Physics 1502Q:

3.1 Electric Fields II
Electric Field Lines

Announcements

- HW due Monday 11:59 PM
- Prelab due before lab
- Midterm acknowledgement assignment (available on HuskyCT) due Saturday 11:59pm
- Reading Assignment due Sunday 11:59 PM
- First paper-based quiz this Thursday during class time
  - Based on topics covered in class last week
  - No full work means no full credit

Preview of this week and next week

Su	M	T	W	Th	F	Sa
30 Reading Assignment Due 11:59 PM	31 HW Due 11:59 PM	1 E-Fields II E-Field Lines	2	3 Electric Flux Gauss's Law I Paper quiz in class	4 Lab 3: Gauss's Law Pre-lab 3 Due before lab	5 Midterm Acknowledgment Due 11:59 PM
6 Reading Assignment Due 11:59 PM	7 HW Due 11:59 PM	8 Gauss's Law II	9	10 Electric Potential Energy Electric Potential I Paper quiz in class	11 Lab 4: Electric Potential Pre-lab 4 Due before lab	12

Electric Fields: Summary for Point Charges

The electric field *E* at a point in space is simply the force per unit charge at that point.

$$\boxed{\vec{E} \equiv \frac{\vec{F}}{q_0}}$$

Electric field due to a point charged particle:

$$\boxed{\vec{E} = k\frac{q}{r^2}\hat{r}}$$

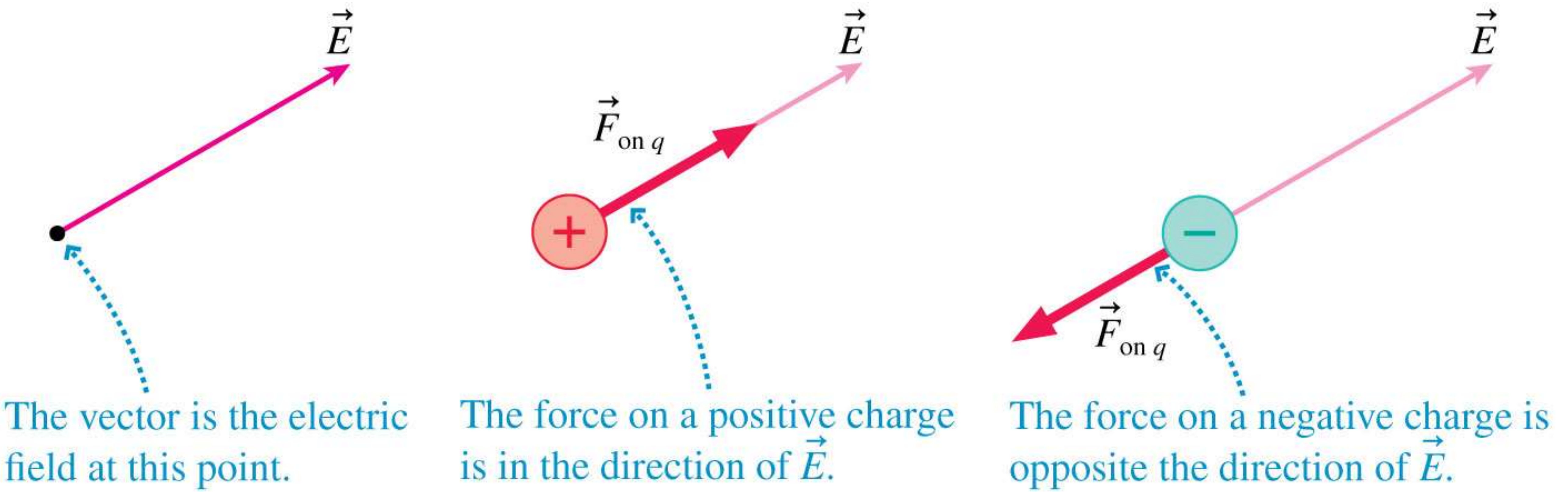
Electric field due to a collection of point charged particles:

$$\boxed{\vec{E} = \sum_i k\frac{q_i}{r_i^2}\hat{r}_i}$$

Note: The SI unit of electric field is the Newton/Coulomb [N/C].

Motion of a Charged Particle in an Electric Field

Consider a particle of charge *q* and mass *m* at a point where an electric field $\vec{E}$ has been produced by *other* charges, the source charges.

