Physics 1502Q:
6.1 Capacitors

Capacitors

LEARNING GOALS

By the end of this unit, you should be able to:

- Identify parallel and series capacitive networks
- Calculate equivalent capacitances for capacitive networks
- Calculate the capacitance of a capacitor that is partially filled with a dielectric.

Capacitance (Recap)

- Capacitance is defined for any pair of spatially separated conductors

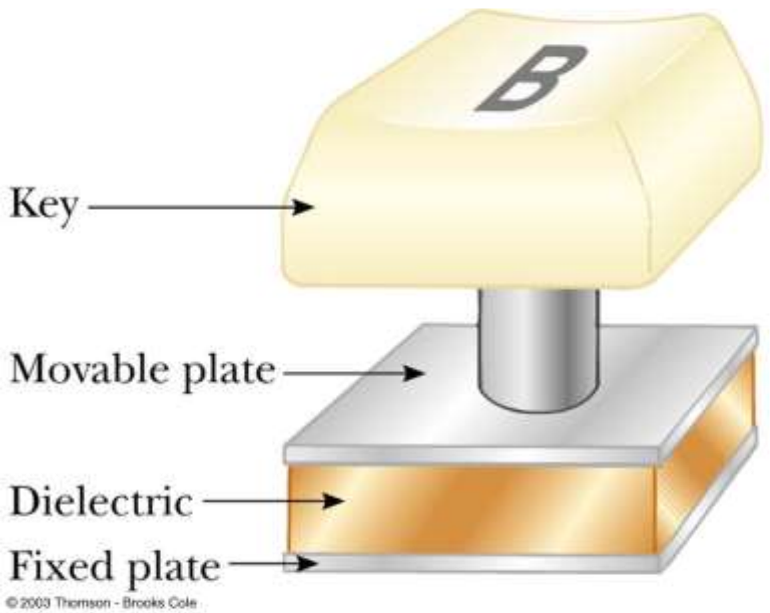
$$C \equiv \frac{Q}{\Delta V}$$

- SI units (Farad): [C/V] = [F]

- Capacitance only depends on the geometry of the conductors

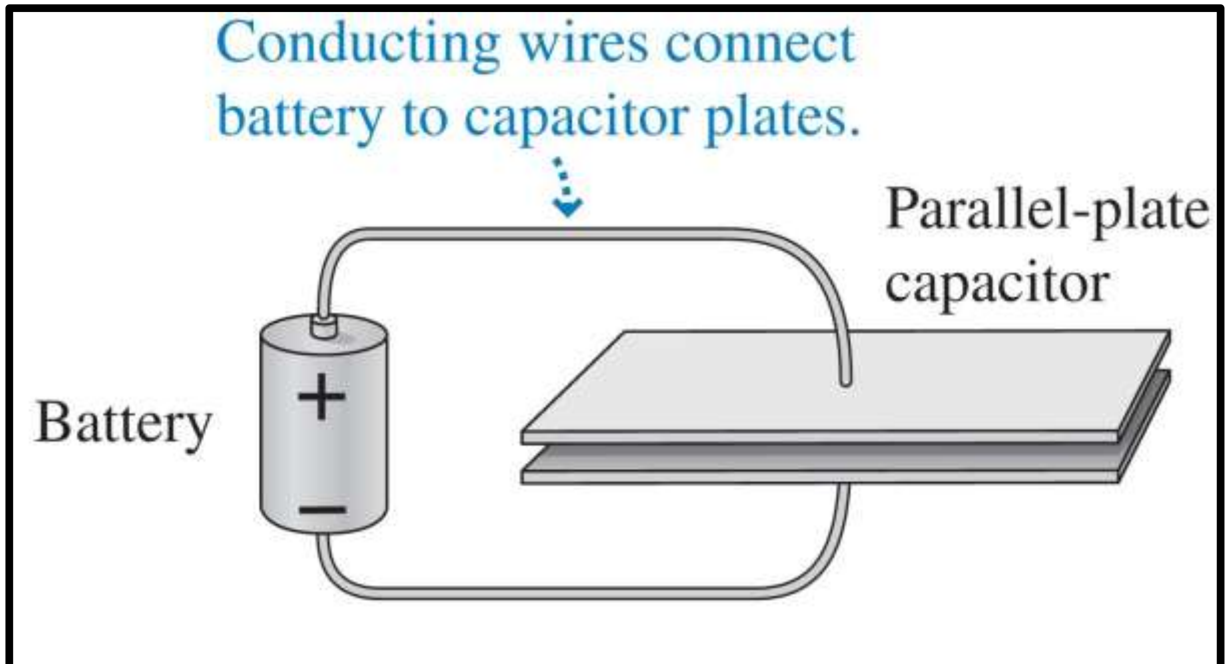
Applications of Capacitors – Keyboards and Touchscreens

- Computers use capacitors in many ways
  - Some keyboards use capacitors at the bases of the keys
  - When the key is pressed, the capacitor spacing decreases and the capacitance increases
  - The key is recognized by the change in capacitance.

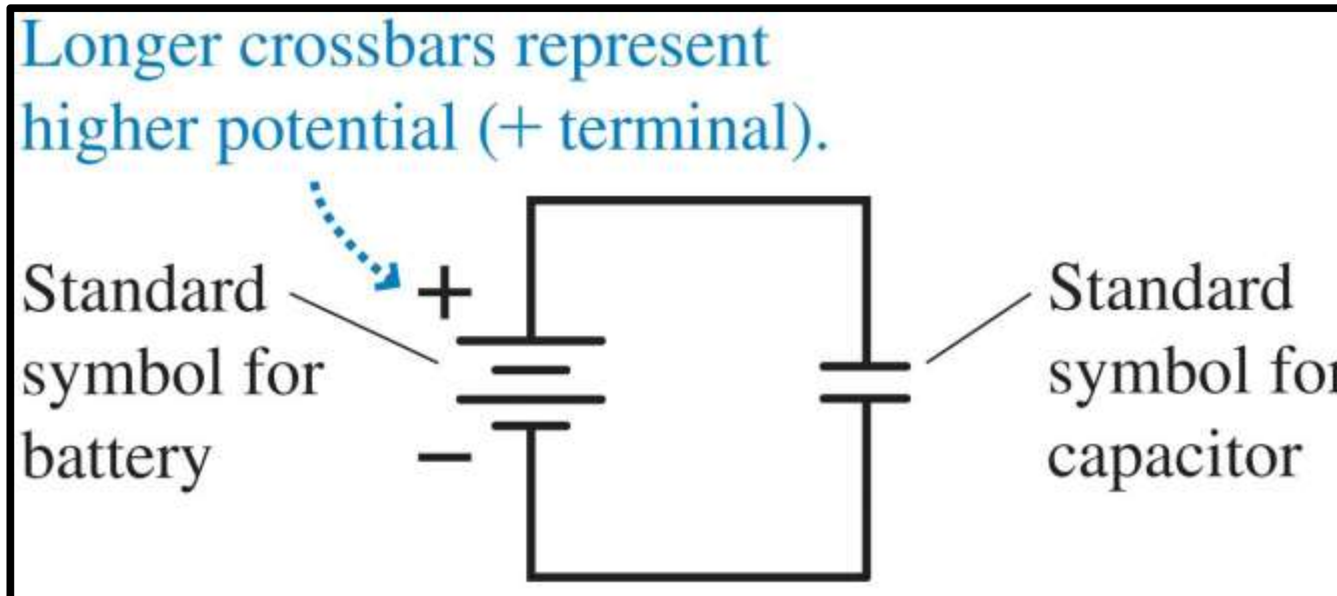


- When you touch a touchscreen, your finger becomes one of the "plates" of the capacitor.

