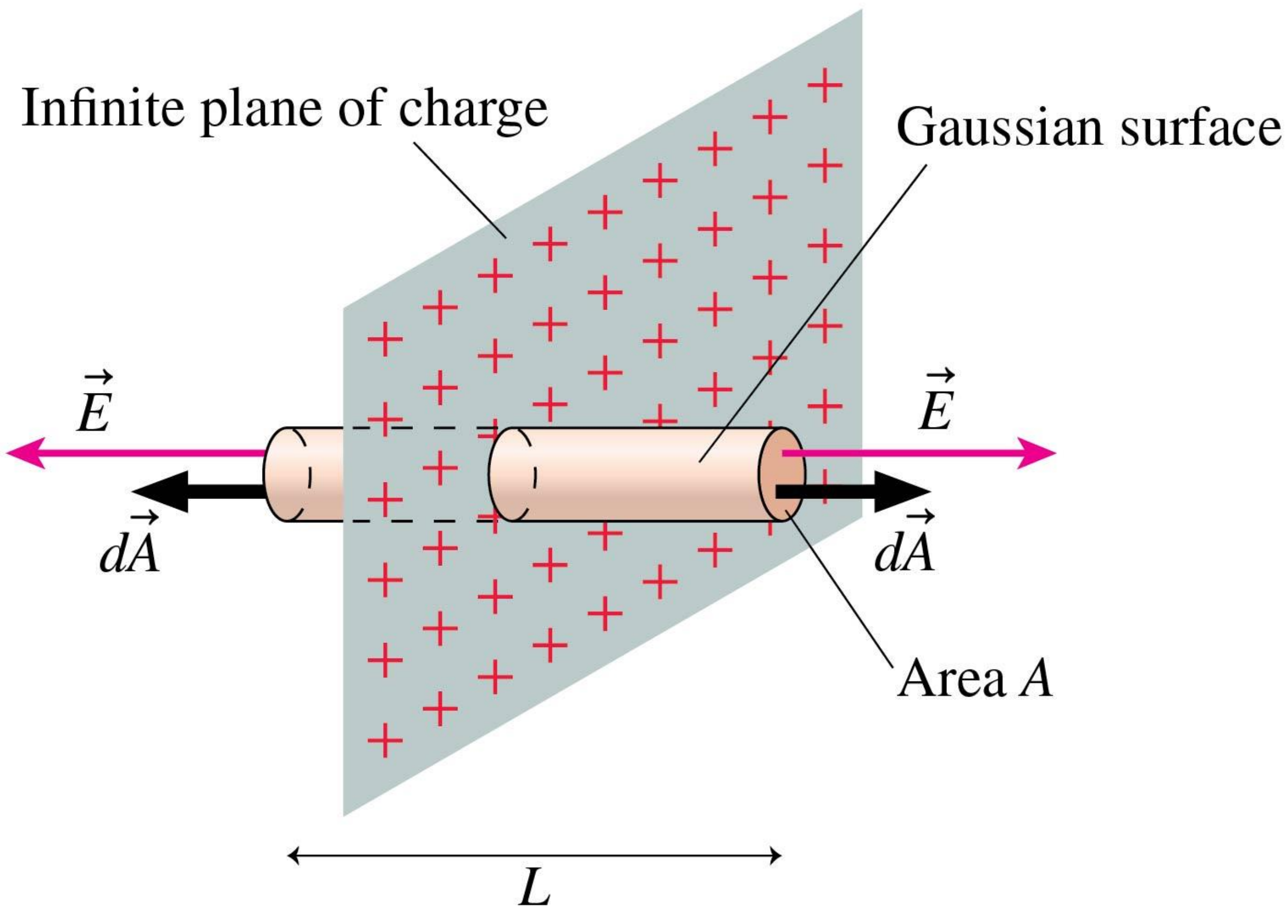
Planar Symmetry

- Planar symmetry involves symmetry with respect to:
  - Translation parallel to the plane.
  - Rotation about any line perpendicular to the plane.
  - Reflection in the plane.



