Physics 1502Q:

3.1 Electric Fields II
Electric Field Lines

Continued …

Announcements & Reminders

- **Clickers:**
  - We are now using these for grades.

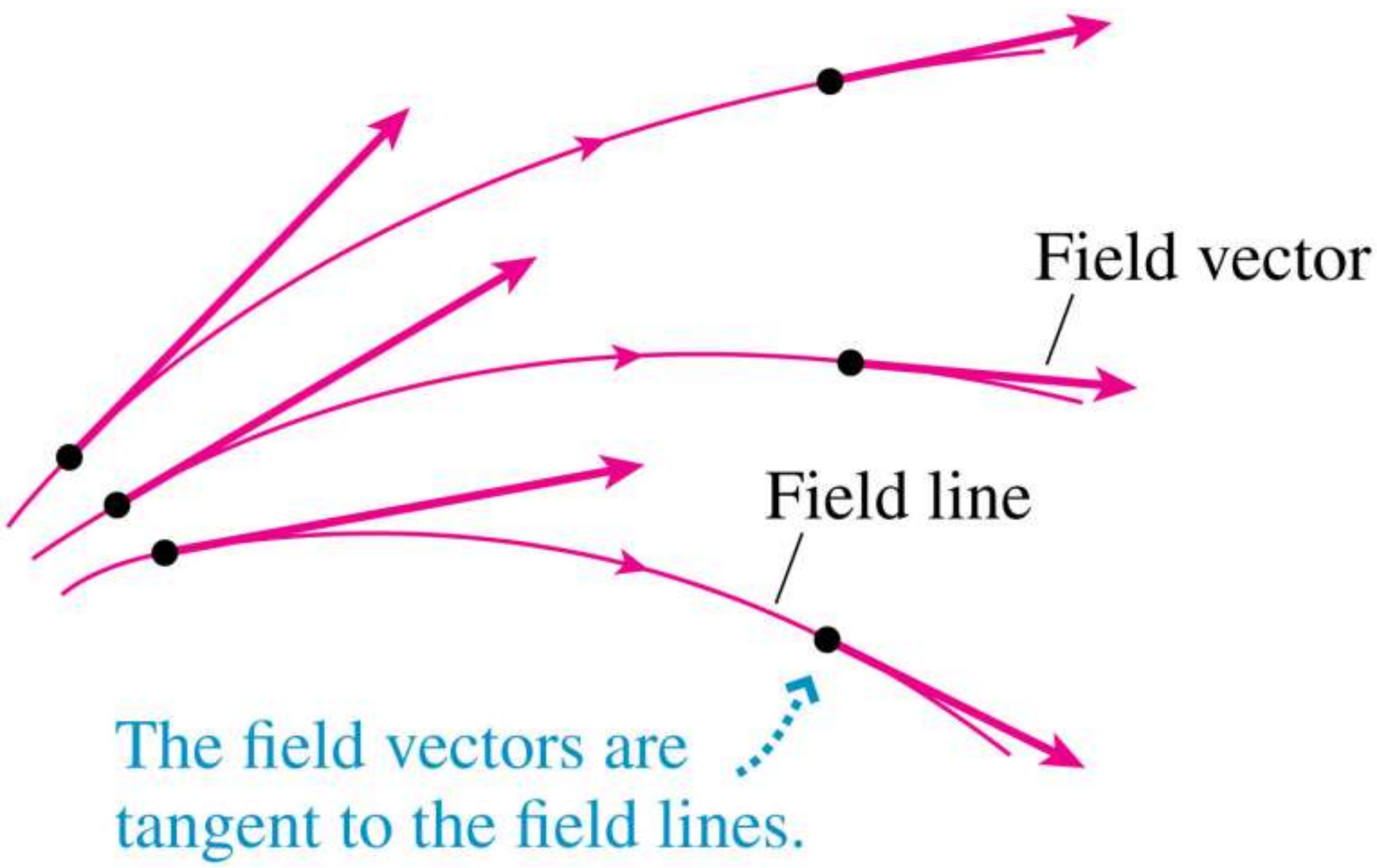
- **Lab Deadlines:**
  - Pre-lab due **at the beginning of your lab this week**
  - Reading Assignment due **Sunday at 11:59 PM**
  - Homework due **Monday at 11:59 PM**

- **Office Hours:**
  - Posted on HuskyCT

Preview of this week and next week

Su	M	T	W	Th	F	Sa
30 Reading Assignment Due 11:59 PM	31 HW Due 11:59 PM	1 E-Fields II E-Field Lines	2	3 Electric Flux Gauss's Law I Paper quiz in class	4 Lab 3: Gauss's Law Pre-lab 3 Due before lab	5 Midterm Acknowledgment Due 11:59 PM
6 Reading Assignment Due 11:59 PM	7 HW Due 11:59 PM	8 Gauss's Law II	9	10 Electric Potential Energy Electric Potential I Paper quiz in class	11 Lab 4: Electric Potential Pre-lab 4 Due before lab	12

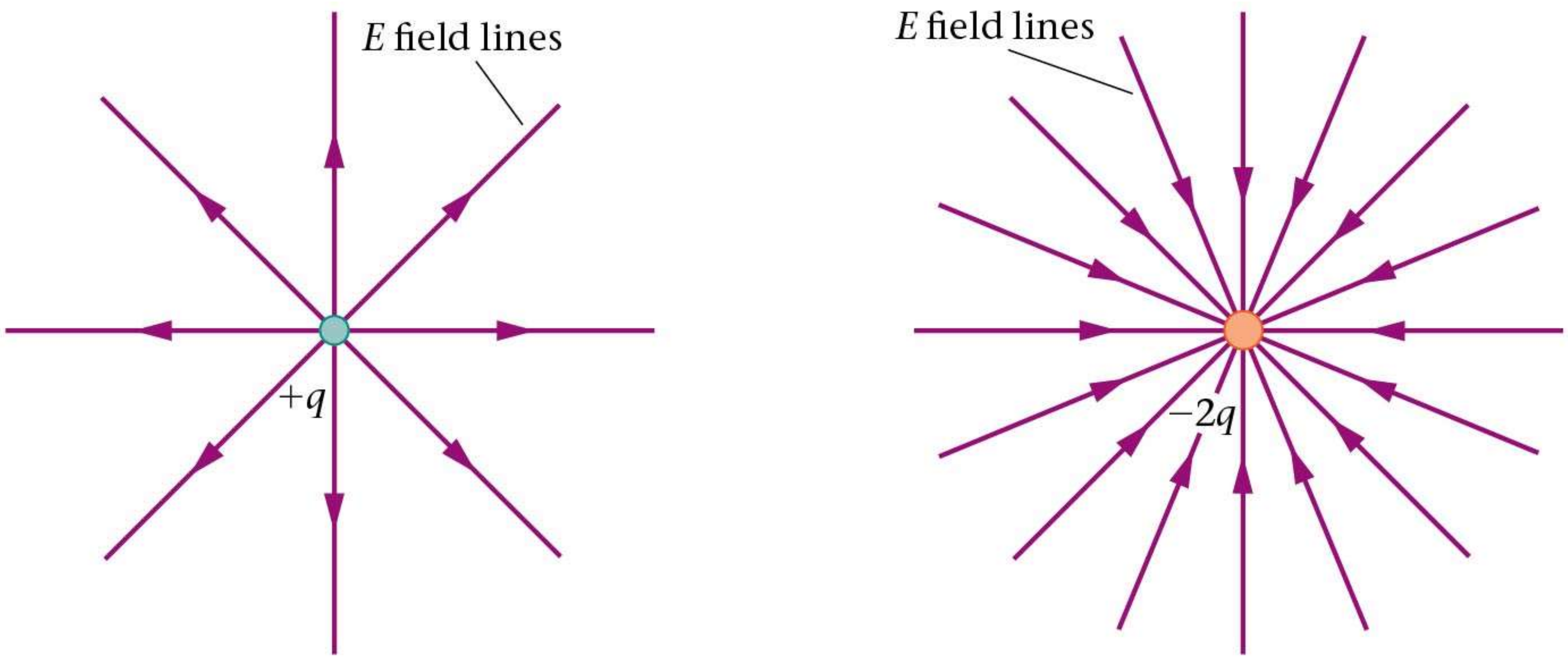
Electric Field Lines



- Electric field lines are **continuous** curves tangent to the electric field vectors.
- Closely spaced field lines indicate a greater field strength.
- Electric field lines start on positive charges and end on negative charges.
- Electric field lines never cross.

Electric Field Lines of Point Charges

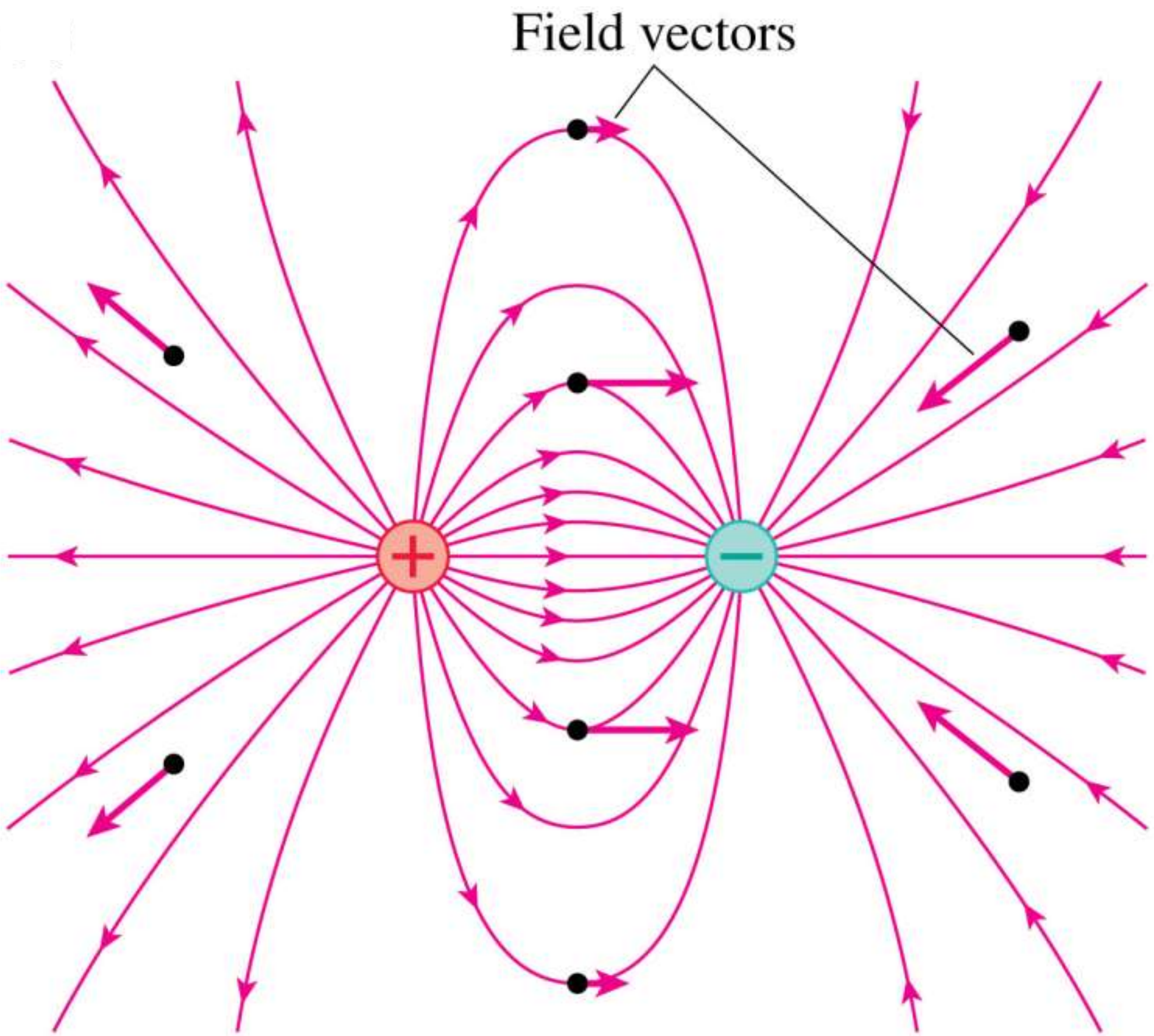
The charge on the right is twice the magnitude of the charge on the left (and opposite in sign), so there are twice as many field lines, and they point toward the charge rather than away from it.