Circuit Diagrams

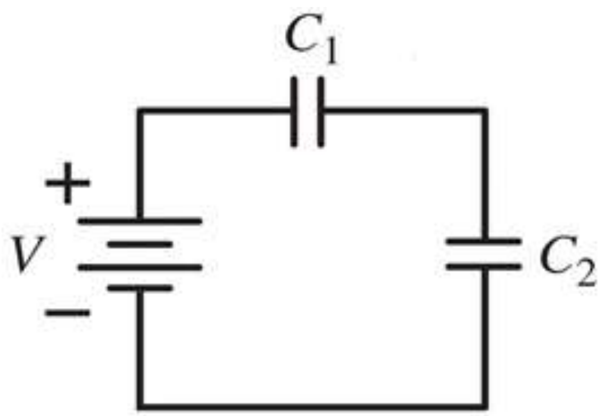
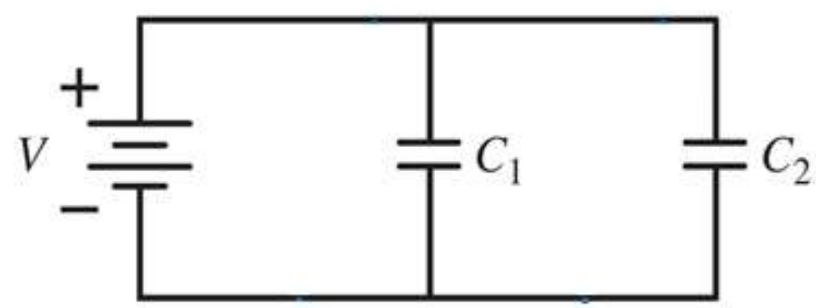


(a) Realistic representation of circuit



(b) Schematic circuit diagram

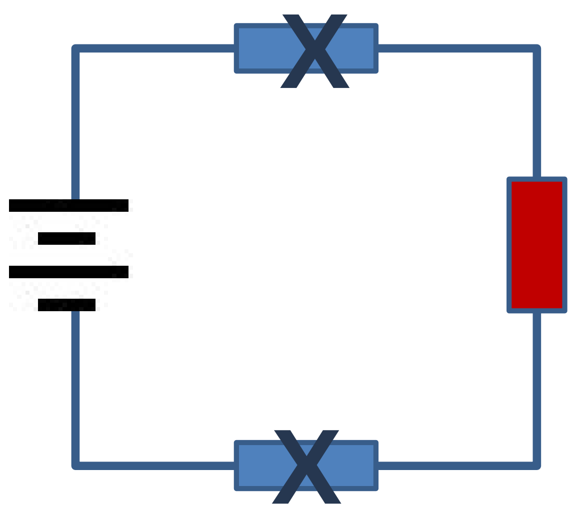
Series and Parallel Circuits

Series Circuit	Parallel Circuit
	
All components are connected with the same wire	There is a junction in the wire separating components
Q is the same for all devices $Q = Q_1 = Q_2 = \cdots$	Q splits between devices $Q = Q_1 + Q_2 + \cdots$
ΔV splits between devices $\Delta V = \Delta V_1 + \Delta V_2 + \cdots$	ΔV is the same for all devices $\Delta V = \Delta V_1 = \Delta V_2 = \cdots$
$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\right)^{-1}$	$C_{\text{eq}} = C_1 + C_2 + C_3 + \ldots$

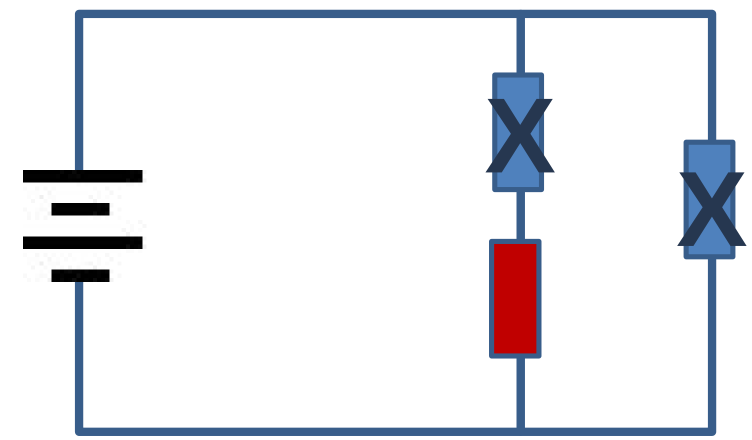
Clicker Question: Series Circuits

For which circuits are the X (blue) devices in **SERIES**?

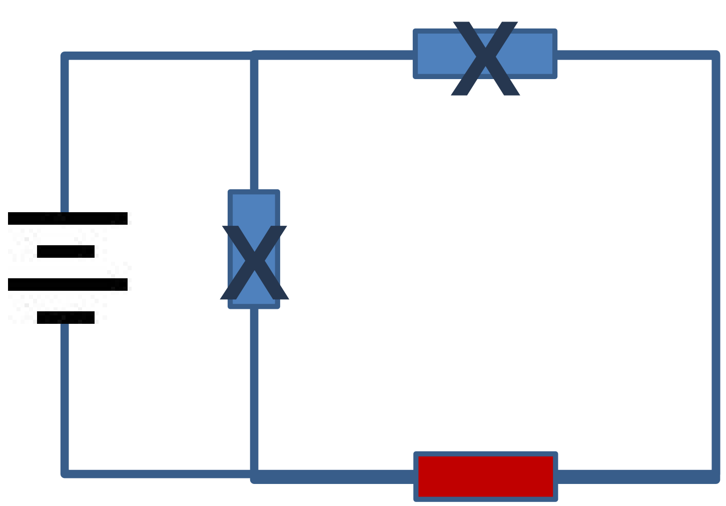
A)



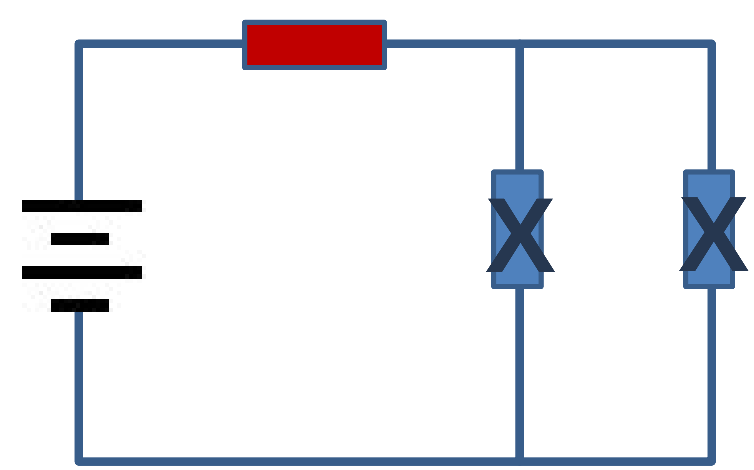
C)



B)

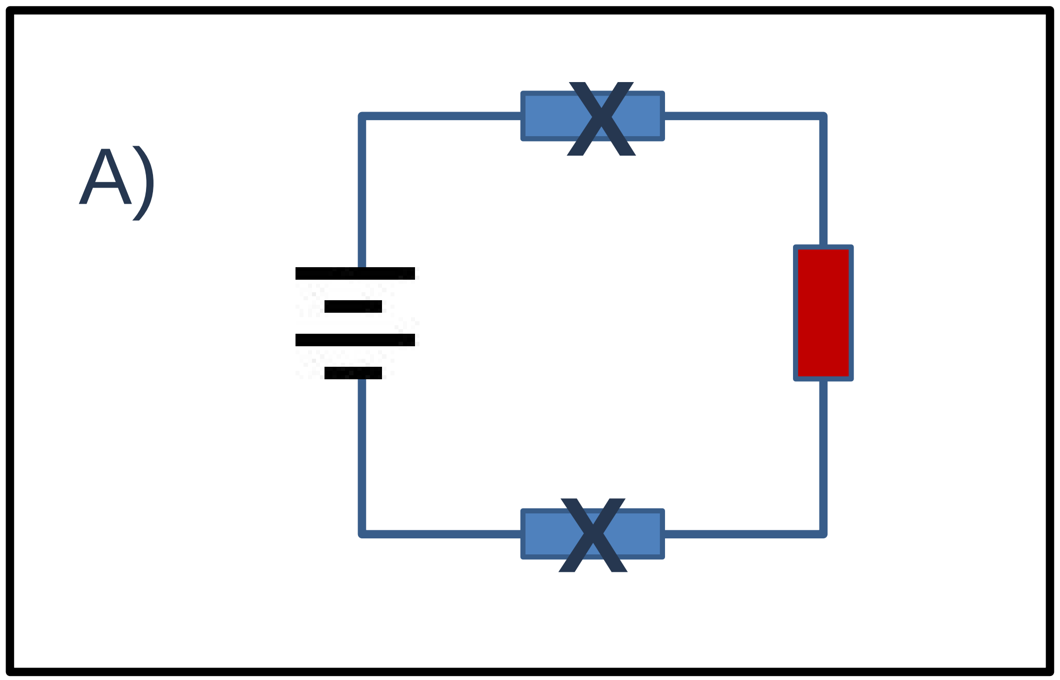


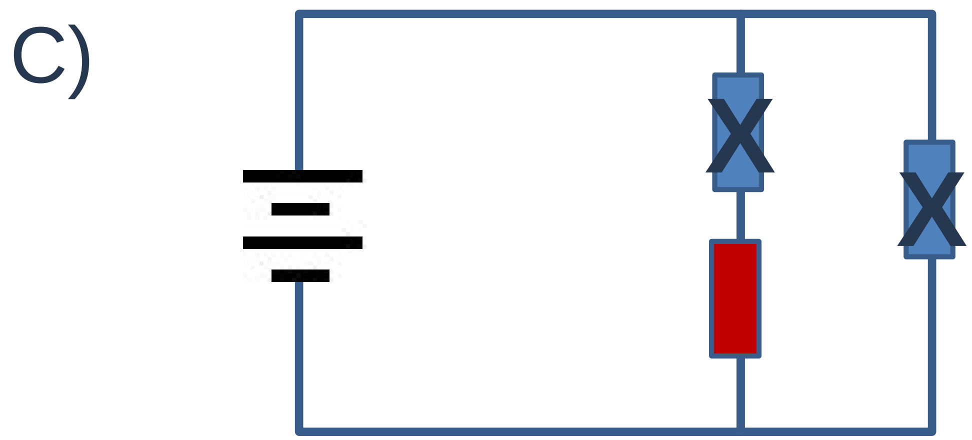
D)