Gauss's Law - Planar Symmetry

An infinite sheet of charge has uniform charge density of σ. For the gray Gaussian surface (radius *r*, length *L*), find the electric flux flowing through it and the electric field at the surface of the Gaussian surface



$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$\Rightarrow \Phi_e = E \oint dA = E(2\cancel{A}) = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

Total area = 2A because the electric field lines go through the FRONT and BACK surfaces of the Gaussian cylinder.

$$\boxed{E = \frac{s}{2e_0}}$$

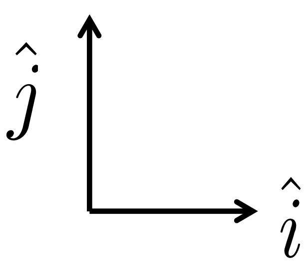
The electric Field due to a large sheet of charge

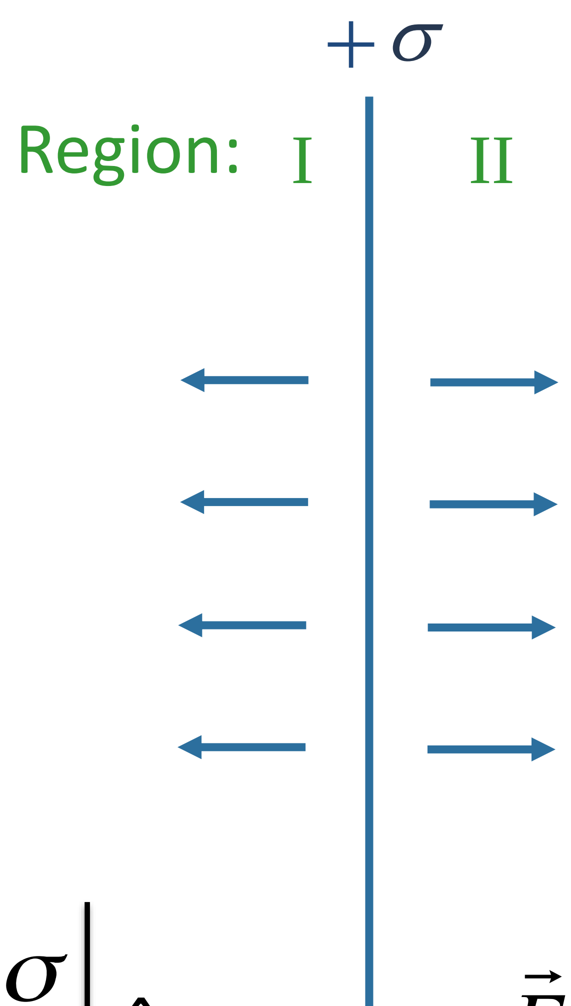
Activity: Multiple Charged Planes

Case A:

What is the electric field (including direction) in each region?