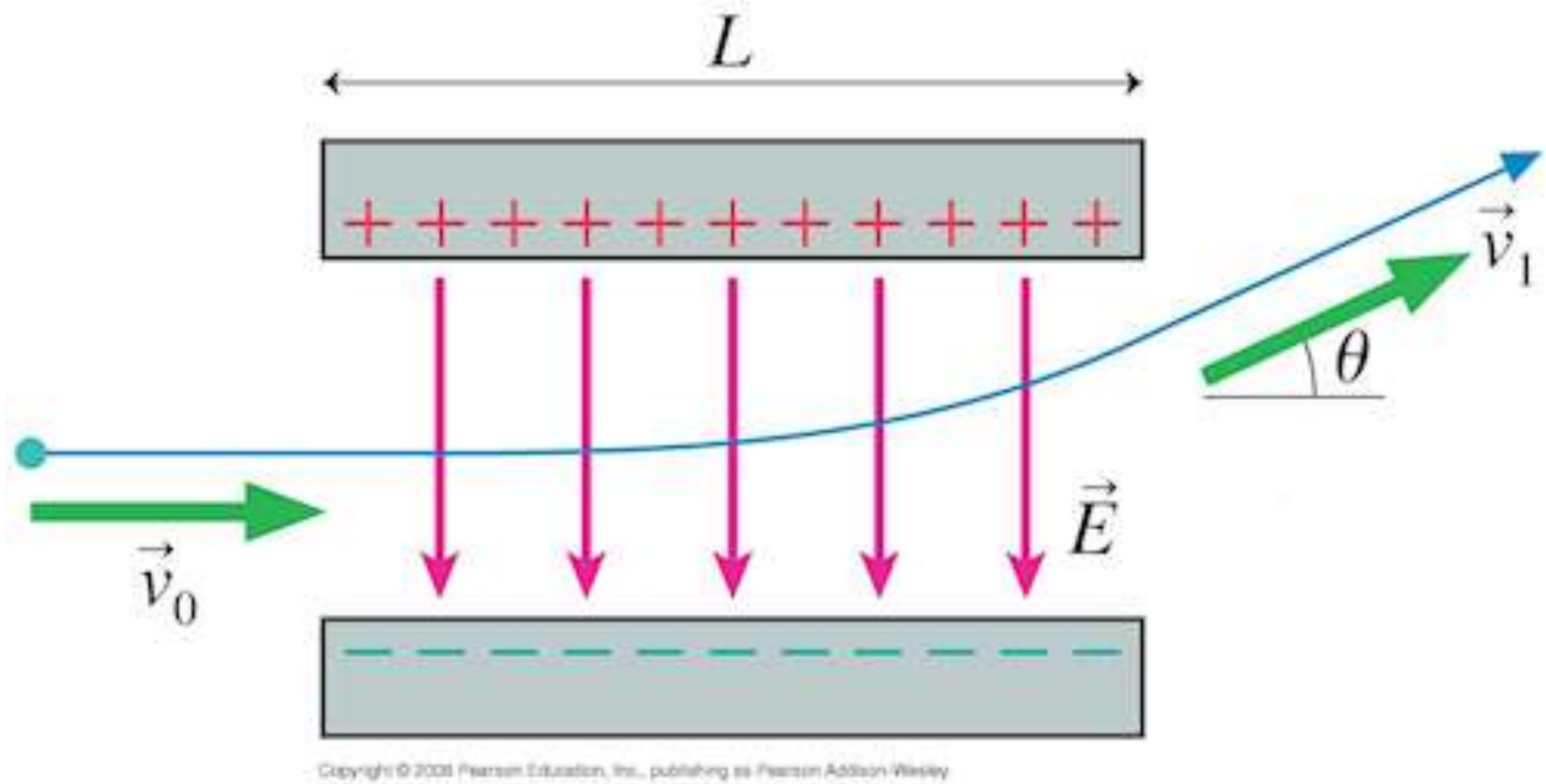


$$\boxed{\vec{F} = q\vec{E}}$$

A practical application of an E-Field is to accelerate or slow down charged particles.

- Positive charges are accelerated along the direction of the E-field.
- Negative charges are accelerated in a direction opposite of the E-field.

Motion of a Charged Particle in an Electric Field Example

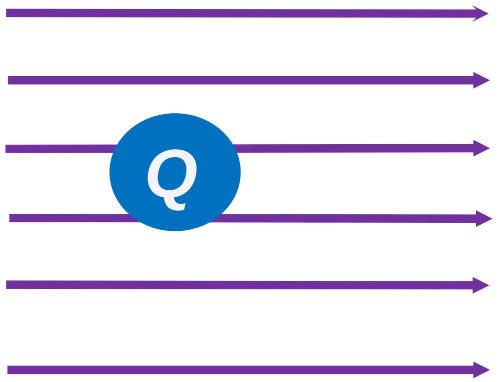


In the picture, a negative charge has its trajectory deflected upward due to a uniform E-field pointing down ($-y$). Since the charge is negative, the electric force acting on it points up ($+y$).

- A vertical E-field can be used to deflect the trajectory of charged particles.
- A vertical E-field can also be used to cancel out the weight (force) of a charged particle, ensuring it travels in a straight line with constant speed: $F_{net} = 0$.

Question: Force in an Electric Field

A **4 C** charge is placed in a uniform electric field in empty space. The charge feels an electric force of **12 N**. If this charge is removed and a **6 C** charge is placed at that point instead, what force will it feel?



A. 12 N
B. 8 N
C. 24 N
D. No force
E. 18 N

Question: Force in an Electric Field

A **4 C** charge is placed in a uniform electric field in empty space. The charge feels an electric force of **12 N**. If this charge is removed and a **6 C** charge is placed at that point instead, what force will it feel?

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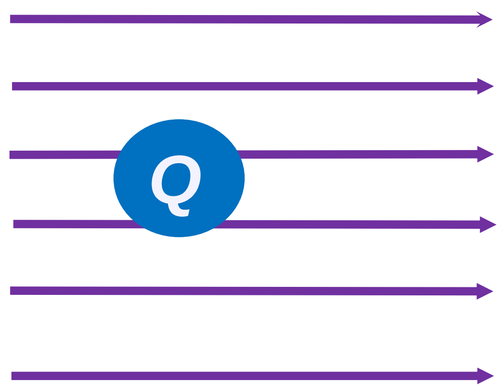


$$\vec{E} = \frac{\vec{F}}{q_0}$$

Since the $4\ \mathrm{C}$ charge feels a force, there must be an electric field present, with magnitude:

$$\boldsymbol{E = F/q = 12\ \mathrm{N}/4\ \mathrm{C} = 3\ \mathrm{N/C}}$$

Once the $4\ \mathrm{C}$ charge is replaced with a $6\ \mathrm{C}$ charge, this new charge will feel a force of:

$$\boldsymbol{F = qE = (6\ \mathrm{C})(3\ \mathrm{N/C}) = 18\ \mathrm{N}}$$

Continuous Charge Distributions

Infinitesimal piece of charge

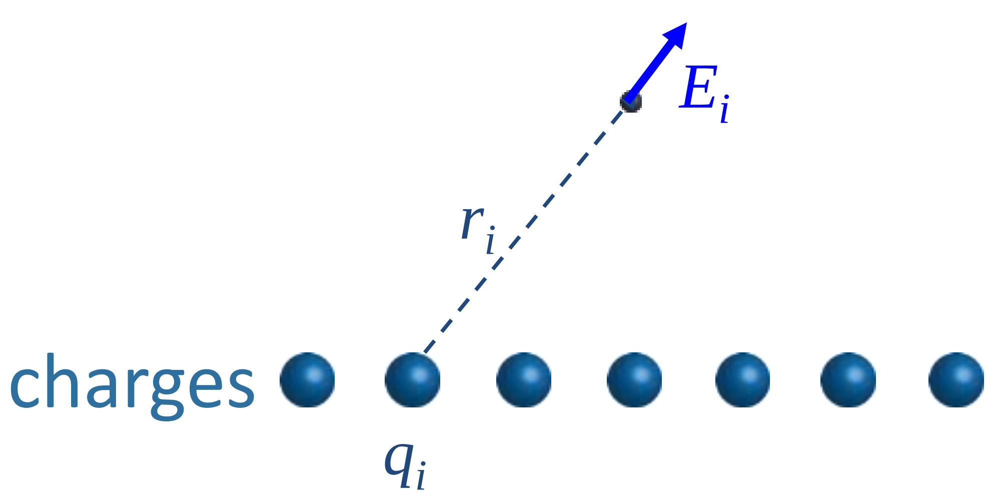
$$\vec{E} = \sum_i k \frac{q_i}{r_i^2} \hat{r}_i \quad \longrightarrow \quad \vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