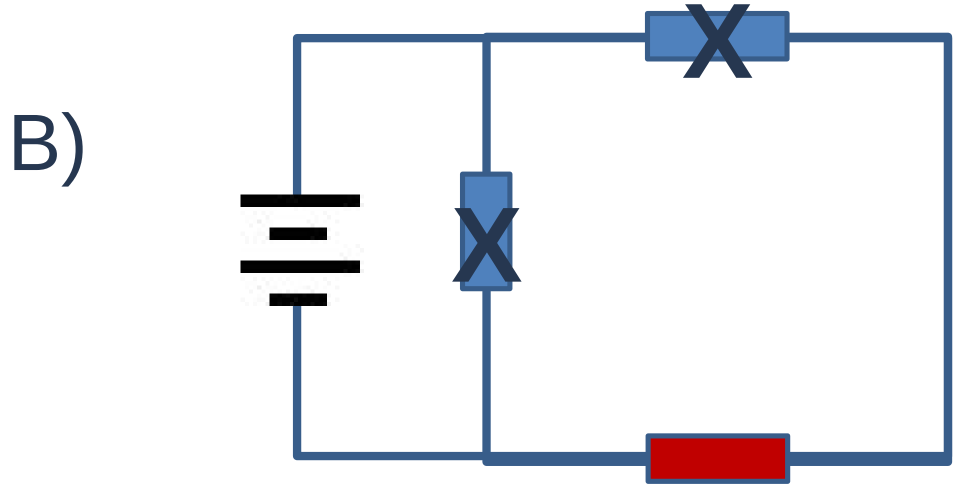


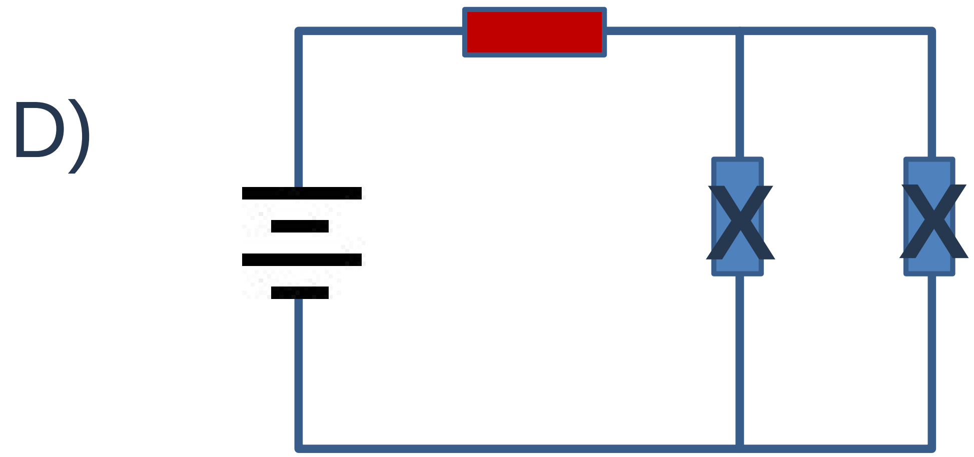
Clicker Question: Series Circuits Answer

For which circuits are the X (blue) devices in **SERIES**?



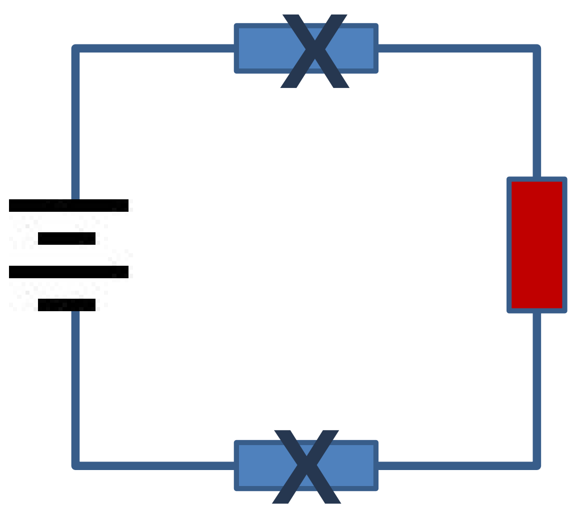




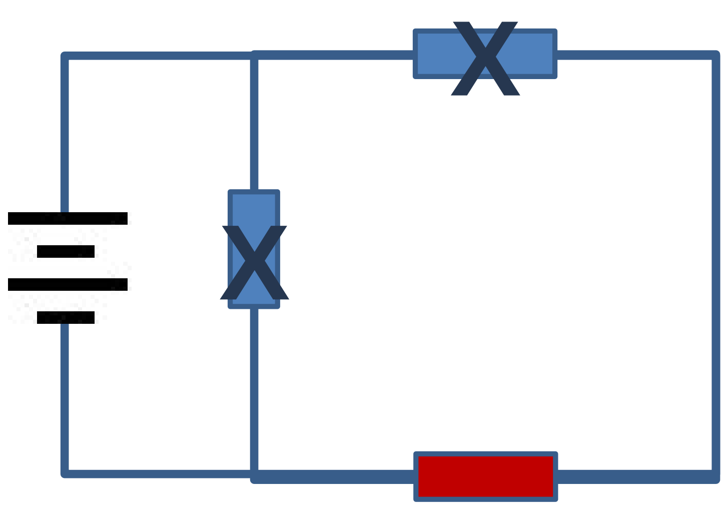


Clicker Question: For which circuits are the X (blue) devices in **PARALLEL**

A)



B)



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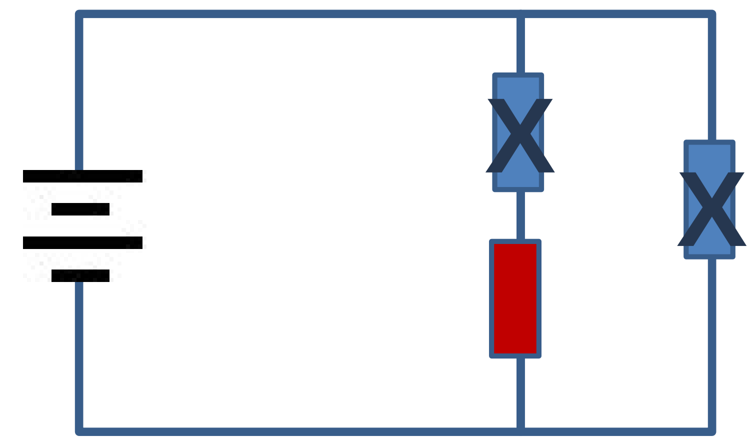
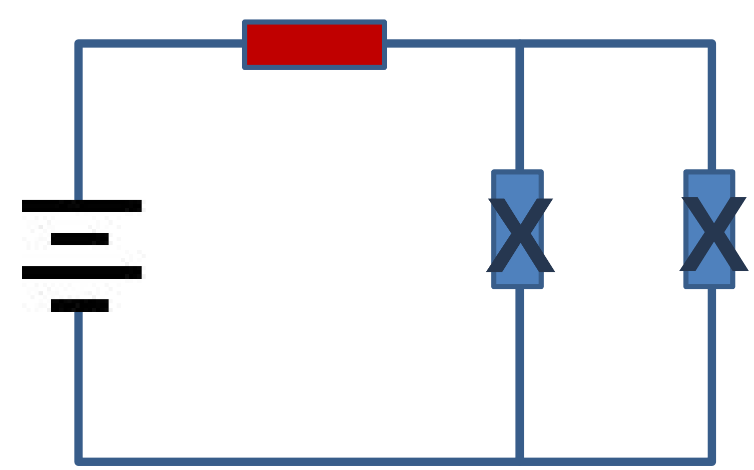
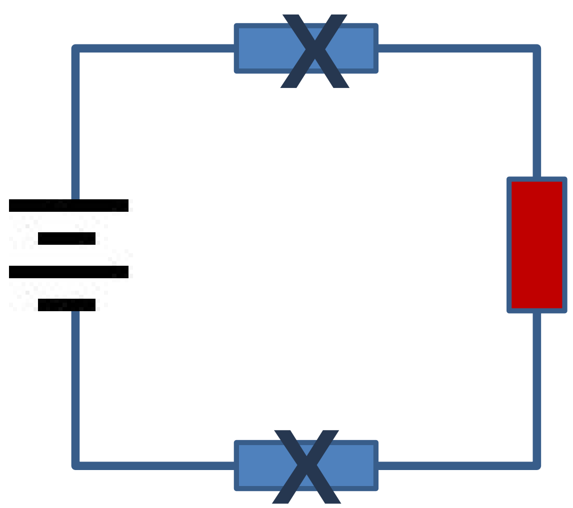