(a) *E* field lines point away from positive charges

(b) *E* field lines point toward negative charges

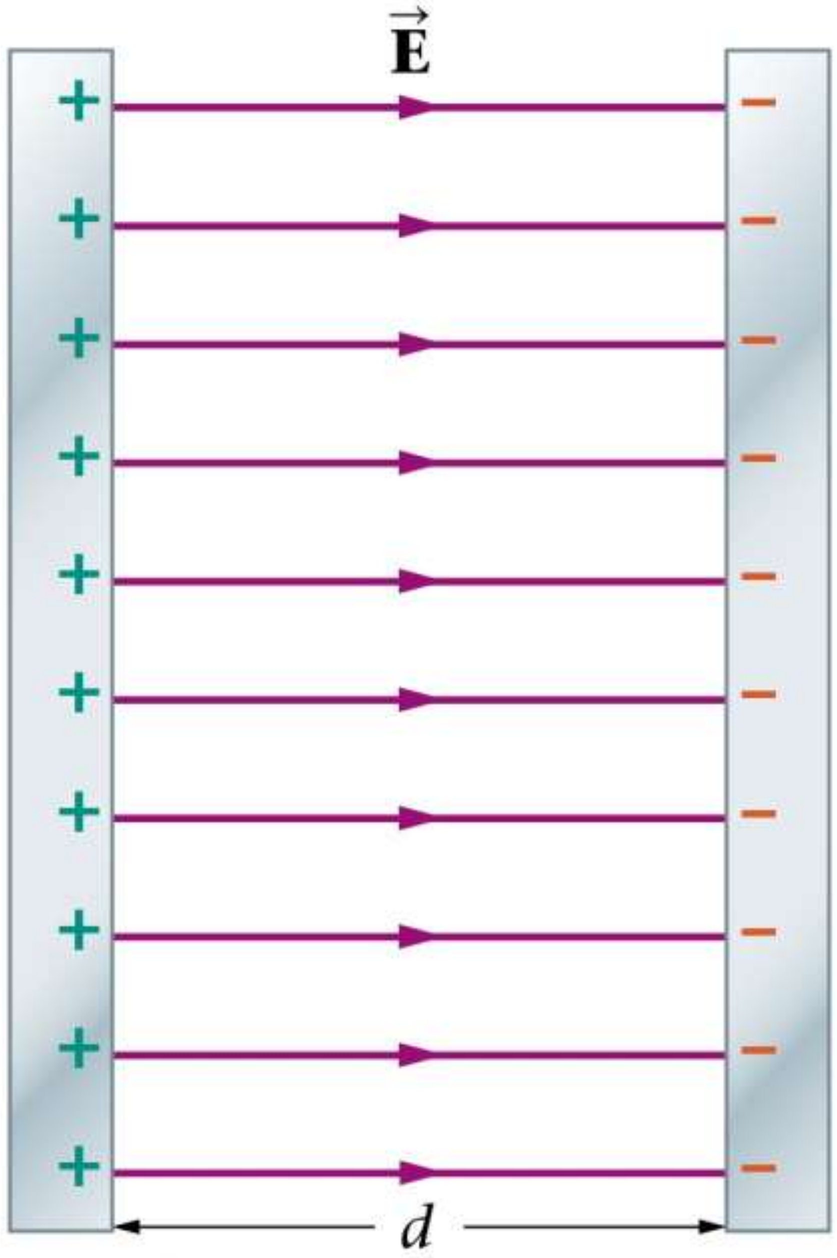
Electric Field Lines of a Dipole Configuration



This figure represents the electric field of a dipole using electric field lines.

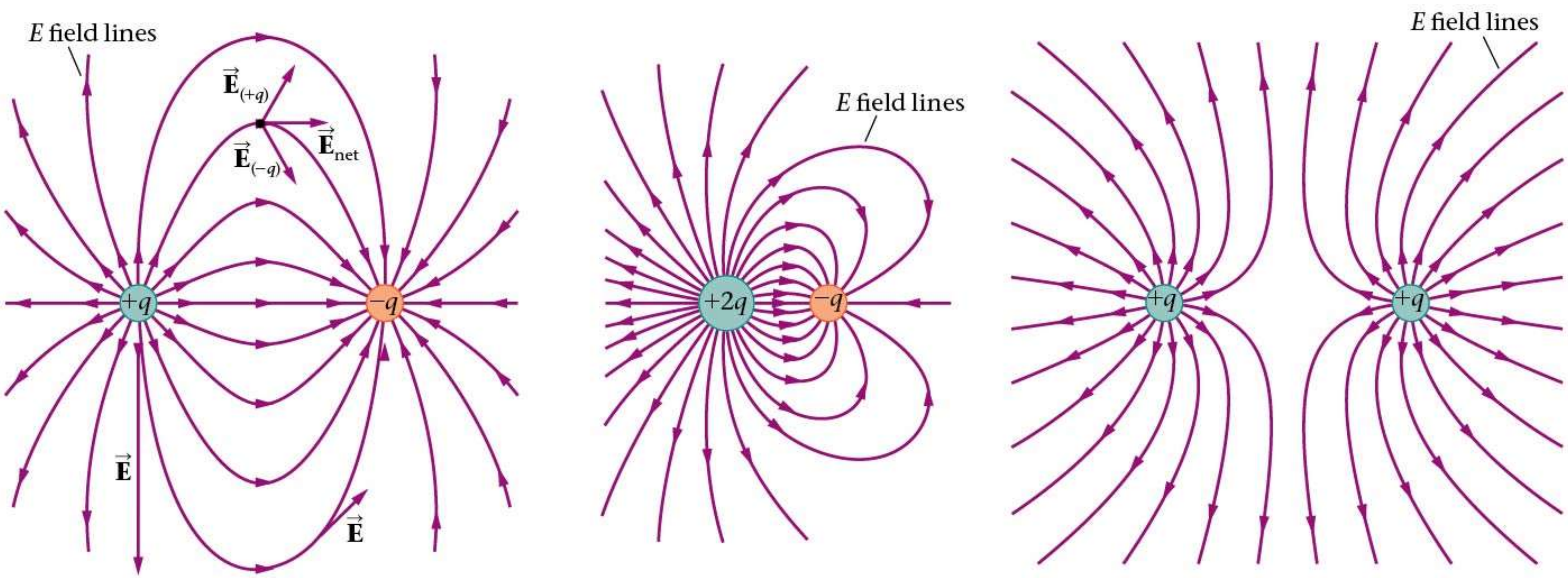
Electric Field Lines of a Parallel Plate Configuration

- The device consists of plates of positive and negative charge
- The total electric field between the plates is constant
- The field outside the plates is zero
- This configuration is commonly used in capacitors



Electric Field Lines with Multiple Charges

Combinations of charges. Note that, while the lines are less dense where the field is weaker, the field is not necessarily zero where there are no lines. In fact, there is only one point within the figures below where the field is zero—can you find it?

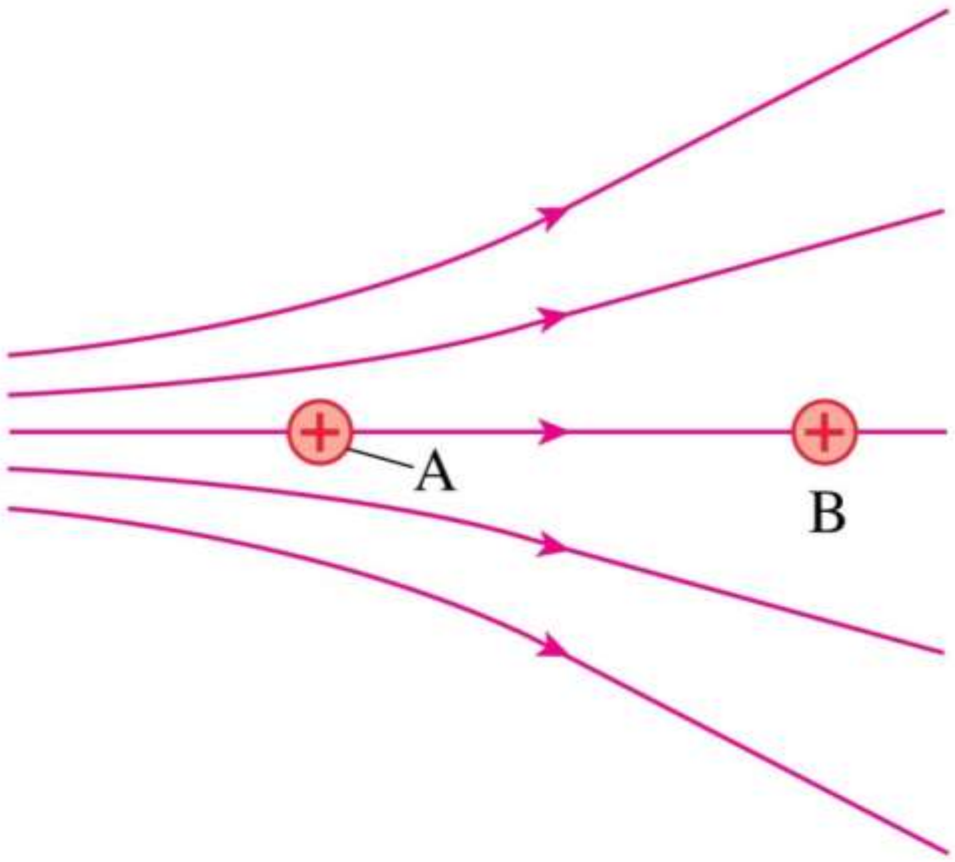


(a) (b) (c)

Dipole

Question: Electric Field Lines

Two protons, A and B, are in the electric field shown in the figure. Which proton has the larger acceleration?