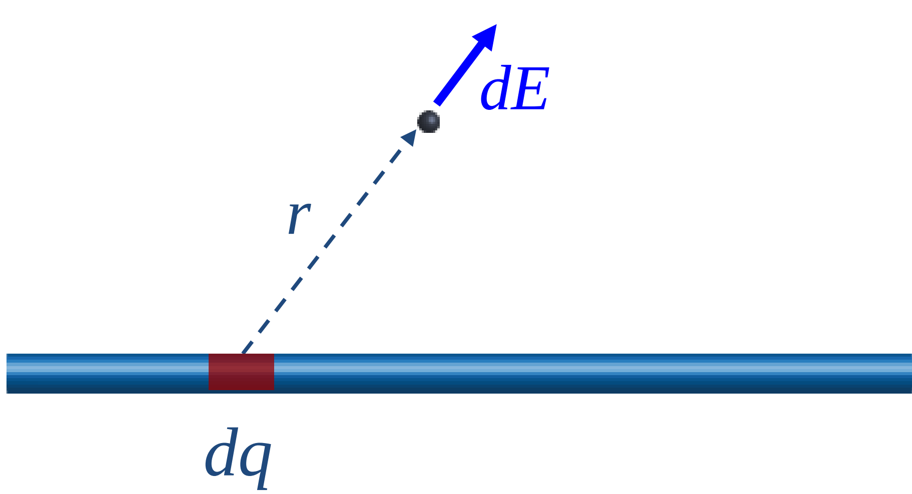
Electric field of point charges

Break object into an infinite number of point charges → Summation becomes an integral

r is vector from dq to the point at which E is found

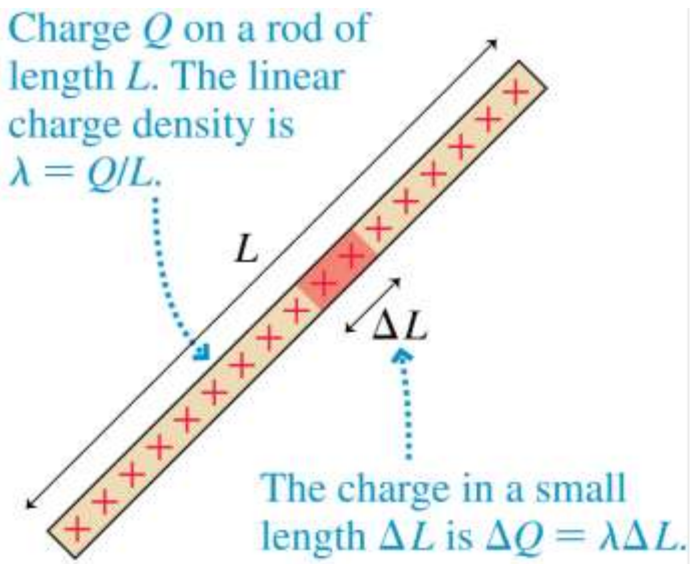
Linear Example:





Charge Density

- Linear $(\lambda = Q/L)$ Coulombs/meter

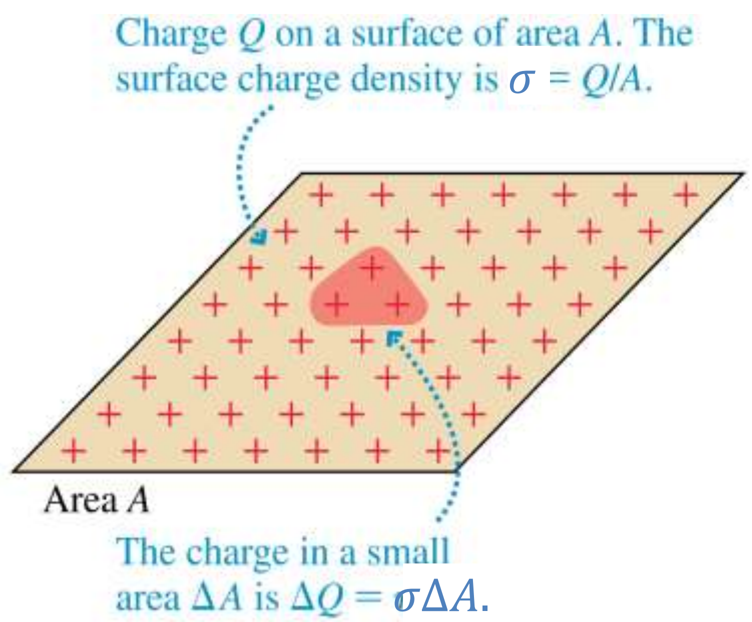


Some Geometry:

$$A_{sphere} = 4\pi R^2 \qquad A_{cylinder} = 2\pi RL$$

$$V_{sphere} = \frac{4}{3}\pi R^3 \qquad V_{cylinder} = \pi R^2 L$$

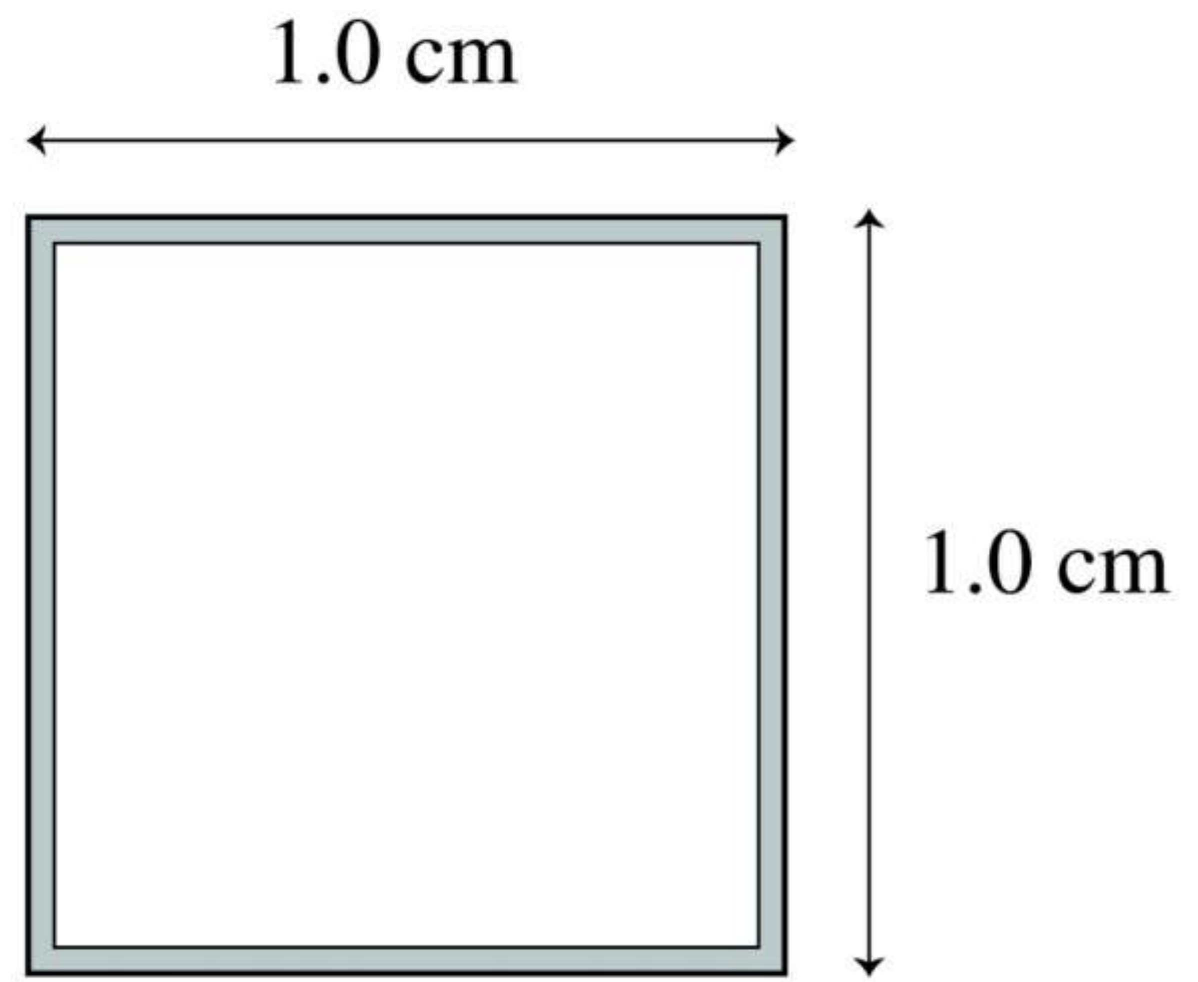
- Surface $(\sigma = Q/A)$ Coulombs/meter^2