$$\boxed{E = \frac{s}{2e_0}}$$





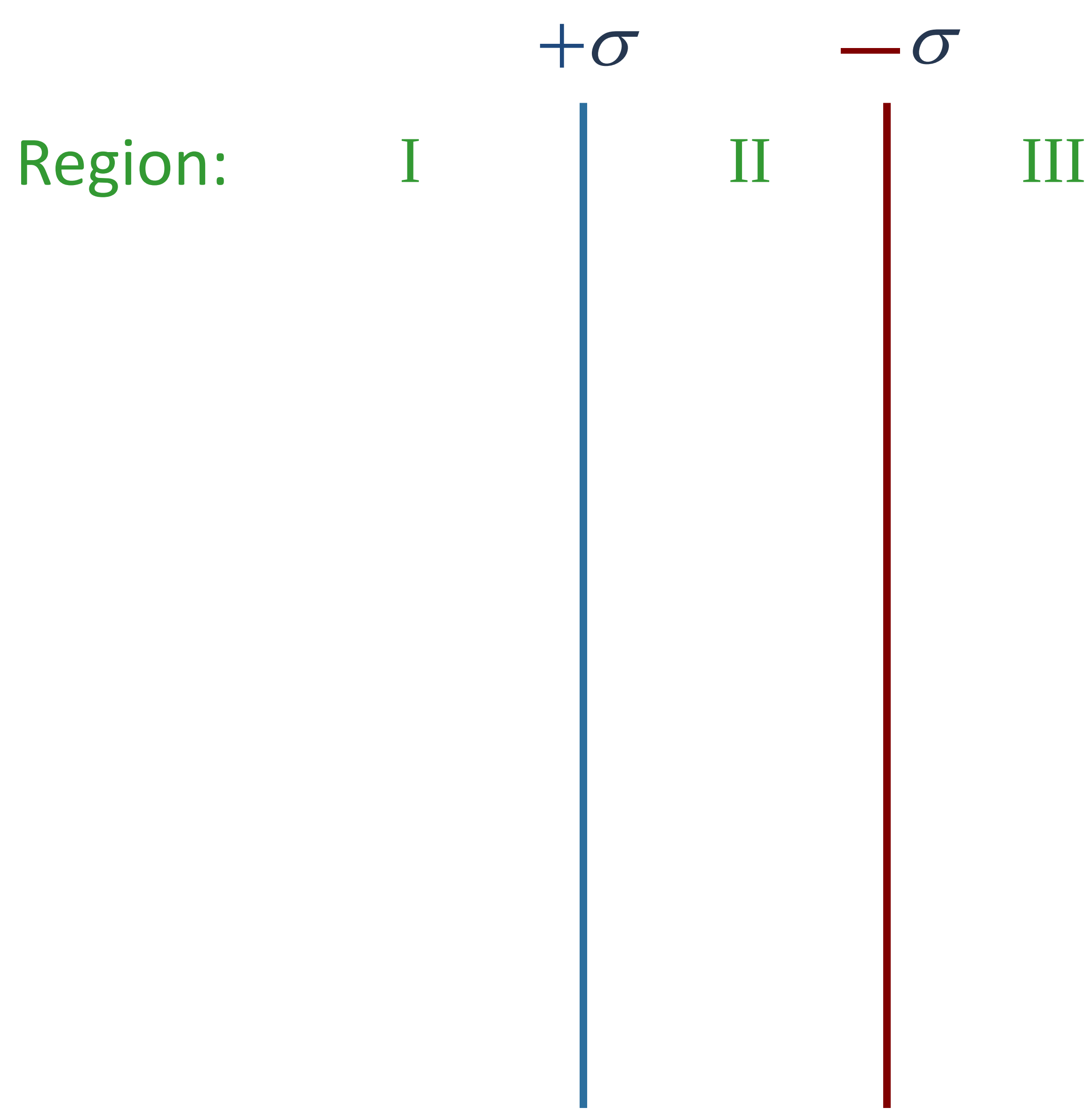
$$\vec{E}_{\mathrm{I}} = -\frac{|\sigma|}{2\varepsilon_0}\hat{\imath} \qquad \vec{E}_{\mathrm{II}} = \frac{|\sigma|}{2\varepsilon_0}\hat{\imath}$$

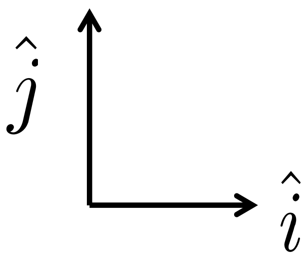
Group Activity: Multiple Charged Planes

Case B:

What is the electric field (including direction) in each region?