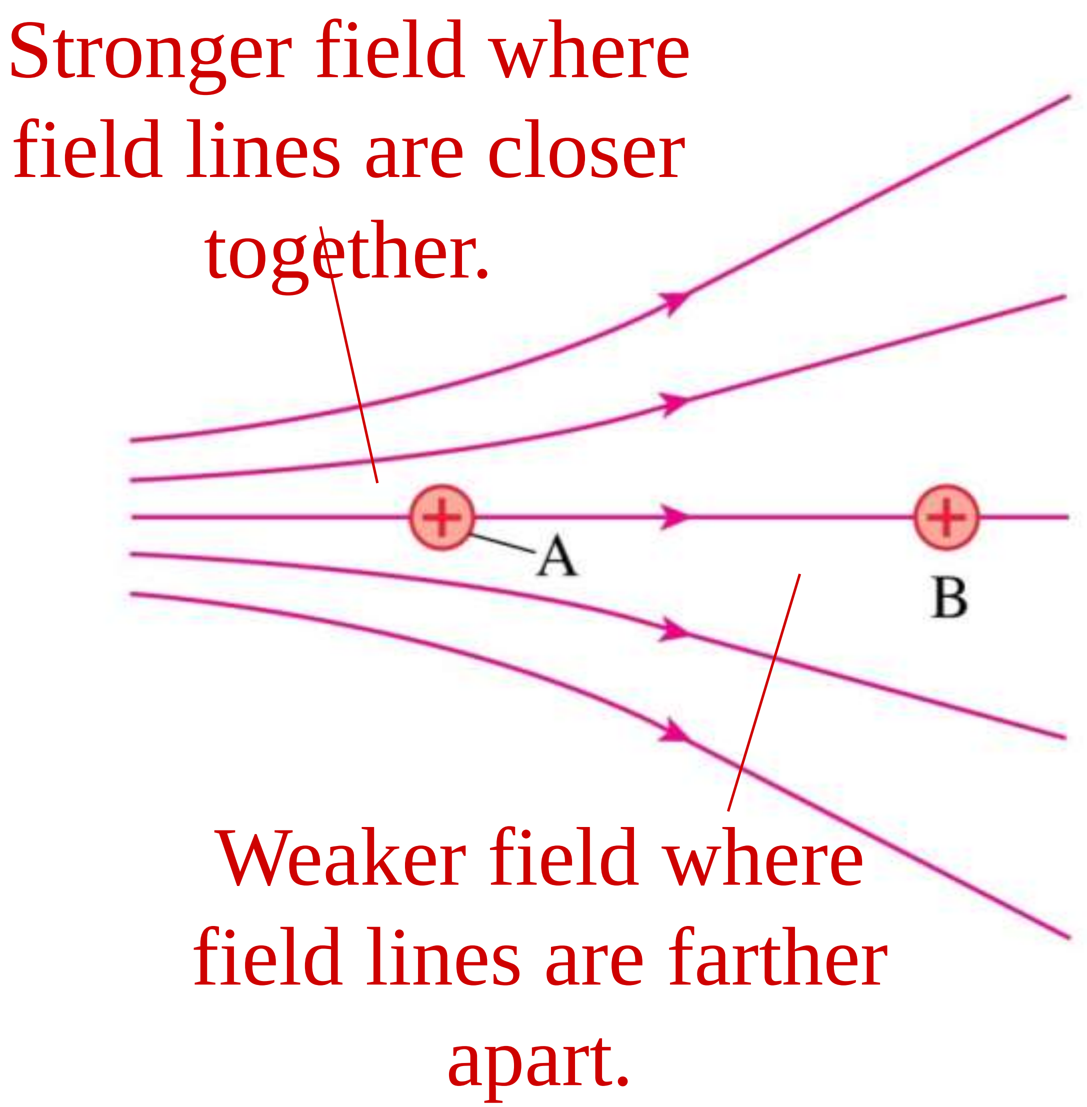
A. Proton A

B. Proton B

C. Both have the same acceleration.

Question: Electric Field Lines

Two protons, A and B, are in the electric field shown in the figure. Which proton has the larger acceleration?



A. Proton A

B. Proton B

C. Both have the same acceleration.

Physics 1502Q:

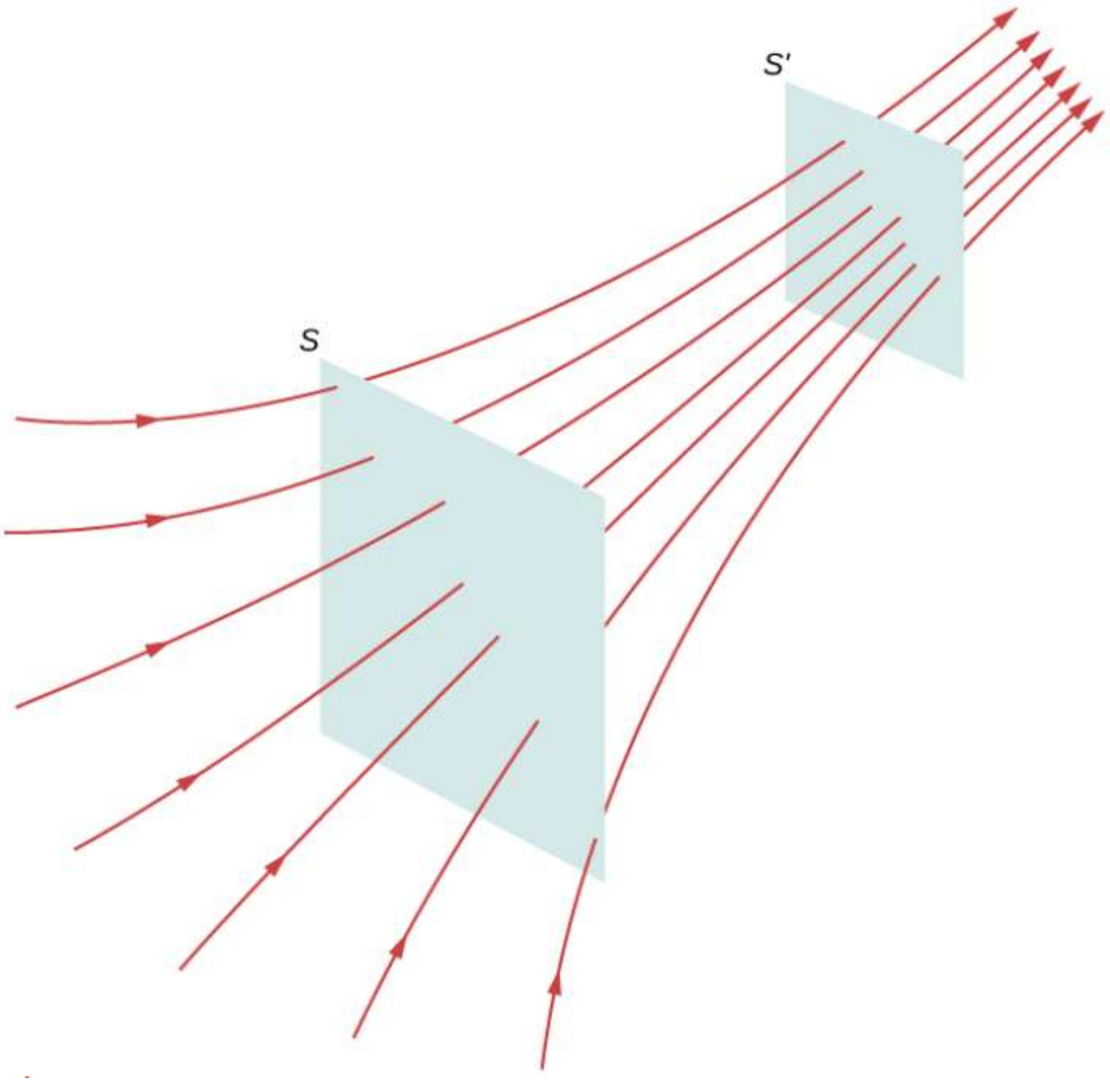
3.2 Electric Flux
Gauss’s Law I

Electric Flux

Gives us a way to quantify the electric field density

<u>Geometric Interpretation:</u>

The Electric Flux counts Electric Field lines



Electric Flux Through a Surface

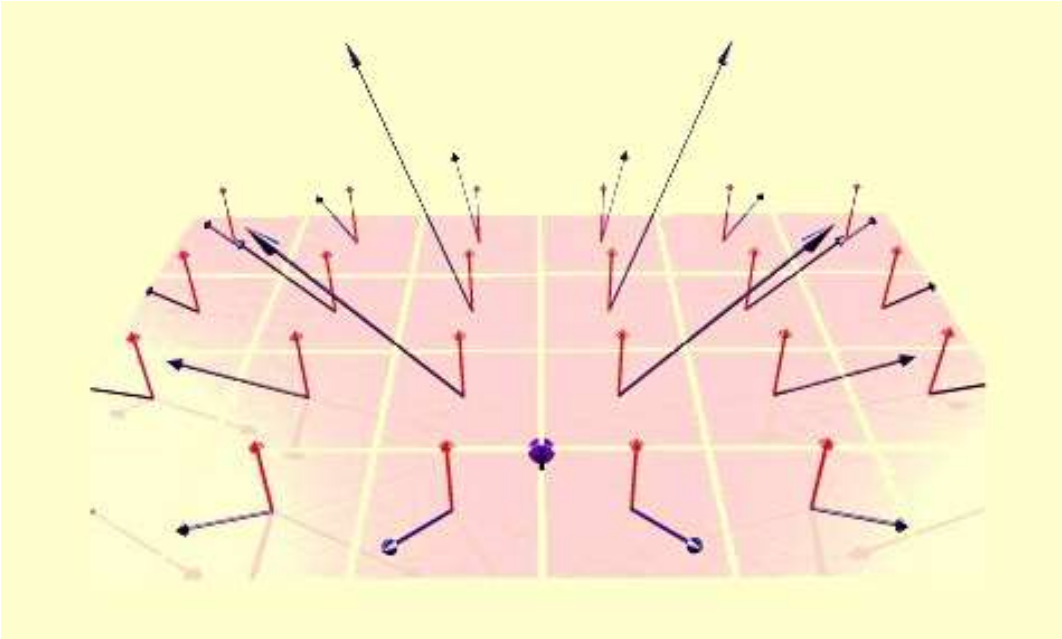
Dot product

$$\Phi_E = \int_S \vec{E} \cdot d\vec{A}$$

A small piece of area on surface *S*

Flux through surface *S*

Electric field going through the piece of area, dA



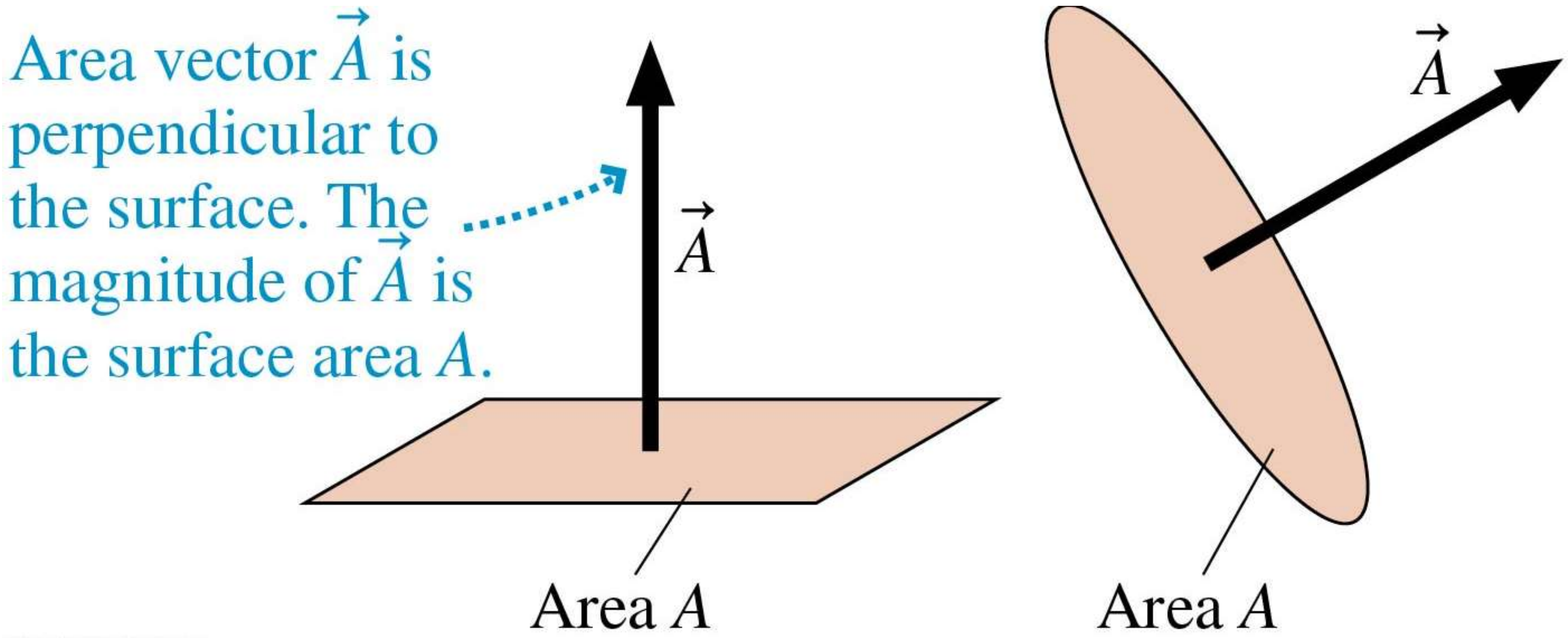
SI units for electric flux: Nm^2/C

If the Electric Field is <u>uniform</u> in a given region of space, then

$$\Phi_E = \vec{E} \cdot \int_S d\vec{A} = \vec{E} \cdot \vec{A}$$

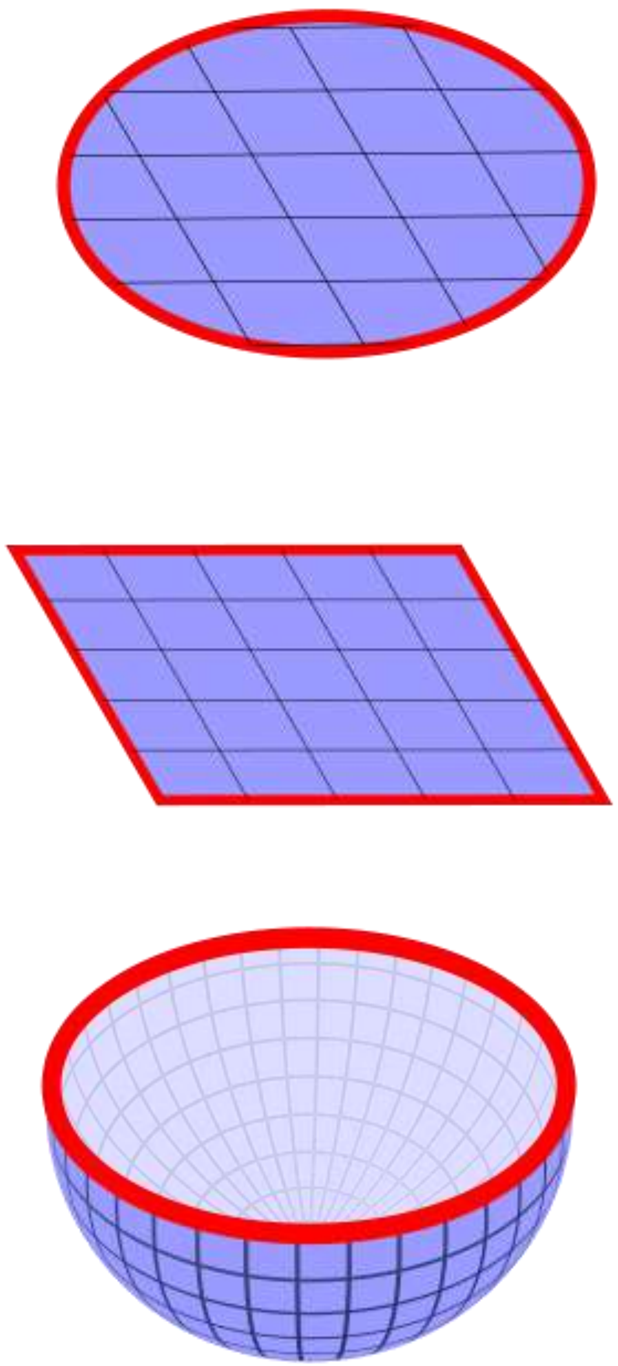
Area Vector

- Define an area vector $\vec{A} = A\hat{n}$ to be a vector in the direction of $\hat{n}$, perpendicular to the surface, with a magnitude A equal to the area of the surface.
- Vector $\vec{A}$ has units of m^2.



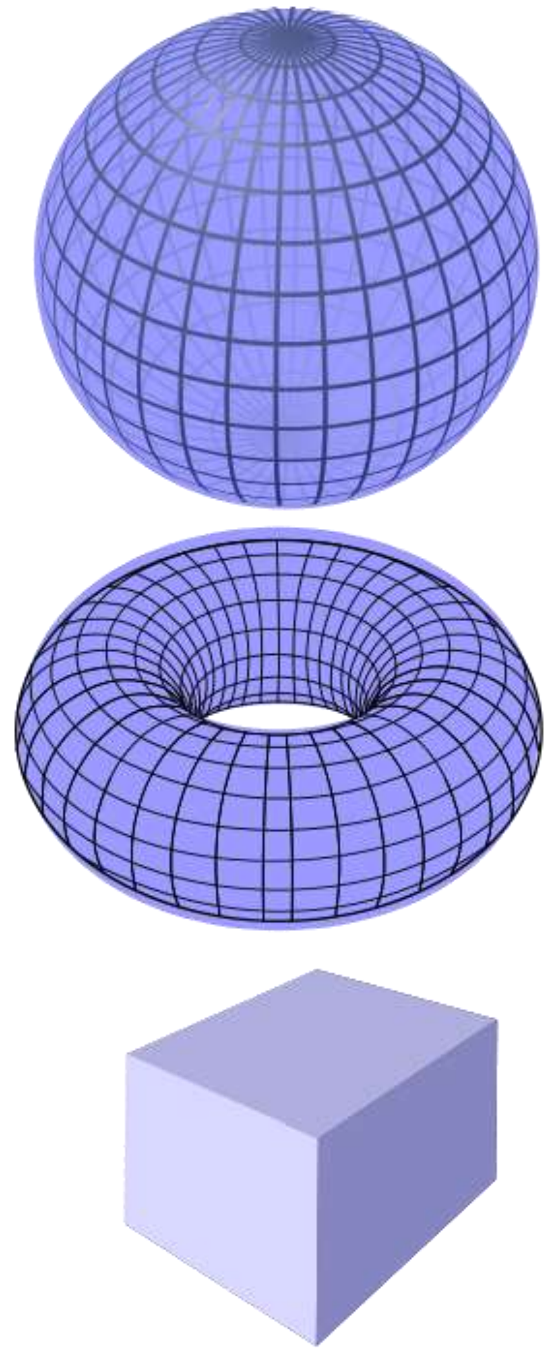
Flux: Surface vector directions…

Open surfaces



We select the direction by choosing which orthogonal direction points in $d\vec{A}$

Closed surfaces

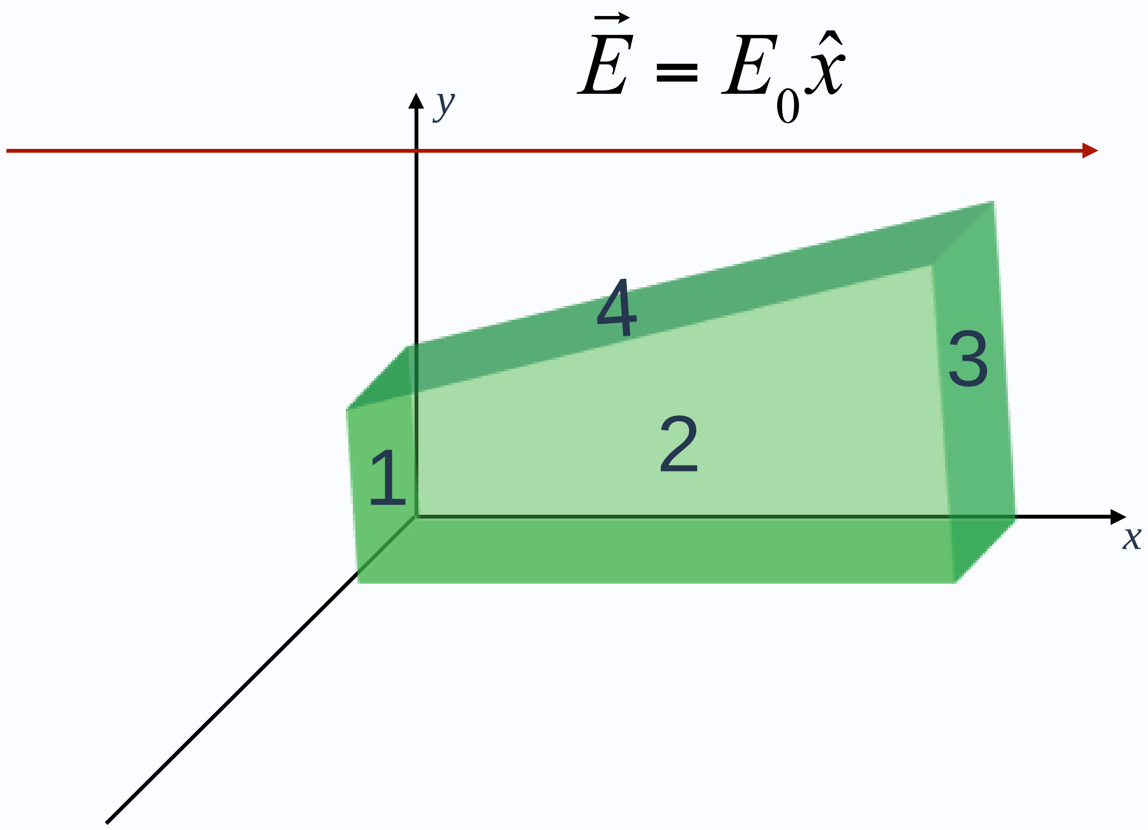


Flux magnitude & direction are defined for us

For a closed surface, $d\vec{A}$ points outward

Practice: Trapezoid in Constant E-Field

Define $\Phi_{E,n}$ = Flux through face *n.* Circle the correct answer for electric flux through each surface.