- Volume $(\rho = Q/V)$ Coulombs/meter^3

Question: Linear Charge Density

If $8\ \mathrm{nC}$ of charge are placed on the square loop of wire shown below, the linear charge density will be



A. $800\ \mathrm{nC/m}$
B. $400\ \mathrm{nC/m}$
C. $200\ \mathrm{nC/m}$
D. $8\ \mathrm{nC/m}$
E. $2\ \mathrm{nC/m}$

Question: Linear Charge Density

If 8 nC of charge are placed on the square loop of wire shown below, the linear charge density will be

A. 800 nC/m
B. 400 nC/m
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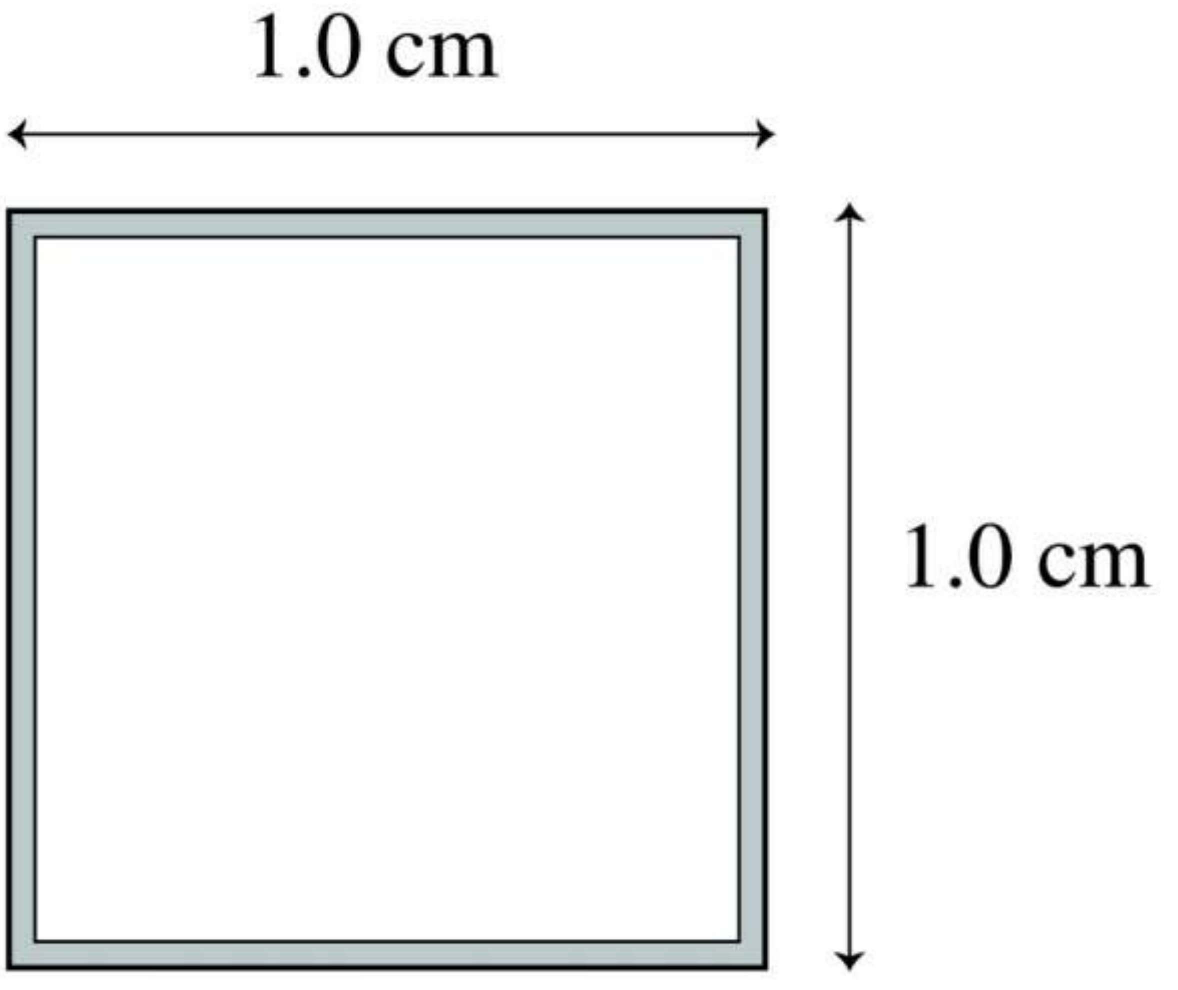


Question: Charge density

- Linear $(\lambda = Q/L)$ Coulombs/meter
- Surface $(\sigma = Q/A)$ Coulombs/meter^2
- Volume $(\rho = Q/V)$ Coulombs/meter^3

Some Geometry

$$A_{sphere} = 4\pi R^2 \qquad A_{cylinder} = 2\pi RL$$

$$V_{sphere} = \tfrac{4}{3}\pi R^3 \qquad V_{cylinder} = \pi R^2 L$$

What has more net charge?

A) A sphere w/ radius 2 meters and *volume* charge density ρ = 2 C/m^3

B) A sphere w/ radius 2 meters and *surface* charge density σ = 2 C/m^2

C) Both A) and B) have the same net charge.

Question: Charge density

- Linear $(\lambda = Q/L)$ Coulombs/meter
- Surface $(\sigma = Q/A)$ Coulombs/meter^2
- Volume $(\rho = Q/V)$ Coulombs/meter^3

Some Geometry

$$A_{sphere} = 4\pi R^2 \qquad A_{cylinder} = 2\pi RL$$

$$V_{sphere} = \tfrac{4}{3}\pi R^3 \qquad V_{cylinder} = \pi R^2 L$$

What has more net charge?