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$





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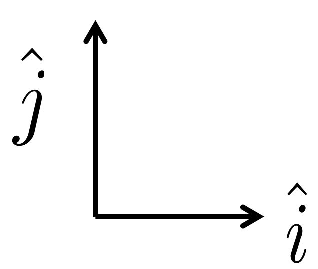
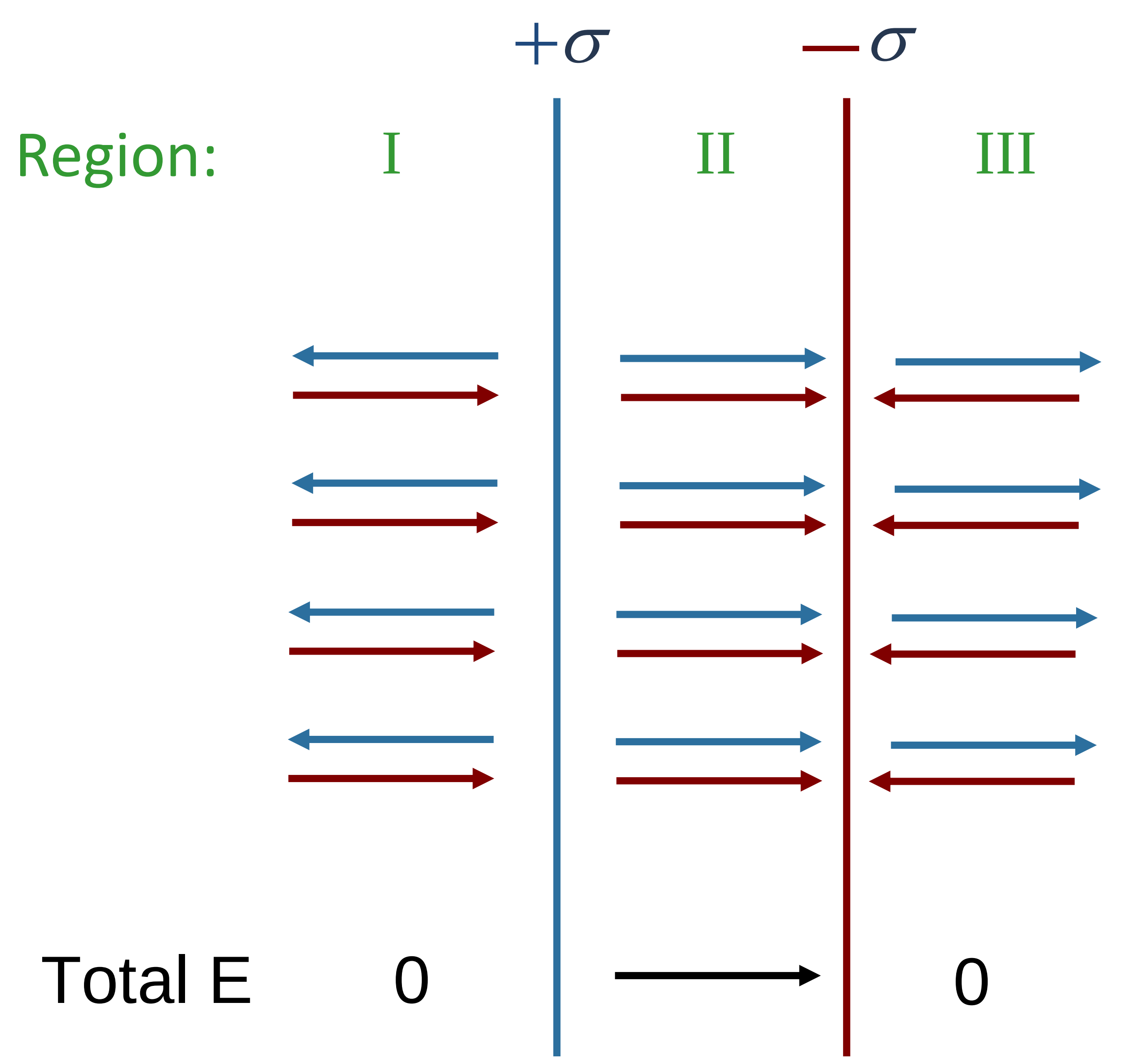
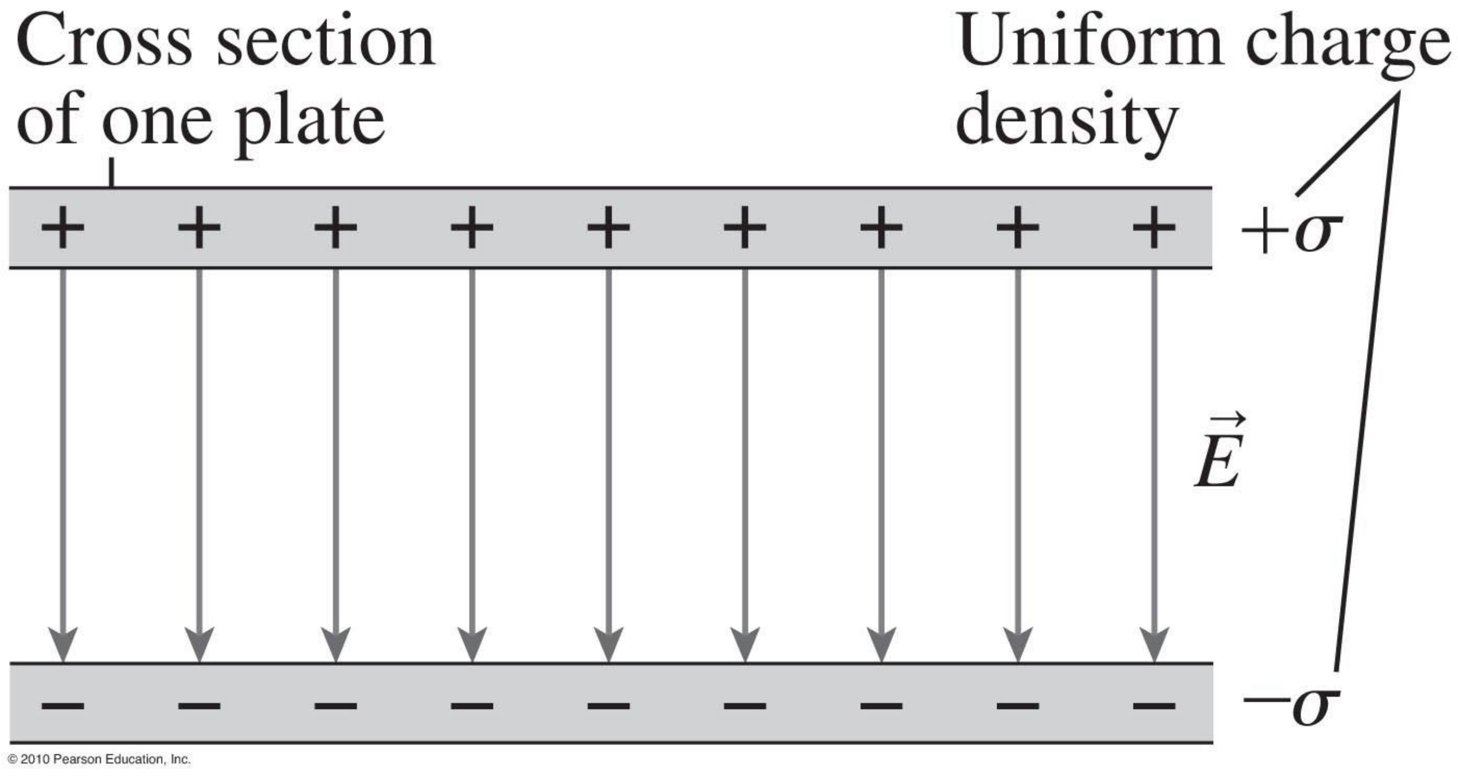