D)

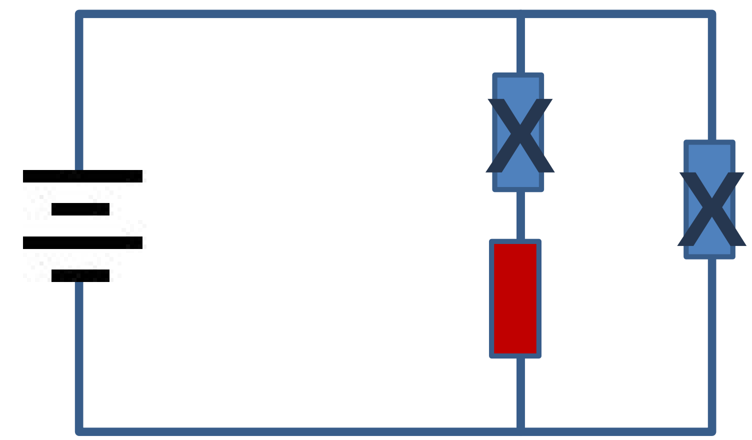


Clicker Question: For which circuits are the X (blue) devices in **PARALLEL**

A)



C)



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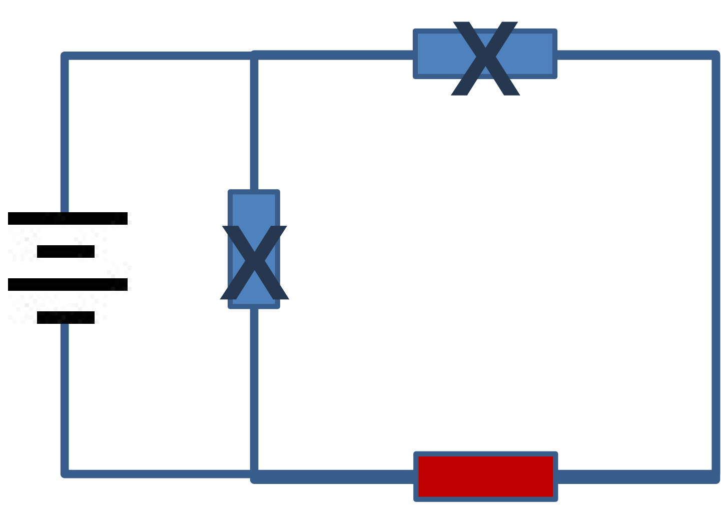

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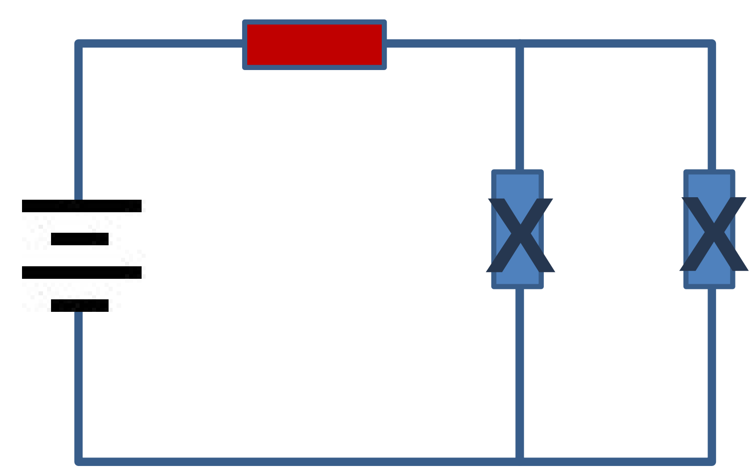
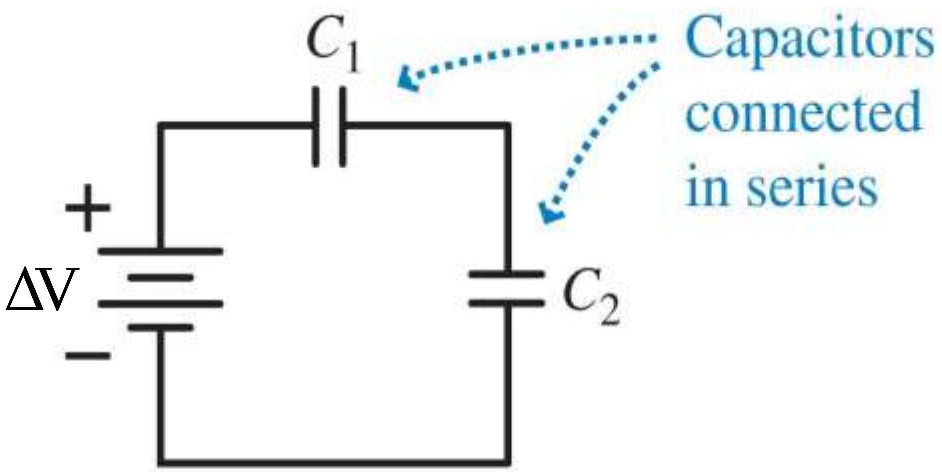
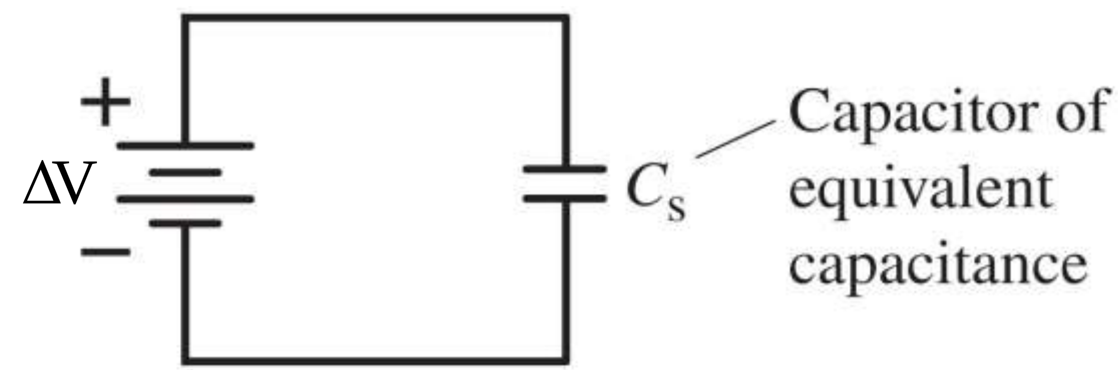


Series Capacitor Circuit



(a) Capacitors connected in series to a battery



(b) The two capacitors are replaced by one of equivalent capacitance C_S.