Surface 1:

A. Negative
B. Zero
C. Positive

Surface 2:

A. Negative
B. Zero
C. Positive

Surface 3:

A. Negative
B. Zero
C. Positive

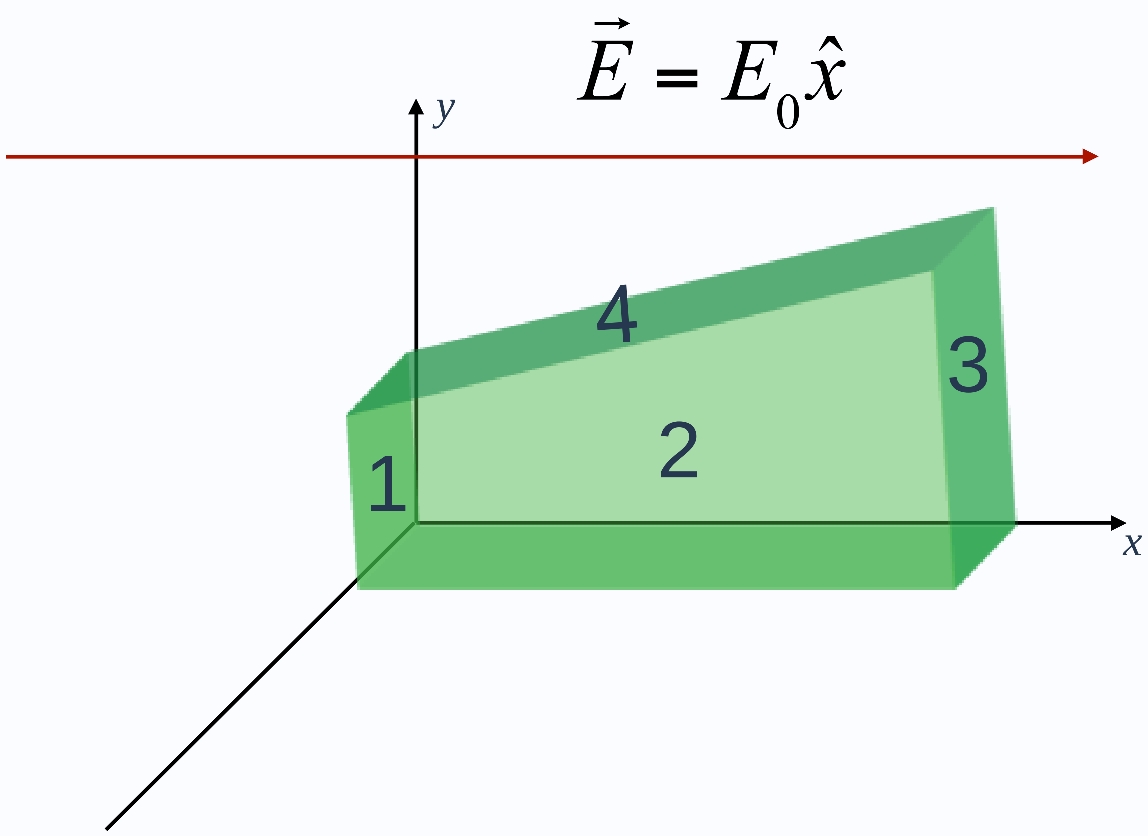
Surface 4:

A. Negative
B. Zero
C. Positive

Practice: Trapezoid in Constant E-Field

Answer

Define $\Phi_{E,n}$ = Flux through face *n.* Circle the correct answer for electric flux through each surface.



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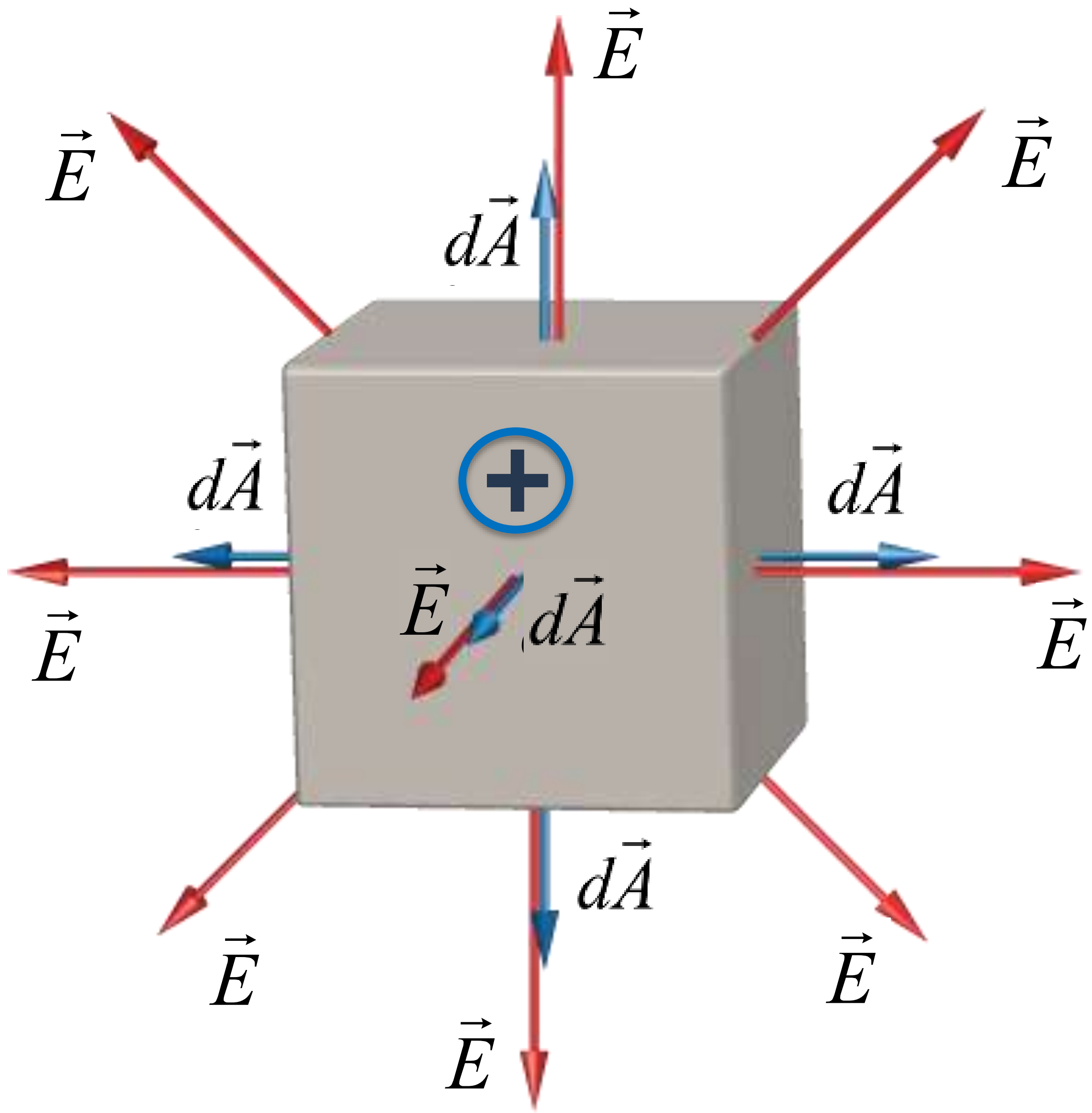
Electric Flux Through a Closed Surface

For a closed surface, $d\vec{A}$ points outward

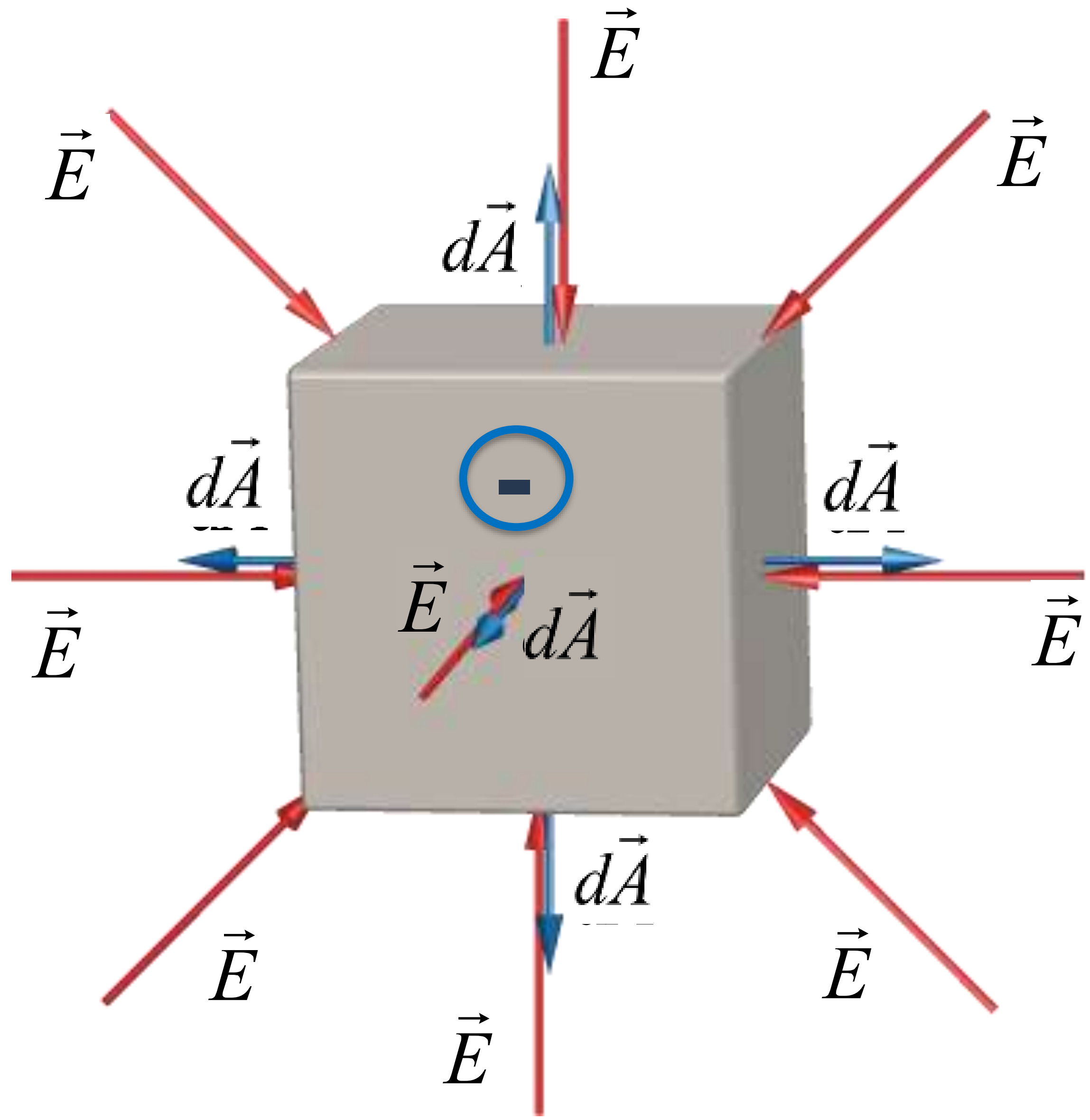


Direction matters!