$$\vec{E}_{\mathrm{II}} = \frac{|\sigma|}{2\varepsilon_0}\hat{\imath} + \frac{|\sigma|}{2\varepsilon_0}\hat{\imath} = \frac{|\sigma|}{\varepsilon_0}\hat{\imath}$$

Electric Field between Charged Parallel Plates



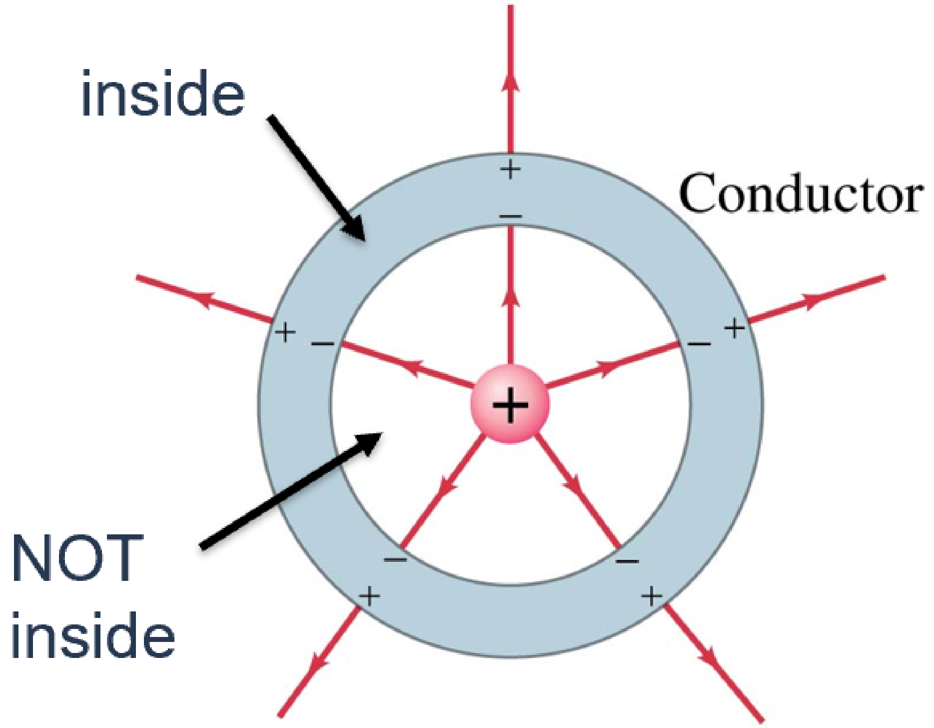
For oppositely charged parallel plates, the net Electric Field strength is:

$$|E_{\text{net}}| = E_{(+)} + E_{(-)} = \frac{s}{2e_0} + \frac{s}{2e_0} = \boxed{\frac{s}{e_0}}$$