Key point: Q is the same for both capacitors

The total potential difference splits across the capacitors:

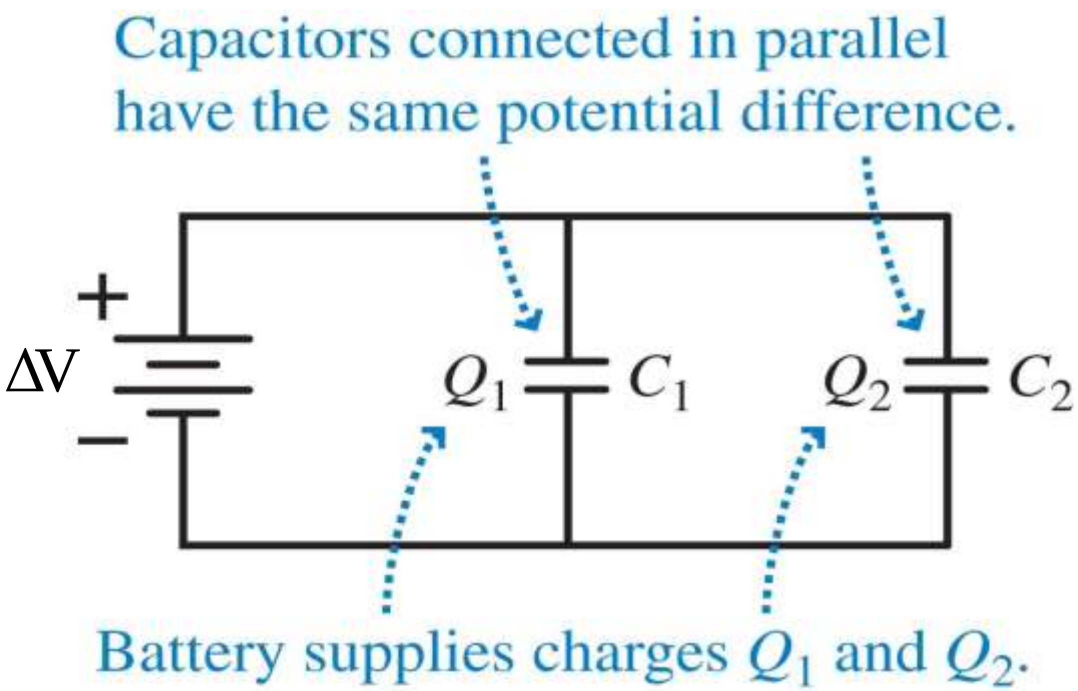
$$\Delta V = \Delta V_1 + \Delta V_2.$$

Sub in

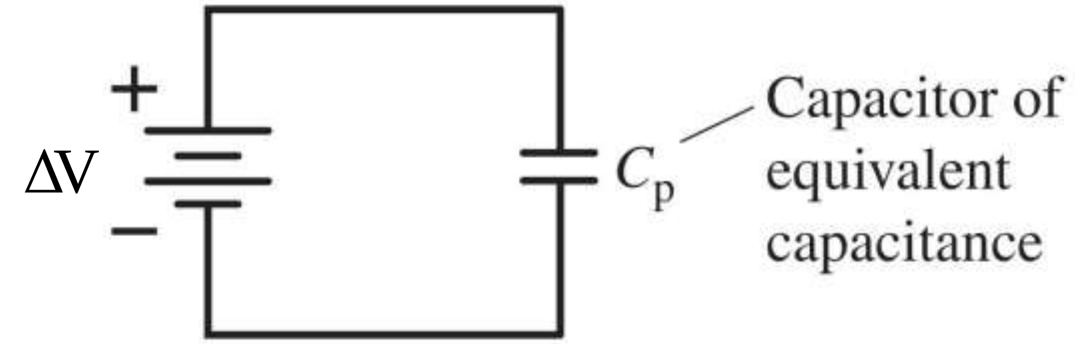
$$C \equiv \frac{Q}{\Delta V} \longrightarrow \frac{\Delta V_C}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Parallel Capacitor Circuit



(a) Capacitors connected in parallel to a battery



(b) The two capacitors are replaced by one of equivalent capacitance C_p.

Key point: ΔV is the same for both capacitors

The total charge splits across the capacitors:

$$Q_{total} = Q_1 + Q_2 = C_1\Delta V + C_2\Delta V = \underbrace{(C_1 + C_2)}_{C_p = C_1 + C_2}\Delta V$$

$$\boxed{C_p = C_1 + C_2}$$

Example: Equivalent Capacitance

What is the equivalent capacitance, C_{eq}, of the combination below?

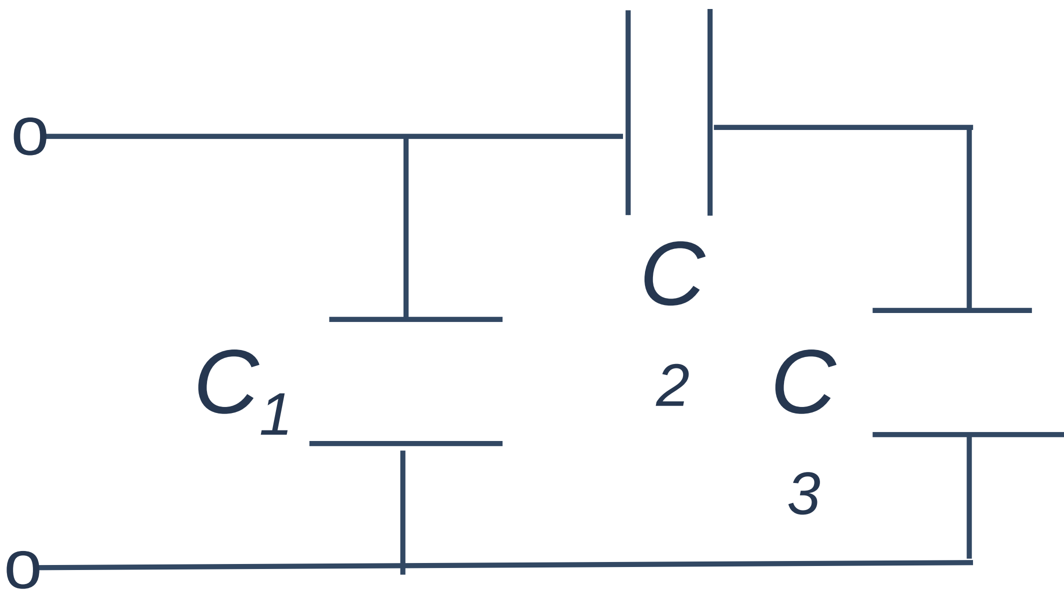
C_2 and C_3 are in series with one another.

The C_2/C_3 combination are in parallel with C_1.

$$C_{eq23} = \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

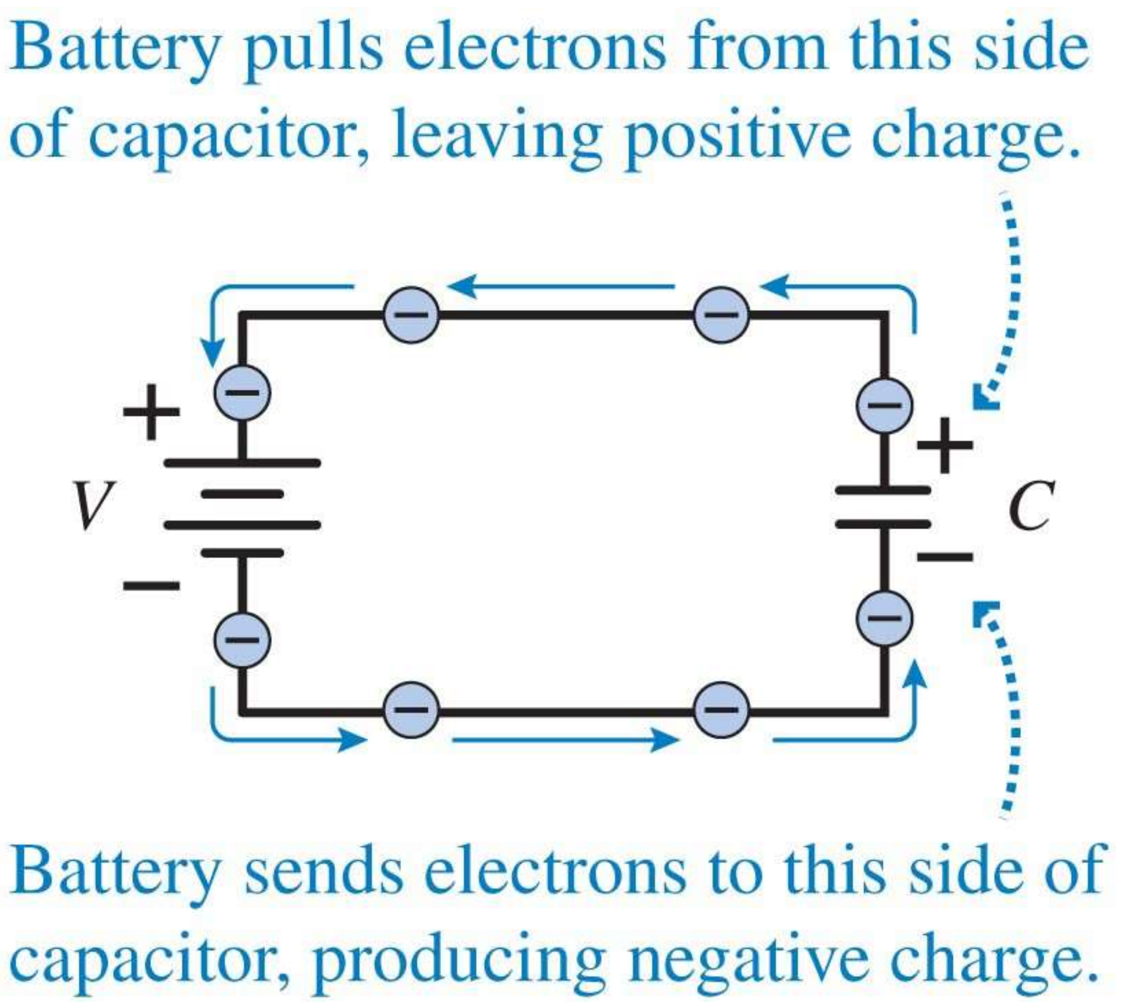
$$C_{eq23} = \left(\frac{C_2 + C_3}{C_2 C_3}\right)^{-1} = \frac{C_2 C_3}{C_2 + C_3}$$