$$\Phi_E = \int_S \vec{E} \cdot d\vec{A} > 0$$

Electric Flux Through a Closed Surface

For a closed surface, $d\vec{A}$ points outward

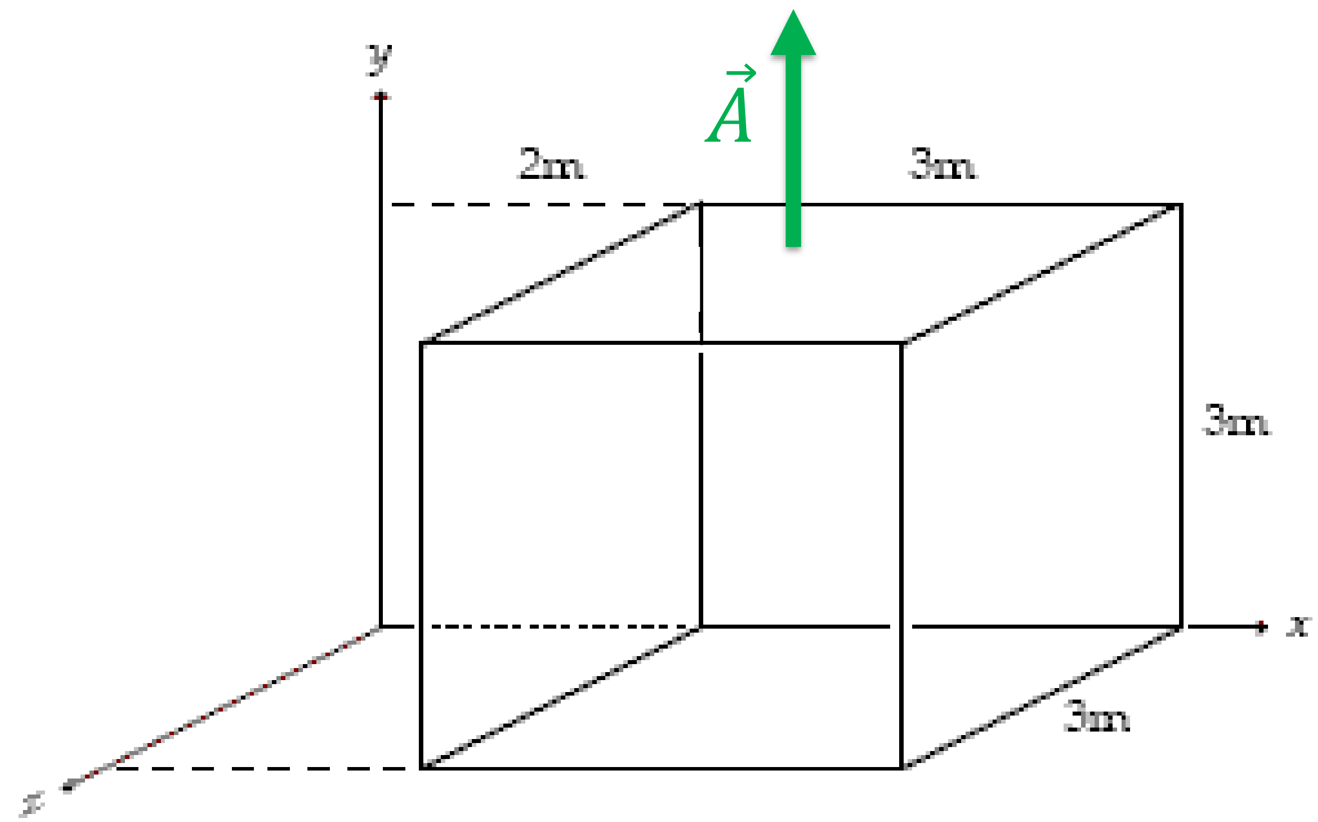


Direction matters!

$$\Phi_E = \int_S \vec{E} \cdot d\vec{A} < 0$$

Example Problem: Electric Flux

The electric field in the region of space shown is given by $\vec{E} = (6\hat{\imath} - 5\hat{\jmath})$ N/C. What is the electric flux through the top face of the cube shown?



$$\vec{A} = A\hat{\jmath} = 9\hat{\jmath}\ \mathrm{m}^2$$

$$\vec{E} = (6\hat{\imath} - 5\hat{\jmath})\ \mathrm{N/C}$$

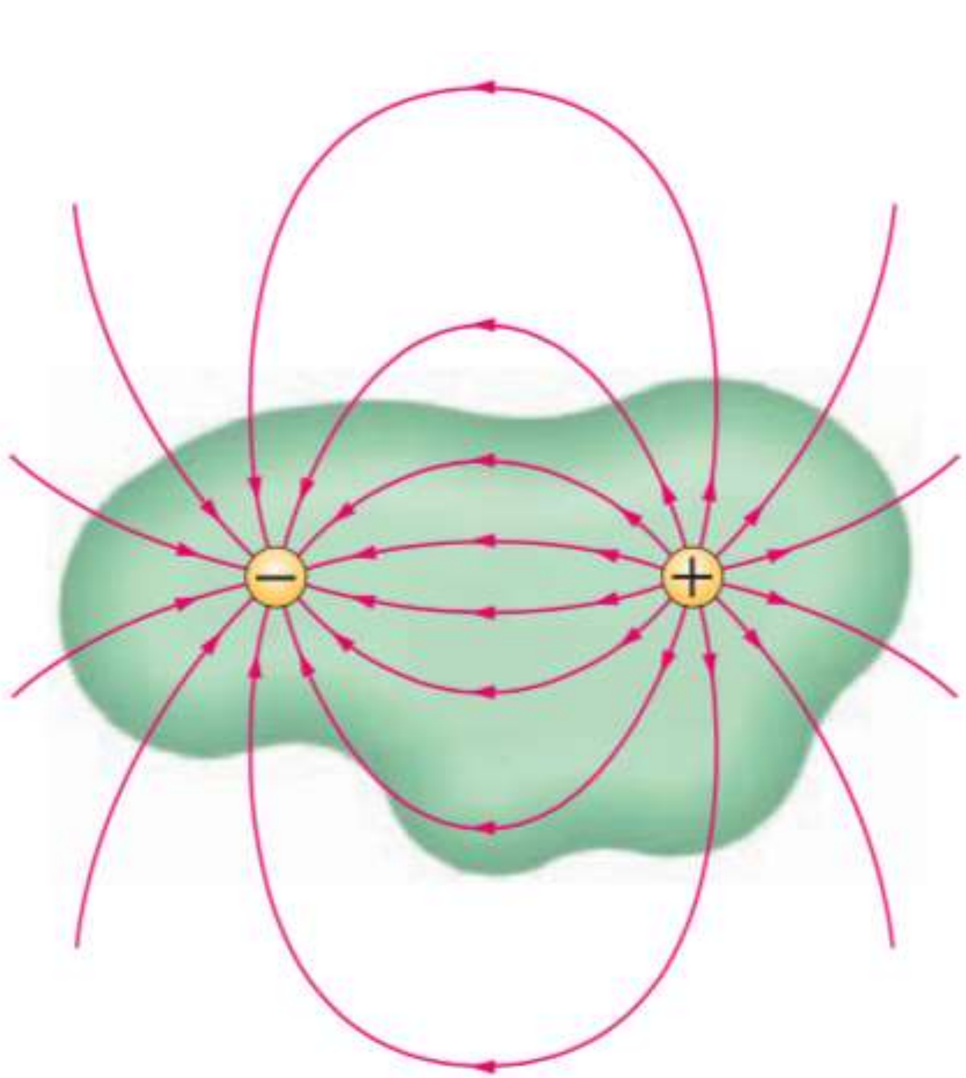
$$\Phi_E = \int \vec{E} \cdot d\vec{A} = \vec{E} \cdot \int d\vec{A} = \vec{E} \cdot \vec{A}$$

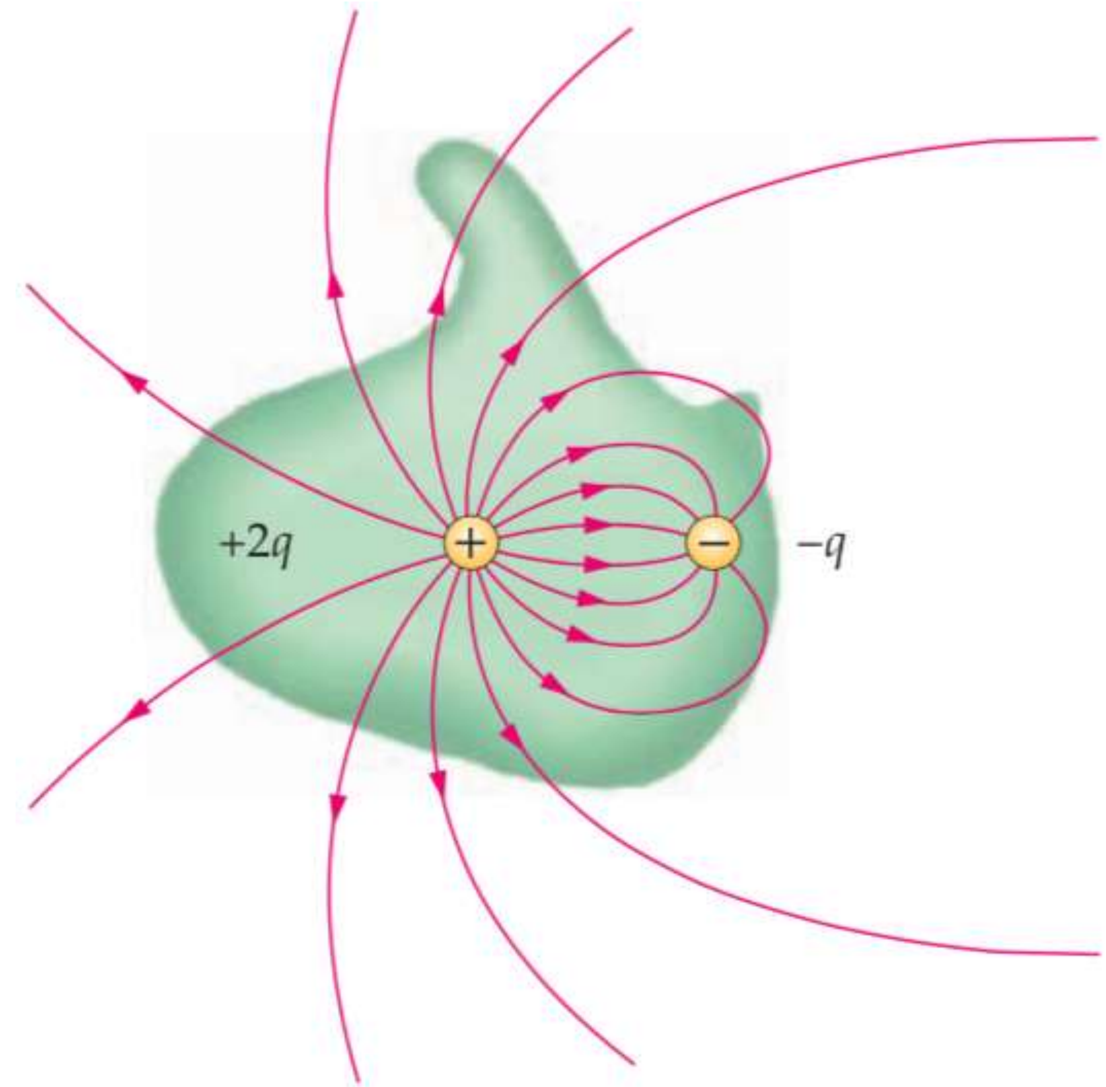
$$= (6\hat{\imath} - 5\hat{\jmath}) \cdot 9\hat{\jmath} = 54\underbrace{(\hat{\imath} \cdot \hat{\jmath})}_{=0} - 45\underbrace{(\hat{\jmath} \cdot \hat{\jmath})}_{=1}\ \ \mathrm{N \cdot m^2/C}$$

$$\boxed{\Phi_E = -45\ \mathrm{N \cdot m^2/C}}$$

Flux will always be scalar!!!

Discussion: Electric Flux Through Closed Surfaces





Φ_E =? = 0

Net charge inside? = 0

Φ_E =? > 0

Net charge inside? +q, >0

- The net number of lines out of any surface enclosing the charges is proportional to the net charge enclosed by the surface.

Electric Flux Activity Discussion

When lines out = lines in $\Phi_{net} = 0$

Net charge inside the surface: $Q_{net} = 0$

When lines out > lines in $\Phi_{net} > 0$

Net charge inside the surface: $Q_{net} > 0$

When lines out < lines in $\Phi_{net} < 0$

Net charge inside the surface: $Q_{net} < 0$

Main Lesson:

The net flux (number of electric lines) for any closed surface is proportional to the net charge enclosed by the surface.