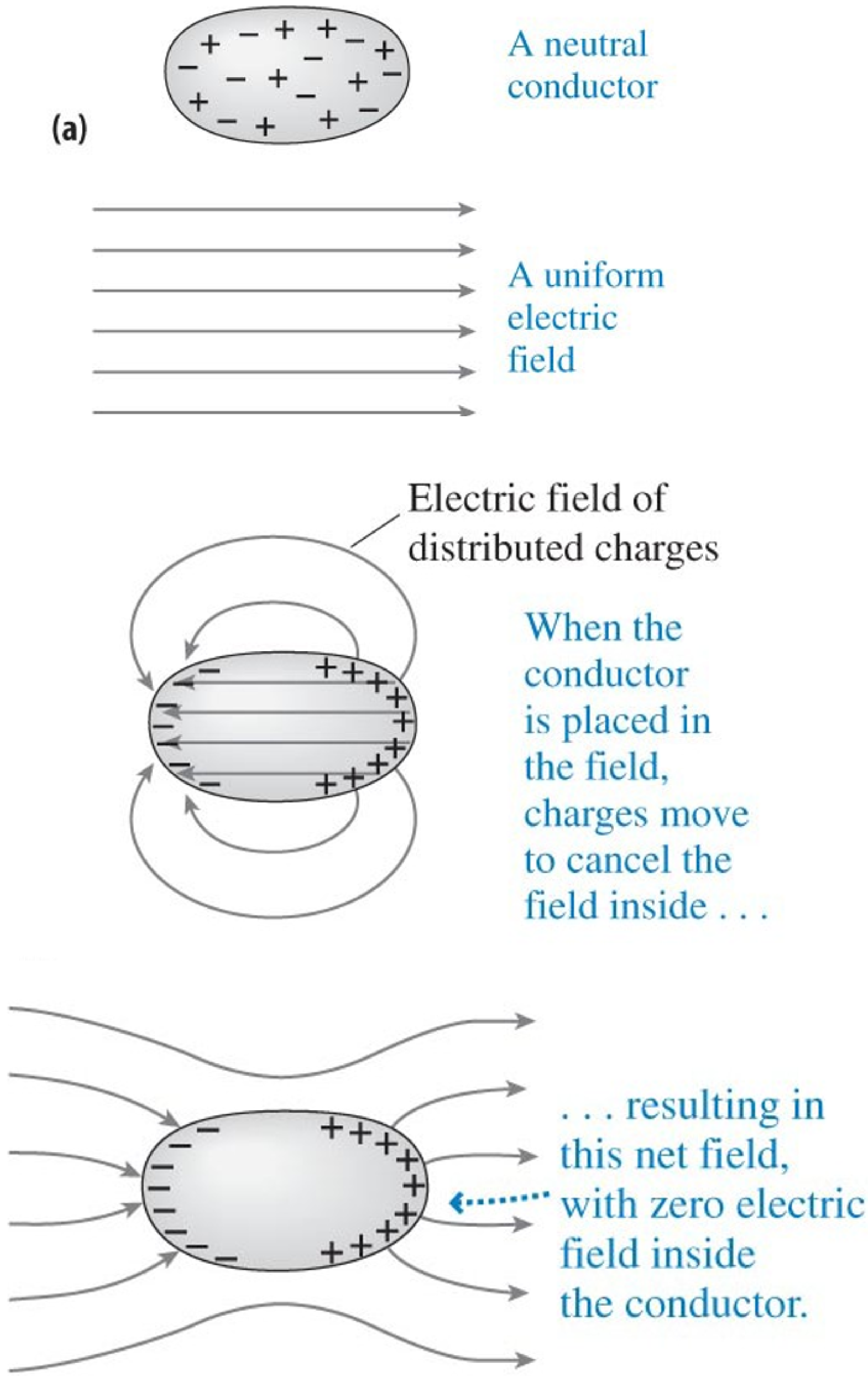
The net Electric Field strength includes contributions from each of the plates. The parallel plates work as a capacitor.