$$C_{eq} = \frac{C_2 C_3}{C_2 + C_3} + C_1$$



Capacitors Require Energy to Charge

- Each electron added to the bottom plate in the picture will feel a potential that it must overcome, requiring work (energy).
- This energy becomes stored as potential energy in the capacitor.
- The larger the charge already present in the plates, the bigger the work required to add an additional charge.



(a) Why charging a capacitor requires work

Energy Stored in a Capacitor (Recap)

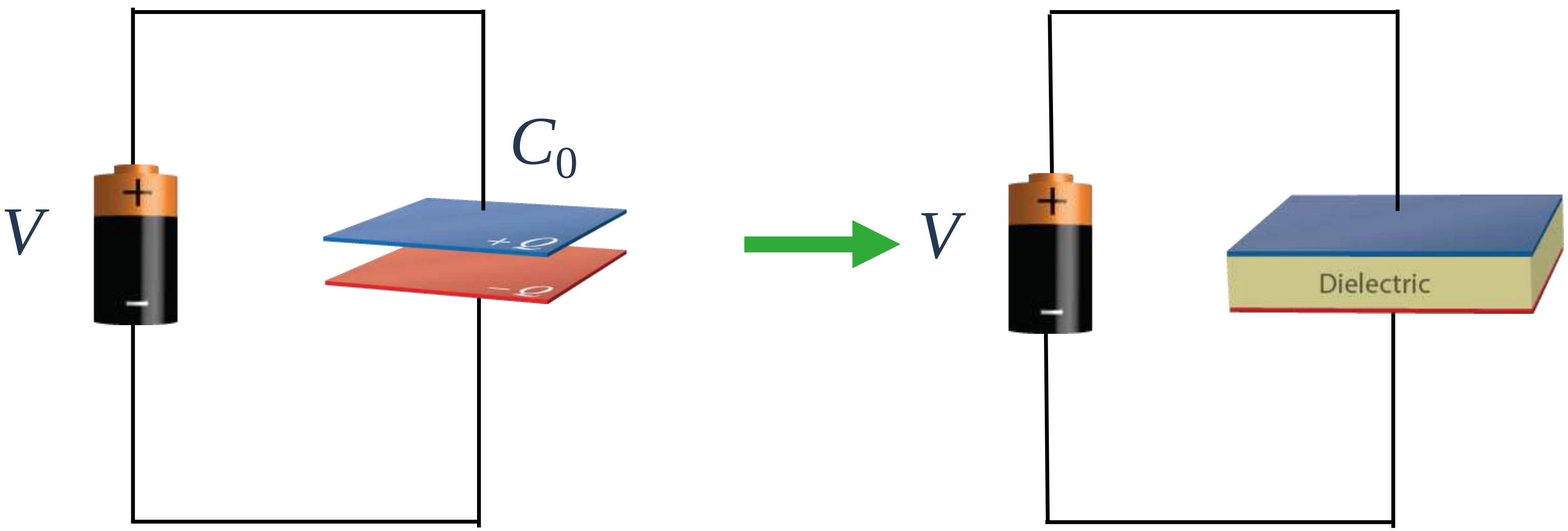
- Capacitors are important elements in electric circuits because of their ability to store energy.

- The charge on the two plates is ±q and this charge separation establishes a potential difference $\Delta V = q/C$ between the two electrodes.

$$U_c = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

- Energy equations are all equivalent and give energy in [J]

Capacitor with a Dielectric (Recap)

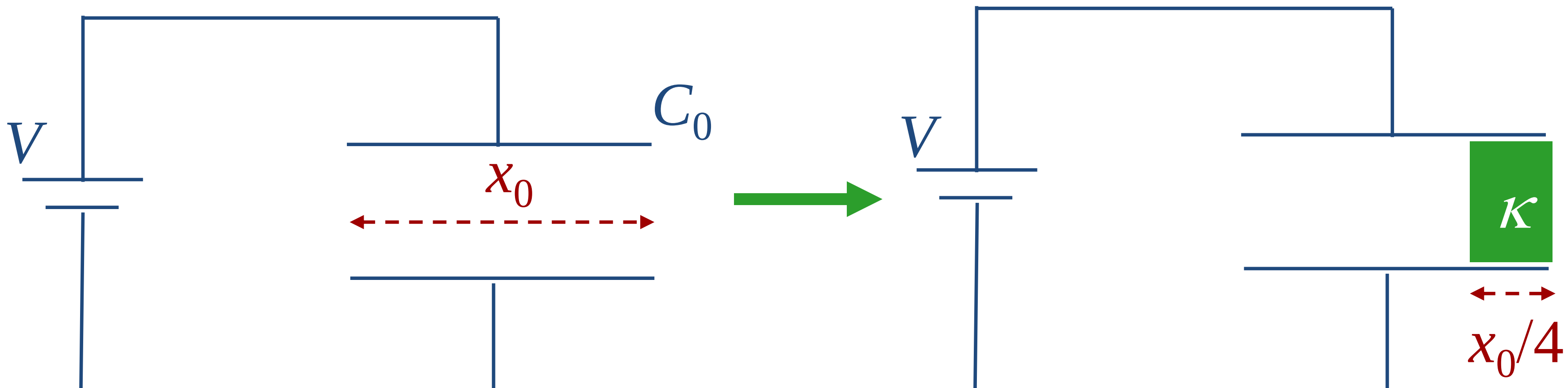


$$C_1 = \kappa C_0$$

By adding a dielectric you are just making a
new capacitor with larger capacitance (factor of κ)

Dielectric
constant

Example Problem: Capacitors

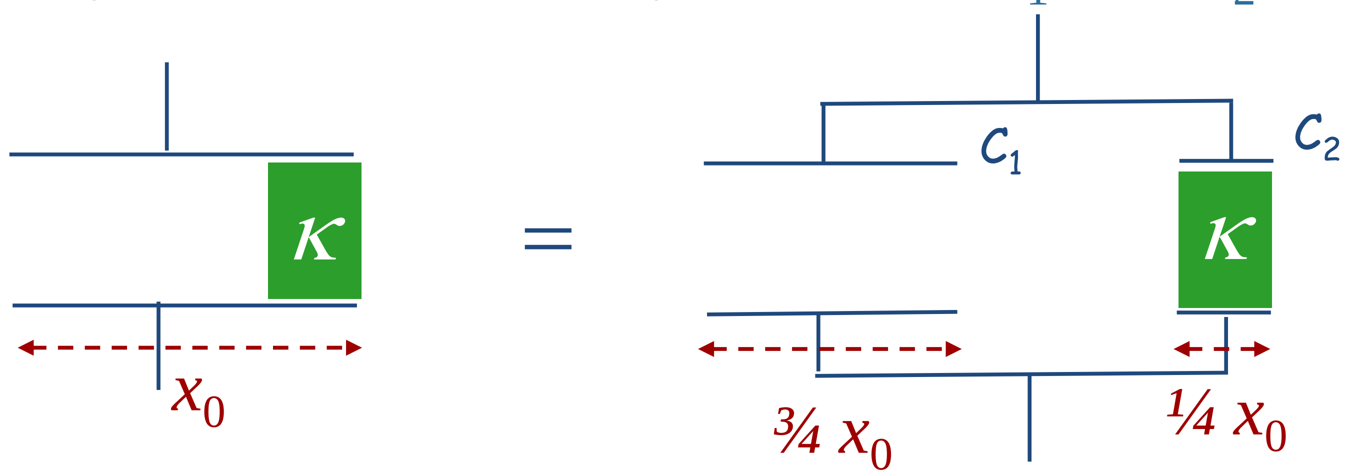


An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage ΔV.