Electric Flux Through A Closed Surface

General rule: There is **no net flux** through a surface from charges outside that surface.

The net flux through a surface will only be nonzero if the surface encloses a charge.

Gauss's Law

"closed loop integral" tells you to integrate over the entire surface

$$\Phi_E = \underbrace{\oint \vec{E} \cdot d\vec{A}}_{\text{Net flux through a closed surface}} = \frac{q_{enc}}{\varepsilon_0}$$

total charge enclosed by the surface

Net flux through a closed surface

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 / N.m^2$
"permittivity constant"

Gauss's law is always true, but it is only useful for finding the electric Field in situations with a high degree of symmetry.

Using Gauss's Law to Find E

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

If the electric field is parallel to the area vector,

$$\oint \vec{E} \cdot d\vec{A} = \oint E dA \cos 0^\circ = \oint E dA = \frac{q_{enc}}{\varepsilon_0}$$

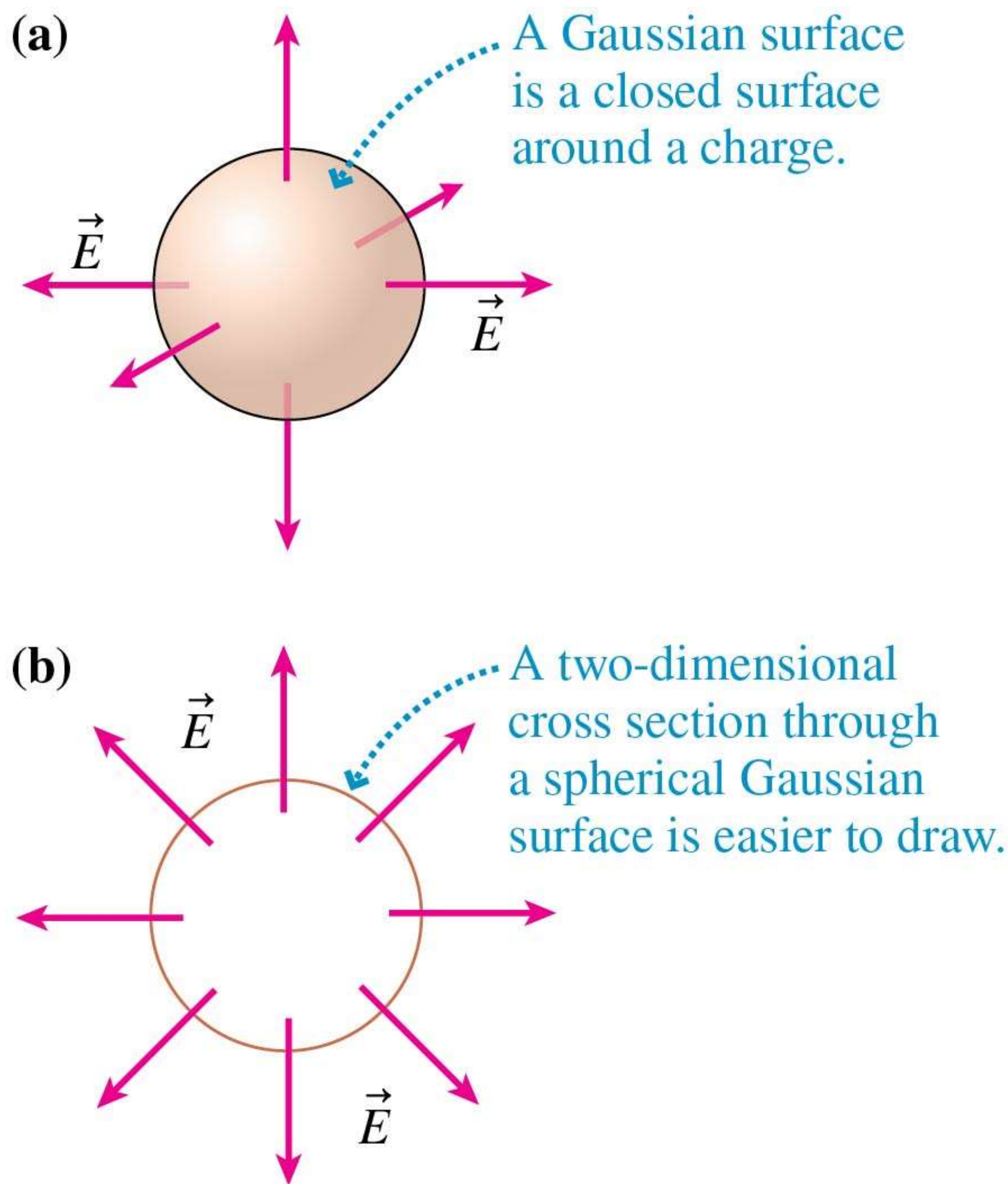
If the electric field is constant everywhere along the surface,

$$E \oint dA = EA = \frac{q_{enc}}{\varepsilon_0}$$

$$\boxed{E = \frac{q_{enc}}{A\varepsilon_0}}$$

So, if we can figure out q_{enc} and the area of the Gaussian surface A, then we know E!

Gaussian Surfaces



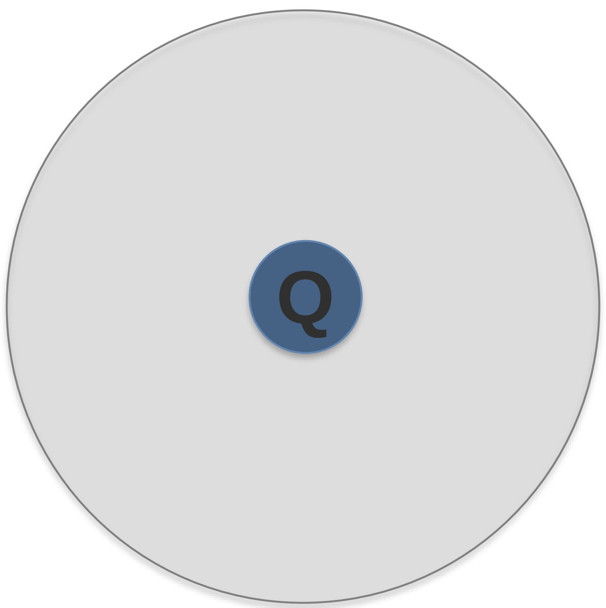
- In order to find E from Gauss's Law, we must create a **Gaussian surface** around the charges and apply Gauss's law to the surface.
- <u>A Gaussian surface is not a physical surface.</u> It need not coincide with the boundary of any physical object (although it could if we wished). It is an imaginary, mathematical surface in the space surrounding one or more charges.
- <u>A Gaussian must be a closed surface</u>
  - The electric field is the same everywhere along all regions of the surface
  - The electric field is parallel to the Area vector at all points along the surface.

Discussion: Determining Gaussian Surfaces

Find the Gaussian surface that would be best for each charge shape.

Hint: Draw the E field created by the charge
- The Gaussian surface must be closed around the charge
- The Gaussian surface must always be parallel to the Area vector
- The electric field must be constant on all parts of the surface.

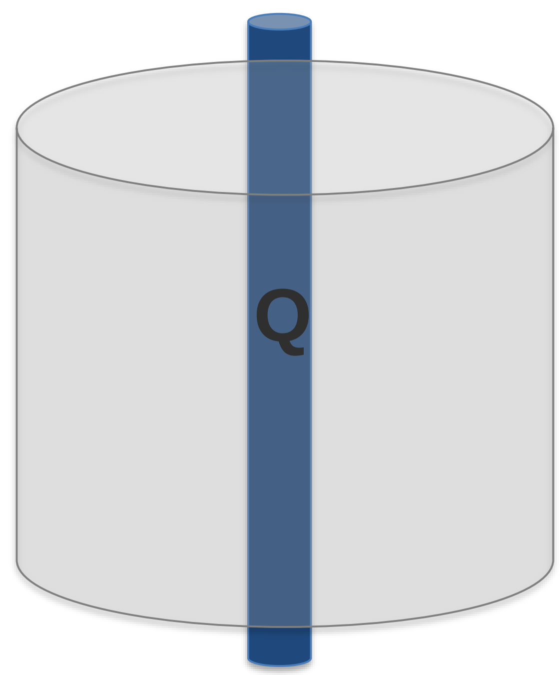
Sphere of charge



Gaussian sphere

$$A = 4\pi r^2$$

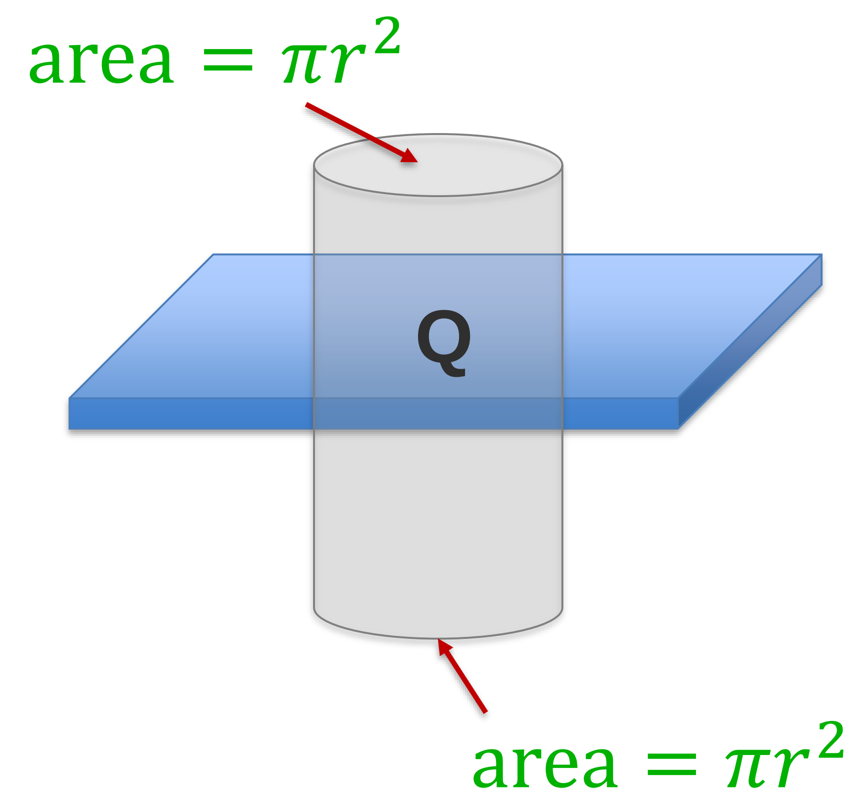
Line/cylinder of charge



Gaussian cylinder

$$A = 2\pi r \mathrm{L}$$

Sheet of charge



Gaussian cylinder

$$A = 2\pi r^2$$

Steps for Using Gauss's Law for Solving for Electric Field

1. Determine the symmetry of the problem (spherical, cylindrical, planar).
    - This will determine the shape of the Gaussian surface and equation for the area.
        - Spherical symmetry $\longrightarrow$ sphere ($A = 4\pi r^2$)
        - Cylindrical symmetry $\longrightarrow$ cylinder ($A = 2\pi r L$)
        - Planar symmetry $\longrightarrow$ cylinder or box (A = top and bottom)

2. Draw the Gaussian surface at the point you want to calculate the E field.
    - Make sure your surface is centered around the charge.