Conductors at equilibrium

When no net motion of charge occurs within a conductor, the conductor is said to be in **electrostatic equilibrium.**

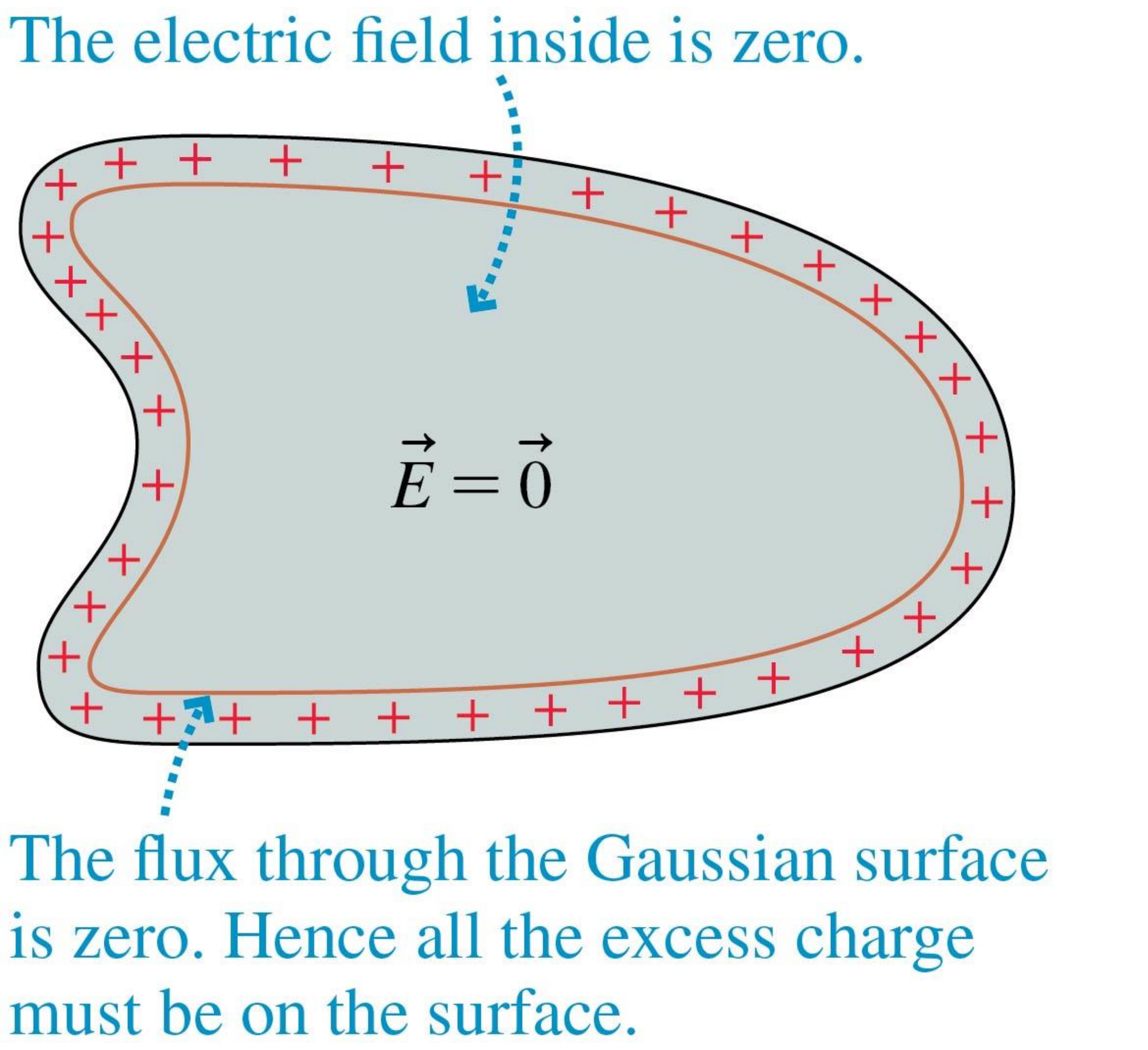


When at equilibrium, the Electric Field inside a conductor is always zero!



Conductors at equilibrium

- The figure shows a Gaussian surface just inside a conductor's surface.
- The electric field must be zero at all points within the conductor, or else the electric field would cause the charge carriers to move, and it wouldn't be in equilibrium.
- By Gauss's Law, $q_{\text{enc}} = 0$



In a conductor at equilibrium, all charge only resides on the surfaces!