A) A sphere w/ radius 2 meters and volume charge density $\rho = 2$ C/m^3

B) A sphere w/ radius 2 meters and surface charge density $\sigma = 2$ C/m^2

C) Both A) and B) have the same net charge.

$$Q_A = \rho V = \rho \tfrac{4}{3}\pi R^3$$

$$Q_B = \sigma A = \sigma 4\pi R^2$$

$$\frac{Q_A}{Q_B} = \frac{r\,\tfrac{4}{3}pR^3}{s\,4pR^2} = \frac{1}{3}\frac{r}{s}R = \frac{2}{3}$$

After plugging in values for ρ,σ and R

Charge Densities for Infinitesimal Amounts

	Charge Density Definition	Infinitesimal Case
Linear Charge	$Q = \lambda L$	$\boldsymbol{dq = \lambda ds}$ $ds = dx, dy, or\ dz$
Surface Charge	$Q = \sigma A$	$\boldsymbol{dq = \sigma dA}$
Volume Charge	$Q = \rho V$	$\boldsymbol{dq = \rho dV}$

Problem-solving pathway to find the electric field due to a continuous distribution of charge:

1) Choose an arbitrary part of the charge (dq). Label its position and draw the displacement vector $\vec{r}$ from the piece of charge to your point of interest.

2) Each small piece, dq, creates an electric field $d\vec{E}$. Draw the electric field due to dq.

$$dE = k\frac{dq}{r^2}$$

3) Write dq in terms of a charge density; $dq \ = \ \lambda ds$ (for linear charge densities), $dq \ = \ \sigma dA$ (for surface charge densities), $dq \ = \ \rho dV$ (for volume charge densities).

- Express ds, dA, or dV in terms of the geometry of your problem

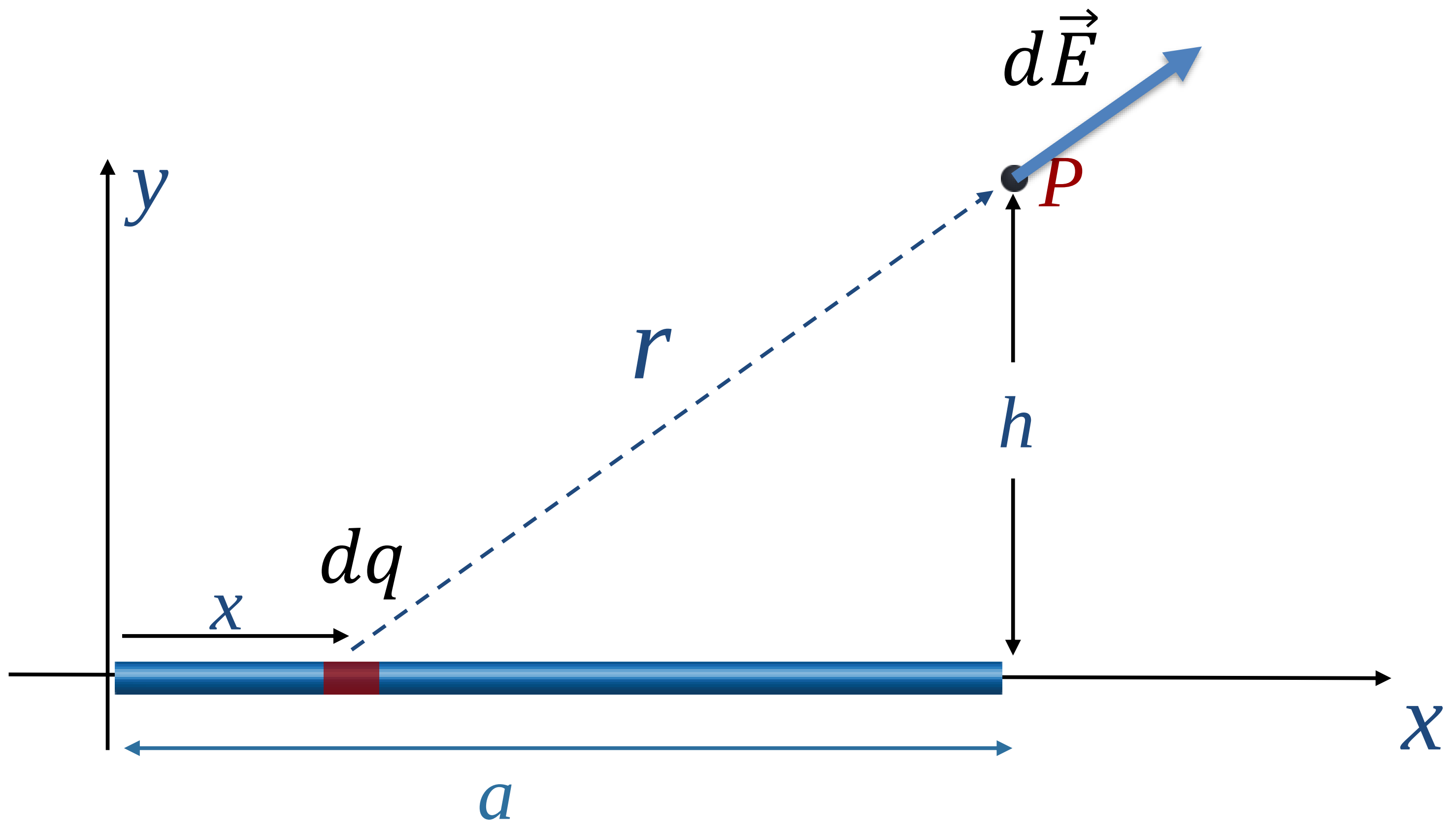
4) Express r in terms of constants and the differential variable (ex. x, y, z).

5) Break $d\vec{E}$ into components if needed (multiply by cosθ or sinθ, and express in terms of constants and the differential variable.

6) Integrate each component of $d\vec{E}$ separately.

Example: Line of Charge - 1

A charge Q is uniformly distributed along the x-axis from the origin to $\boldsymbol{x = a}$. The charge density is $+\boldsymbol{\lambda}$. What is the x-component of the electric field at point $\boldsymbol{P}$ located at $(\boldsymbol{a}, \boldsymbol{h})$?