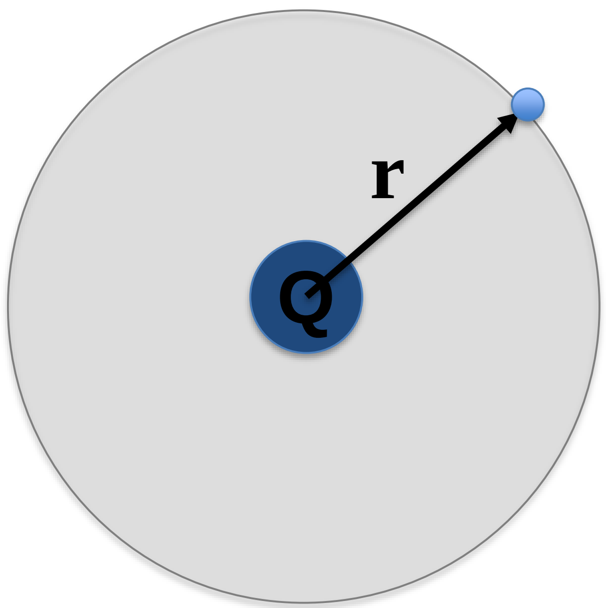
3. Write down Gauss's Law and then simplify.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \Rightarrow E = \frac{q_{enc}}{A\varepsilon_0}$$

4. Determine the charge enclosed by your Gaussian surface.

Gauss's Law Example: Sphere of Charge

Find the electric field at a radius r away from a sphere of charge $+Q$.



Spherical Gaussian surface

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = \oint E dA \cos 0^\circ = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$E \oint dA = EA = \frac{q_{\text{enc}}}{\varepsilon_0}$$

For a spherical surface around a charge $+Q$, we know that $A = 4\pi r^2$

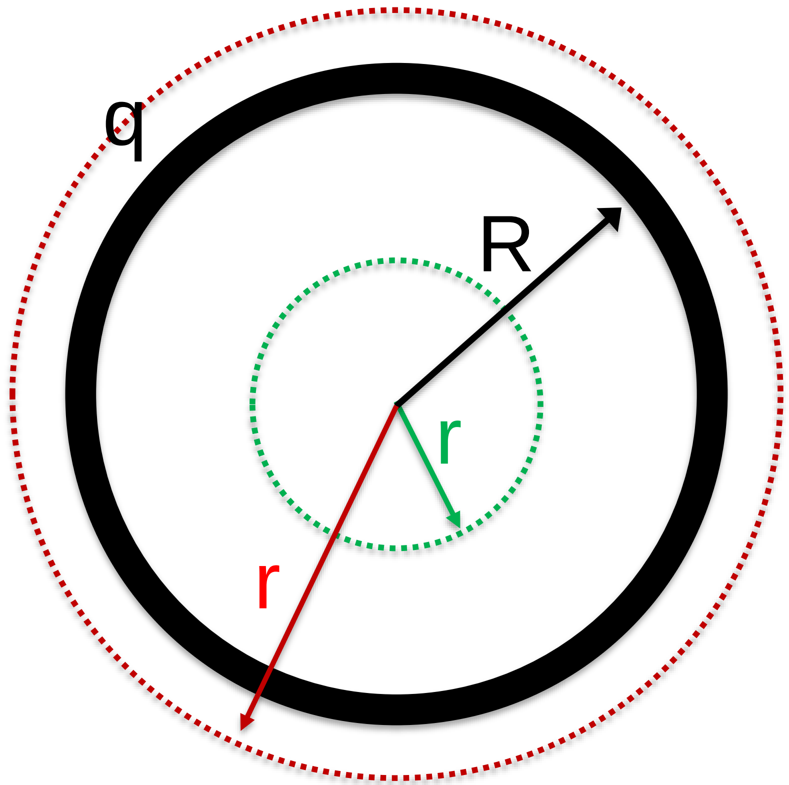
$$E(4\pi r^2) = +\frac{Q}{\varepsilon_0}$$

$$\boxed{E = +\frac{kQ}{r^2} = +\frac{Q}{4\pi\varepsilon_0 r^2}}$$

<u>Note:</u> The equation is the same as for a point charge, even though this is NOT a point charge!!

Gauss's Law Example: Hollow Sphere

A thin, uniformly charged, spherical shell of radius R with total charge q. Find the electric field inside and outside the shell.



For inside the shell, $r < R$ draw a Gaussian surface, and apply Gauss's Law to the surface.

$$q_{enc} = 0 \qquad \oint \vec{E} \cdot d\vec{A} = EA = 0$$

$$\boxed{E = 0}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

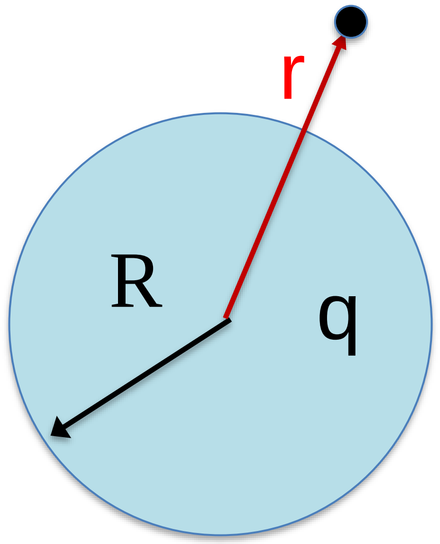
For outside the shell, $r > R$ draw a Gaussian surface, and apply Gauss's Law to the surface.

$$\oint \vec{E} \cdot d\vec{A} = EA = \frac{q_{enc}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.

Spherical Symmetry



Any charged object that has spherical symmetry will create an electric field <u>outside</u> the object, with the form

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{enc}}{r^2}$$

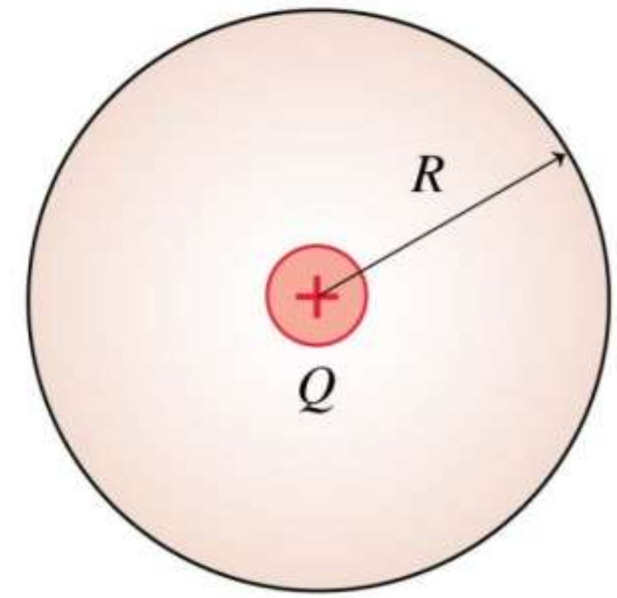
where r is measured from the center.

Question: Spherical Gaussian Surface

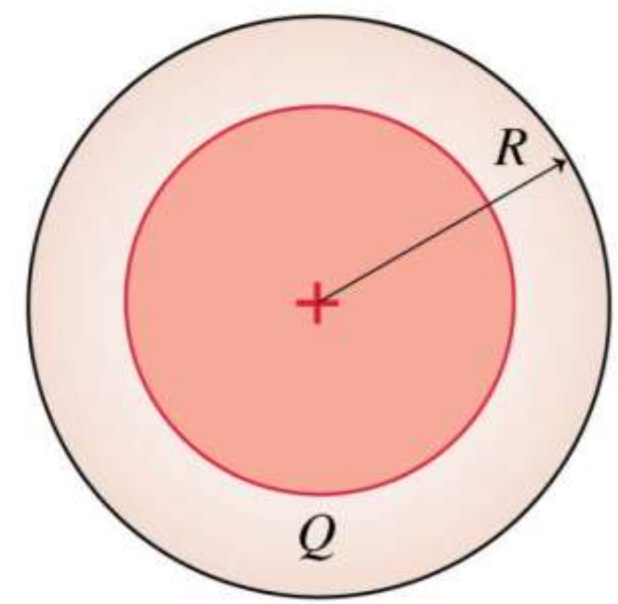
Spherical Gaussian surfaces of equal radius R surround two spheres of equal charge Q. Which Gaussian surface has the larger electric field?

A. Surface A
B. Surface B
C. They have the same electric field.
D. Not enough information to tell.

A.



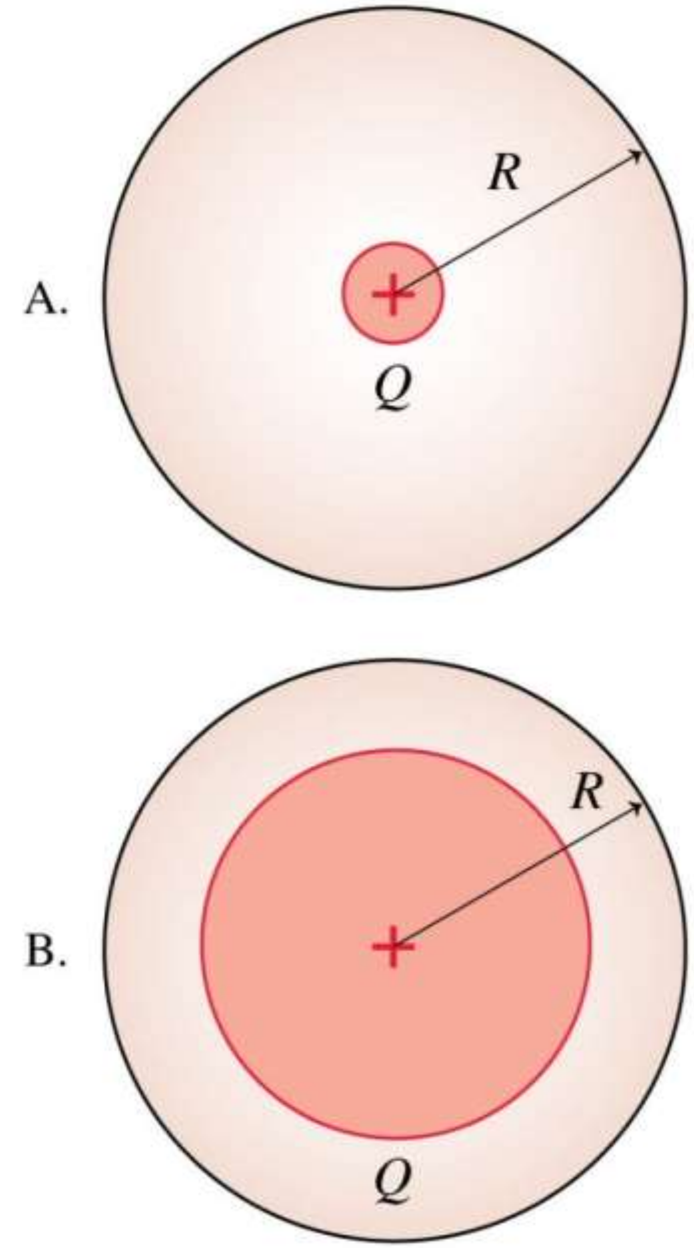
B.



Question: Spherical Gaussian Surface Answer

Spherical Gaussian surfaces of equal radius R surround two spheres of equal charge Q. Which Gaussian surface has the larger electric field?

A. Surface A
B. Surface B
C. $\boxed{\text{They have the same electric field.}}$
D. Not enough information to tell.



TUTORIAL PROBLEMS