Faraday Cage Demo

What happens to the object inside a faraday when place close to a strong electric field?

Will the Faraday cage be able to block the signal emitted by the radio? cell phone?

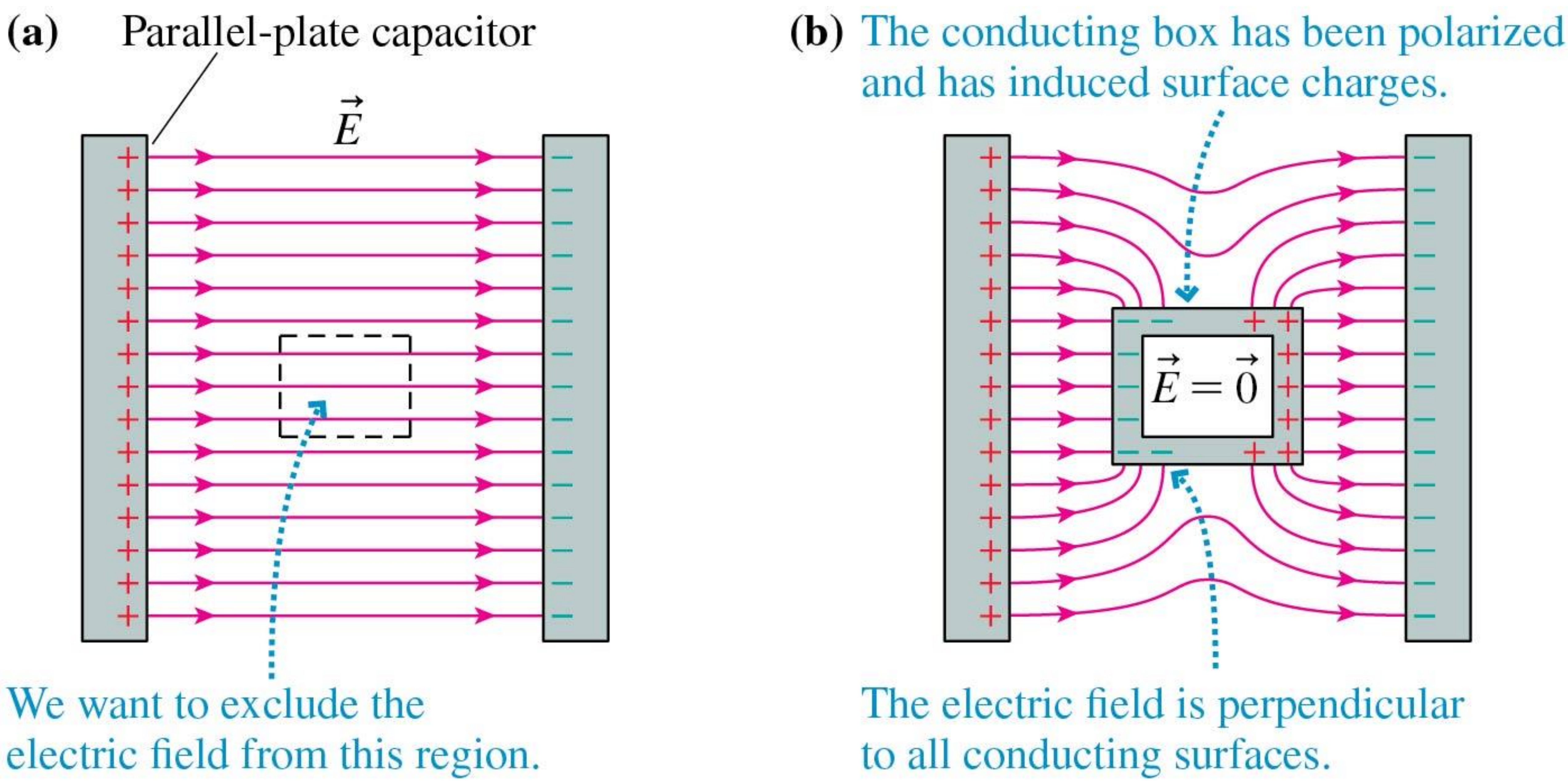
This effect explains why it’s safe to be inside a car during a lightning storm

Faraday Cage Demo

https://youtu.be/QU0fLnucE6A

Faraday Cages

The use of a conducting box, or *Faraday cage*, to exclude electric fields from a region of space is called **screening**.



TUTORIAL PROBLEMS