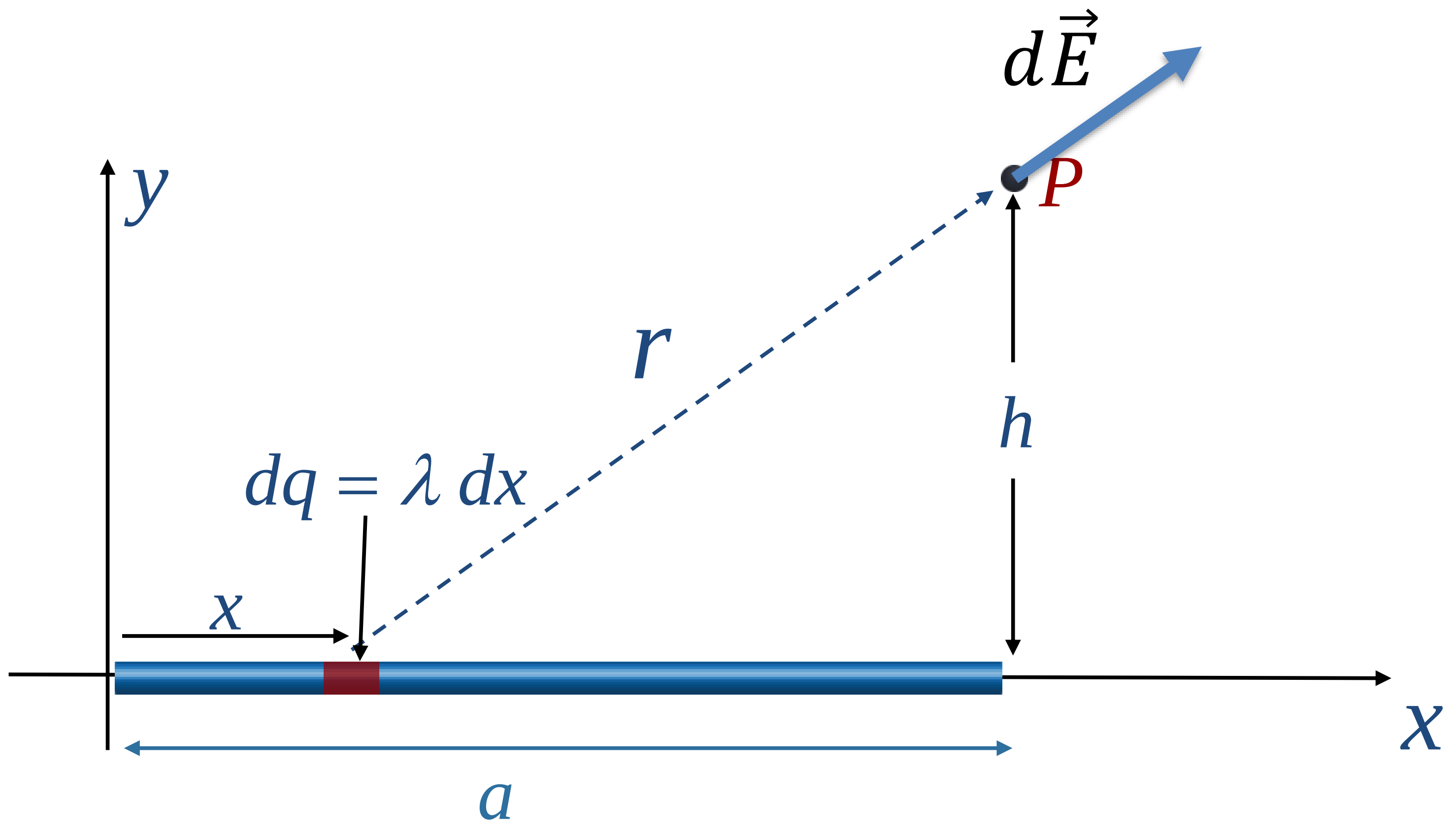
1) Choose an arbitrary element of charge dq and draw the position vector $\vec{r}$ to your point of interest.

$$\overrightarrow{dE} = k\frac{dq}{r^2}\hat{r}$$

2) Draw the electric field $d\vec{E}$ due to the element of charge dq chosen.

Example: Line of Charge -2

A charge Q is uniformly distributed along the x-axis from the origin to $\boldsymbol{x = a}$. The charge density is $+\boldsymbol{\lambda}$. What is the x-component of the electric field at point $\boldsymbol{P}$ located at $(\boldsymbol{a, h})$?



3) Write $dq = \lambda ds$ (for linear charge densities) in terms of the geometry of your problem (Here that means $ds = dx$)