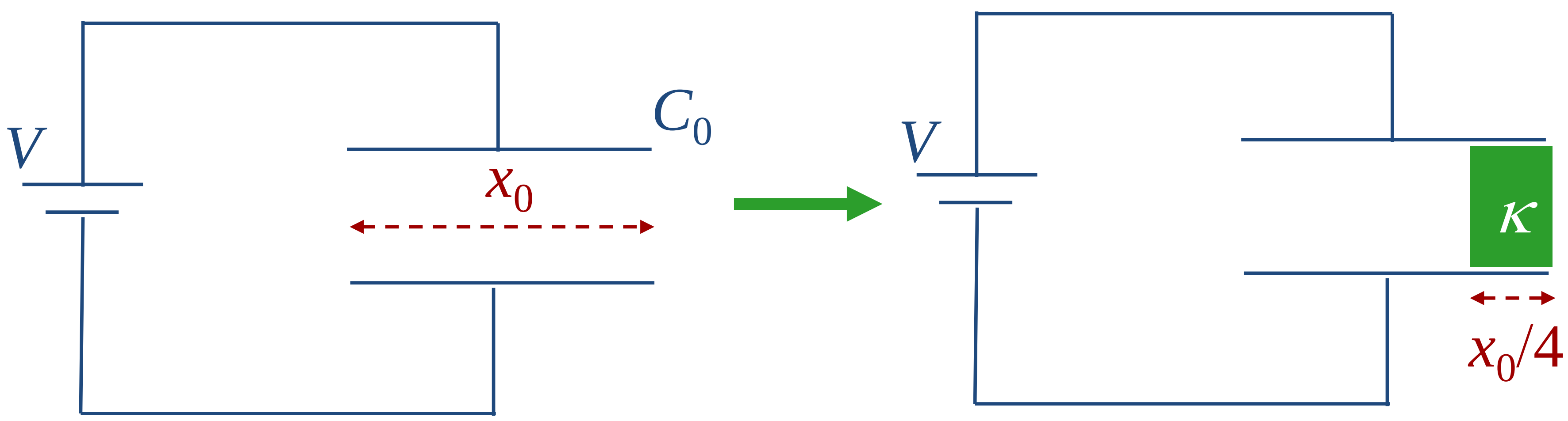
A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor, in terms of C_0?

Can consider capacitor to be two capacitances, C_1 and C_2, in parallel



Example Problem: Capacitors

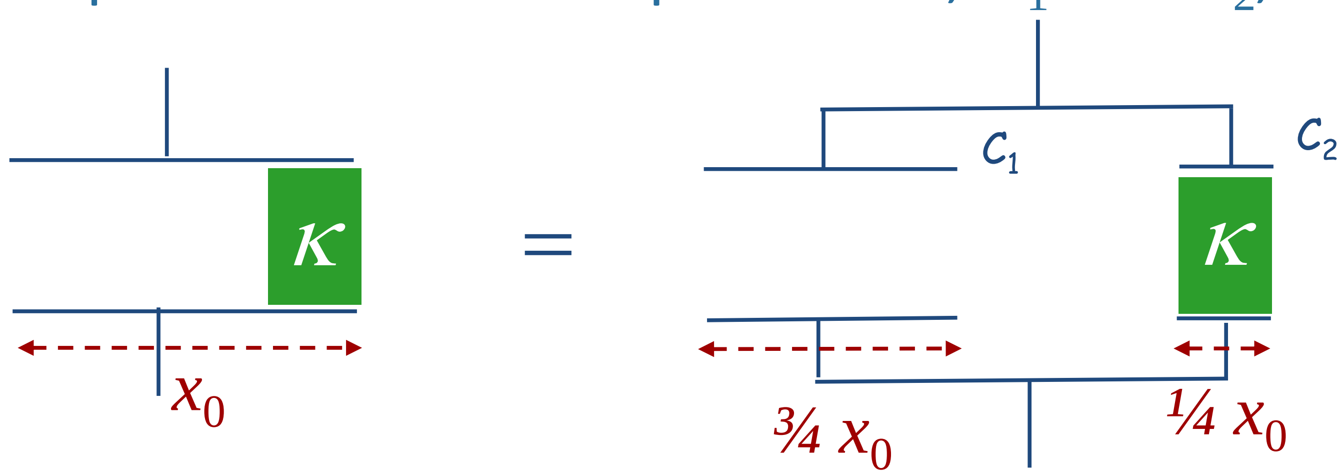


An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

Can consider capacitor to be two capacitances, C_1 and C_2, in parallel



In general.
For parallel plate capacitor: $C = \varepsilon_0 A/d$

What is C_1 ?

A) $C_1 = C_0$ $\boxed{\text{B) } C_1 = {}^3/_4 C_0}$ C) $C_1 = {}^4/_3 C_0$ D) $C_1 = {}^1/_4 C_0$

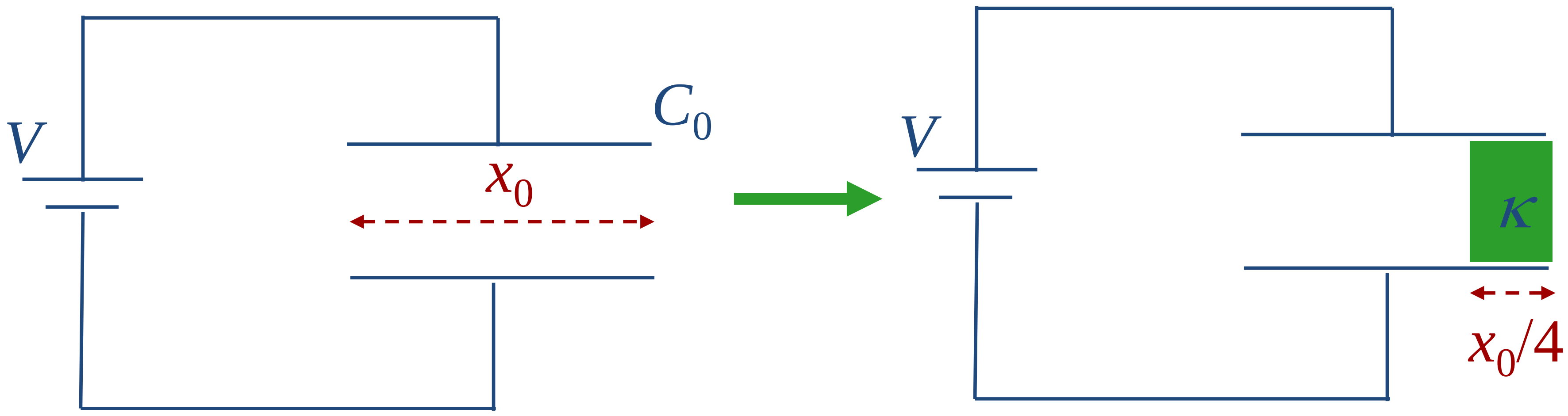
$$\left.\begin{array}{l} A_1 = {}^3/_4 A_0 \\ d_1 = d_0 \end{array}\right] \longrightarrow C_1 = {}^3/_4\,(\varepsilon_0 A_0/d_0) \longrightarrow \boxed{C_1 = {}^3/_4 C_0}$$

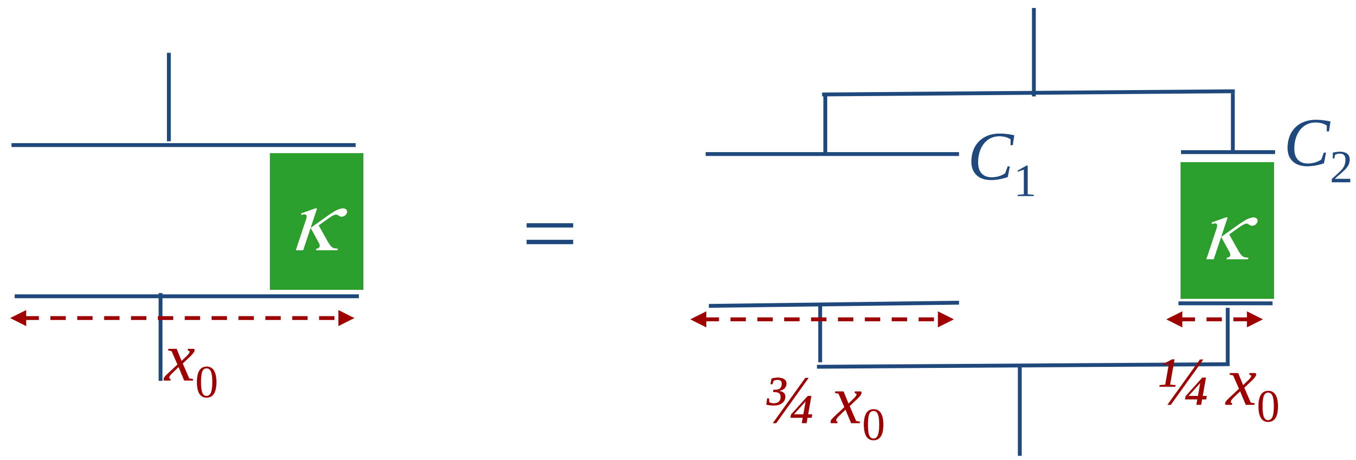
Example Problem: Capacitors

An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?