$$\lambda = \frac{q}{L} \Rightarrow \lambda = \frac{Q}{a} = \frac{dq}{dx}$$

$$dq = \lambda dx$$

4) Express r in terms of your given variables

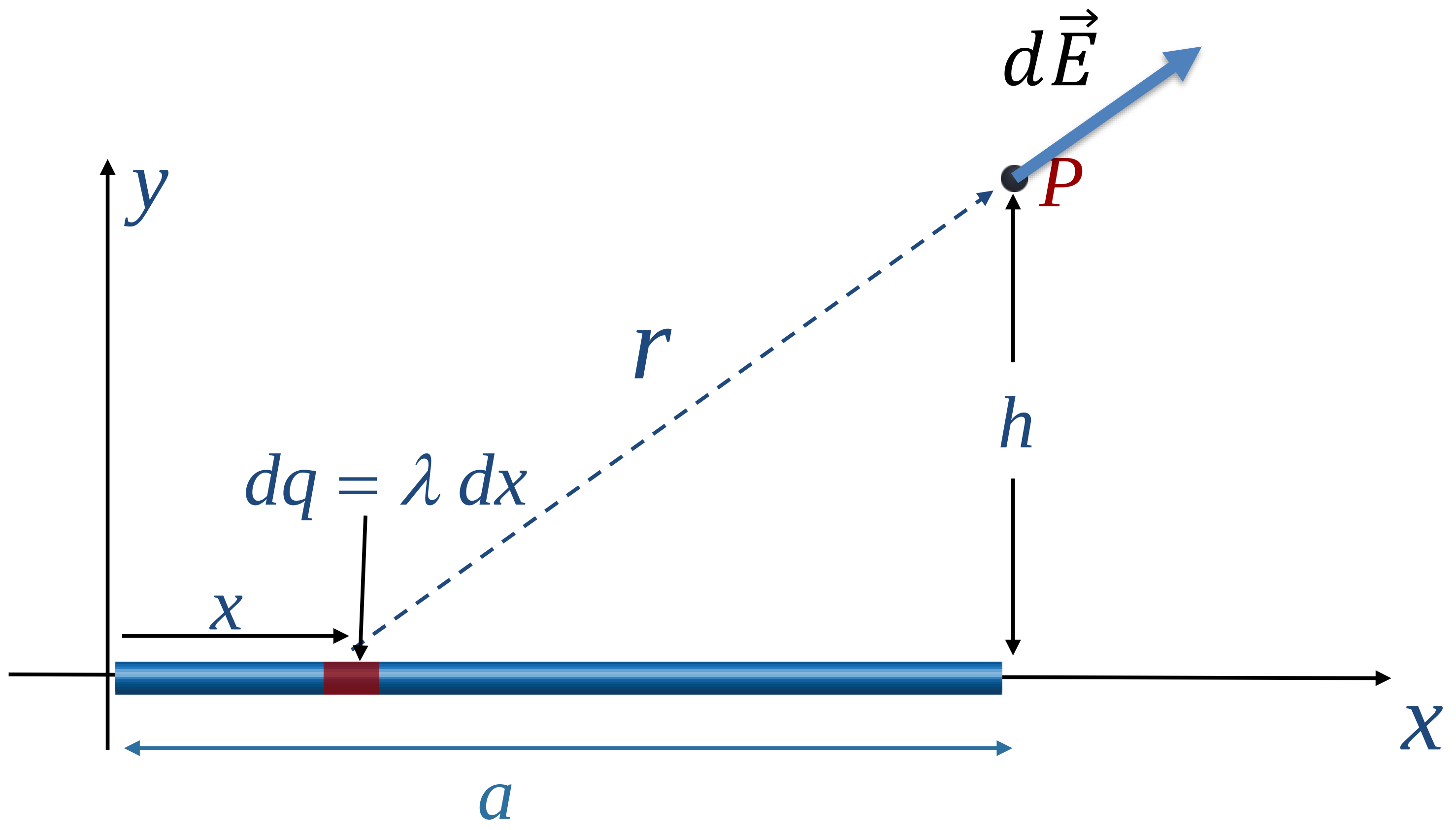
$$c^2 = a^2 + b^2$$

$$r^2 = (a - x)^2 + h^2$$

$$r = \sqrt{(a - x)^2 + h^2}$$

Clicker Question: Line of Charge

A charge Q is uniformly distributed along the x-axis from the origin to $\boldsymbol{x = a}$. The charge density is $+\boldsymbol{\lambda}$. What is the x-component of the electric field at point $\boldsymbol{P}$ located at $(\boldsymbol{a}, \boldsymbol{h})$?

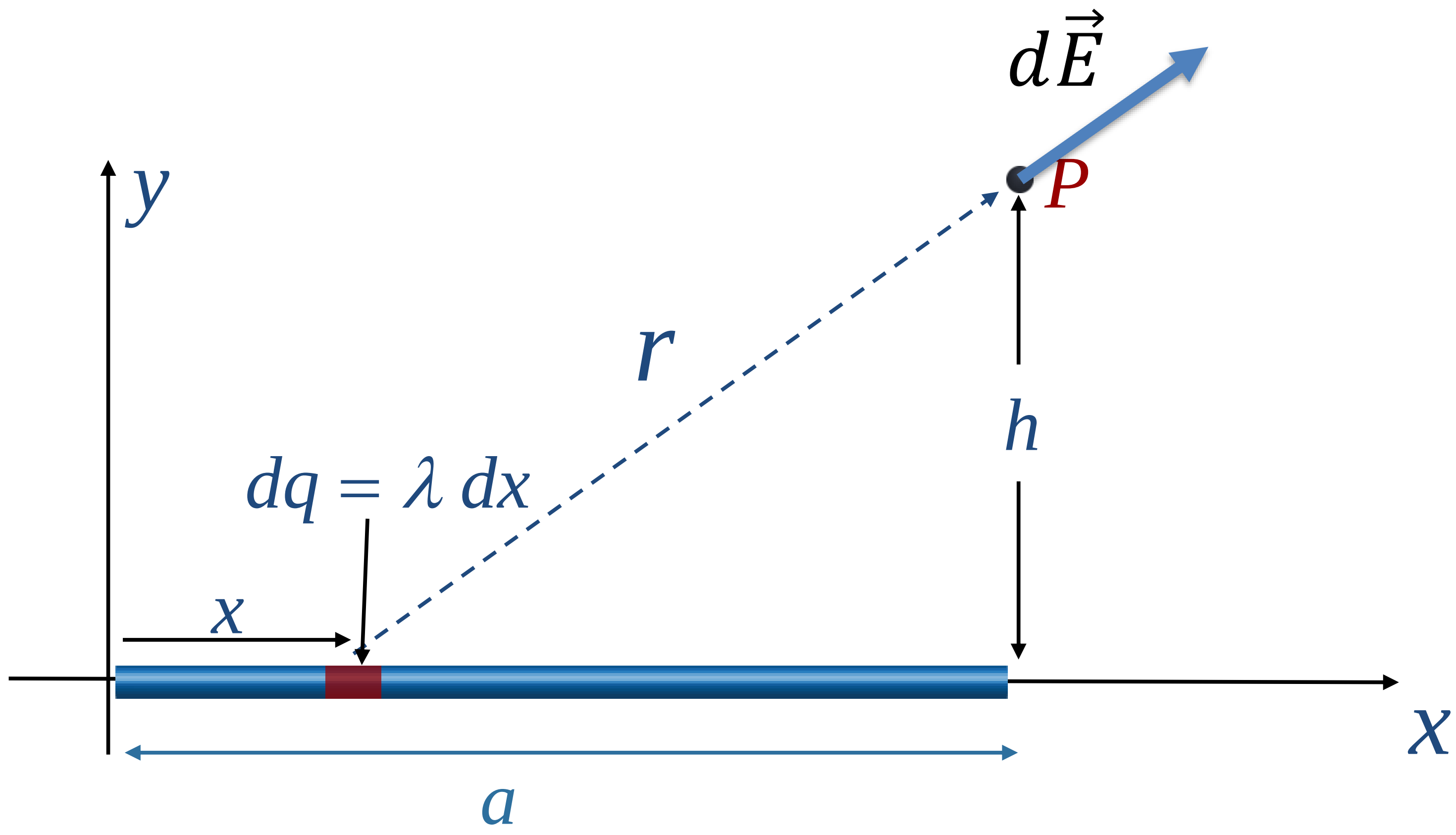


Which is correct? dE/k =

A) $\dfrac{dx}{x^2}$ B) $\dfrac{dx}{a^2+h^2}$ C) $\dfrac{\lambda dx}{a^2+h^2}$ D) $\dfrac{\lambda dx}{(a-x)^2+h^2}$ E) $\dfrac{\lambda dx}{x^2}$

Clicker Question: Line of Charge Answer

A charge Q is uniformly distributed along the x-axis from the origin to $\boldsymbol{x} = \boldsymbol{a}$. The charge density is $+\boldsymbol{\lambda}$. What is the x-component of the electric field at point $\boldsymbol{P}$ located at $(\boldsymbol{a}, \boldsymbol{h})$?