$C_1 = {}^3/_4 C_0$

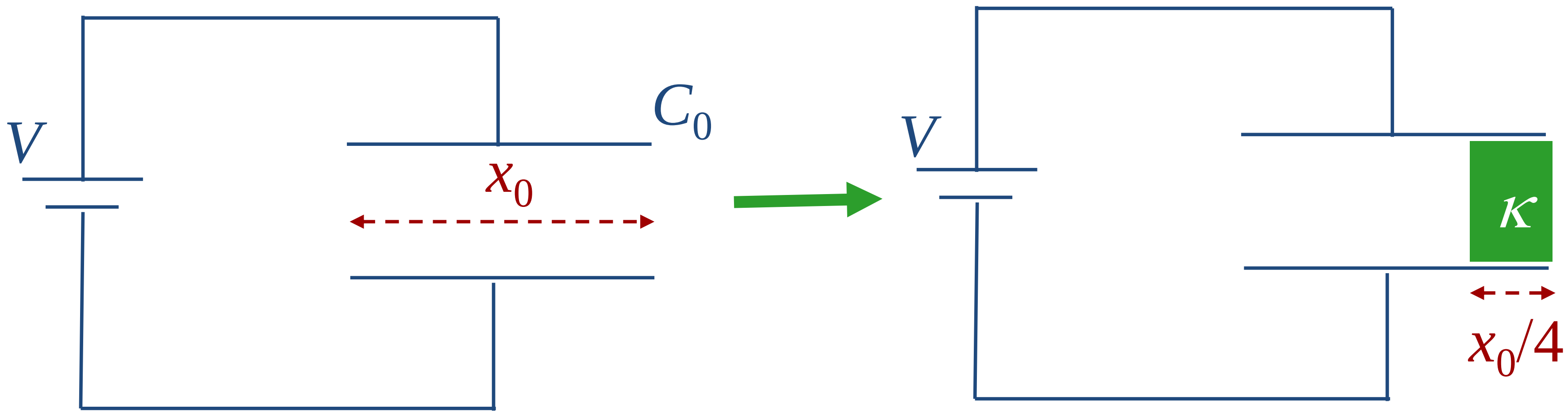
What is C_2 ?

A) $C_2 = \kappa C_0$ B) $C_2 = {}^3/_4\, \kappa C_0$ C) $C_2 = {}^4/_3\, \kappa C_0$ $\boxed{\text{D) } C_2 = {}^1/_4\, \kappa C_0}$

For parallel plate capacitor filled with dielectric: $C = \kappa\varepsilon_0 A/d$

$$\left.\begin{array}{l} A_2 = {}^1/_4 A_0 \\ d_2 = d_0 \end{array}\right] \longrightarrow C_2 = ¼(\kappa\varepsilon_0 A_0/d_0) \longrightarrow C_2 = {}^1/_4\, \kappa C_0$$

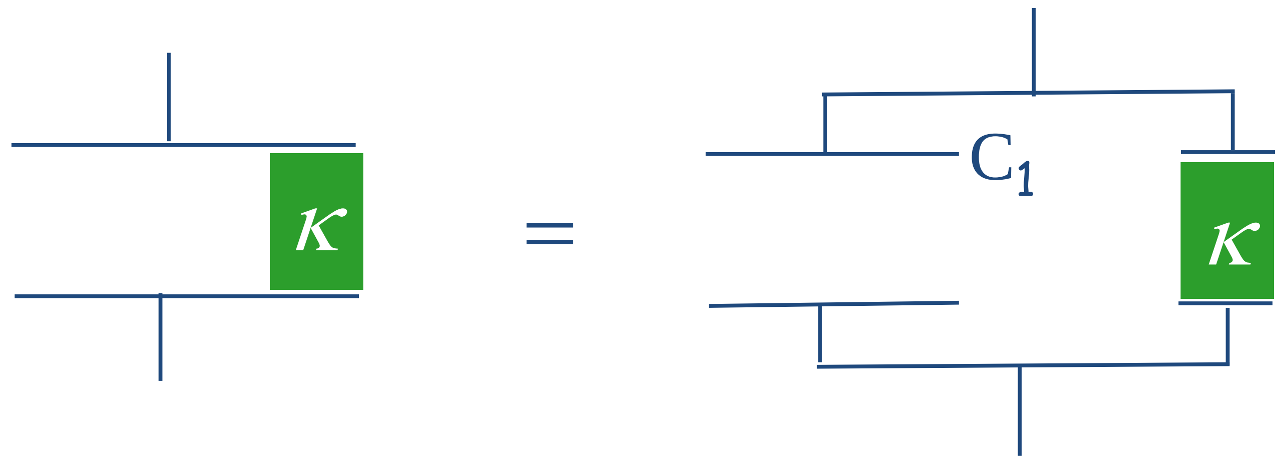
Example Problem: Dielectrics



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?



$C_1 = {}^3/_4 C_0$

$C_2 = {}^1/_4\, \kappa C_0$

C = parallel combination of C_1 and C_2:

$$C = C_1 + C_2$$

$$C = C_0\,({}^3/_4 + {}^1/_4\, \kappa)$$

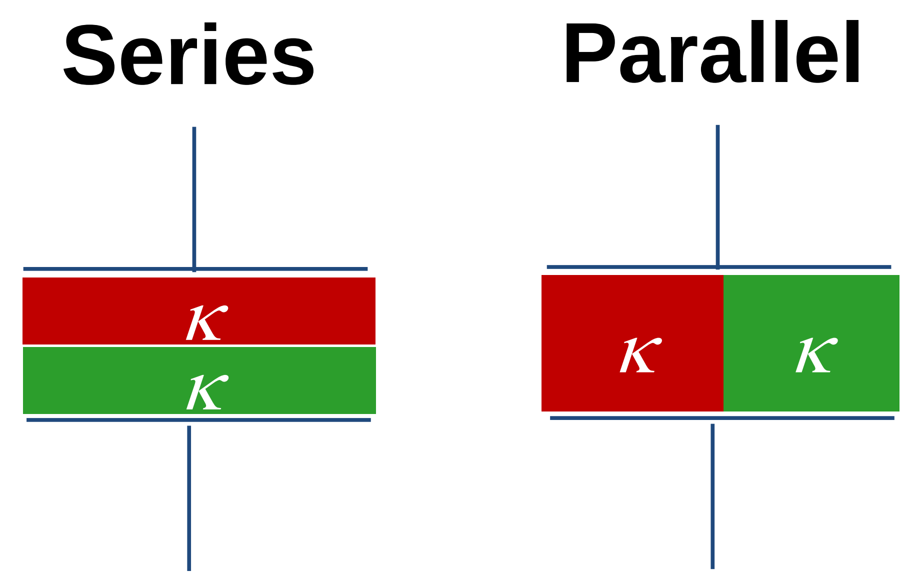
What is Q?

$$C \equiv \frac{Q}{V} \longrightarrow Q = VC$$

$$\boxed{Q_f = VC_0\left(\frac{3}{4} + \frac{1}{4}\kappa\right)}$$

Capacitance Activity

Place the plates as far apart as possible and connect the voltmeter to the plates. Move your hands away from the apparatus and record the capacitance. This is the background capacitance. You will need to zero your voltmeter to remove this background capacitance. This step must be done between ALL your measurements.



Choose two dielectrics and place them inside the capacitor. Predict the total capacitance and then calculate.

	Dielectrics in Series	**Dielectrics in Parallel**
Predicted Capacitance		
Measured Capacitance		

TUTORIAL PROBLEMS