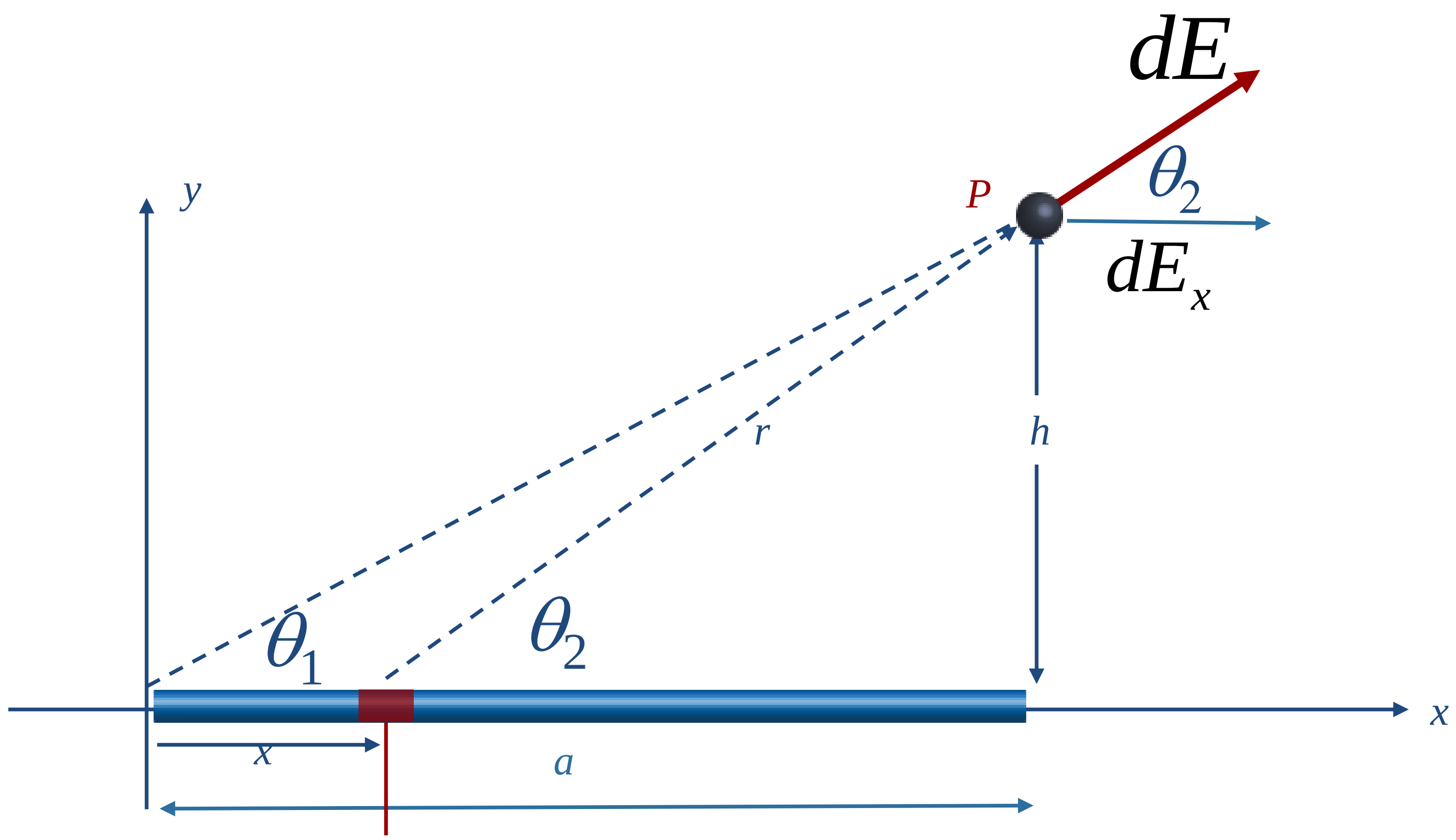
Which is correct? dE/k =

A) $\dfrac{dx}{x^2}$ B) $\dfrac{dx}{a^2+h^2}$ C) $\dfrac{\lambda dx}{a^2+h^2}$ D) $\boxed{\dfrac{\lambda dx}{(a-x)^2+h^2}}$ E) $\dfrac{\lambda dx}{x^2}$

$$\overrightarrow{dE} = k\frac{dq}{r^2}\hat{r} \qquad dq = \lambda dx \qquad r^2 = (a-x)^2 + h^2$$

Example: Line of Charge - 3

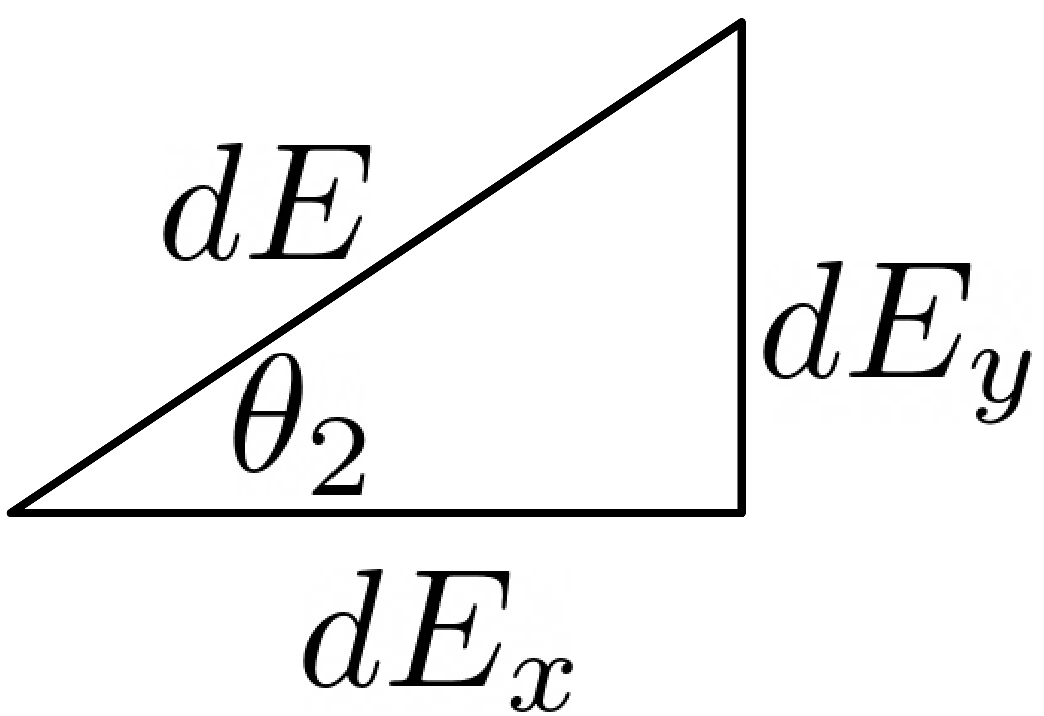
A charge Q is uniformly distributed along the x-axis from the origin to $\boldsymbol{x} = \boldsymbol{a}$. The charge density is $+\boldsymbol{\lambda}$. What is the x-component of the electric field at point $\boldsymbol{P}$ located at $(\boldsymbol{a}, \boldsymbol{h})$?



5) Break $d\vec{E}$ into components if needed

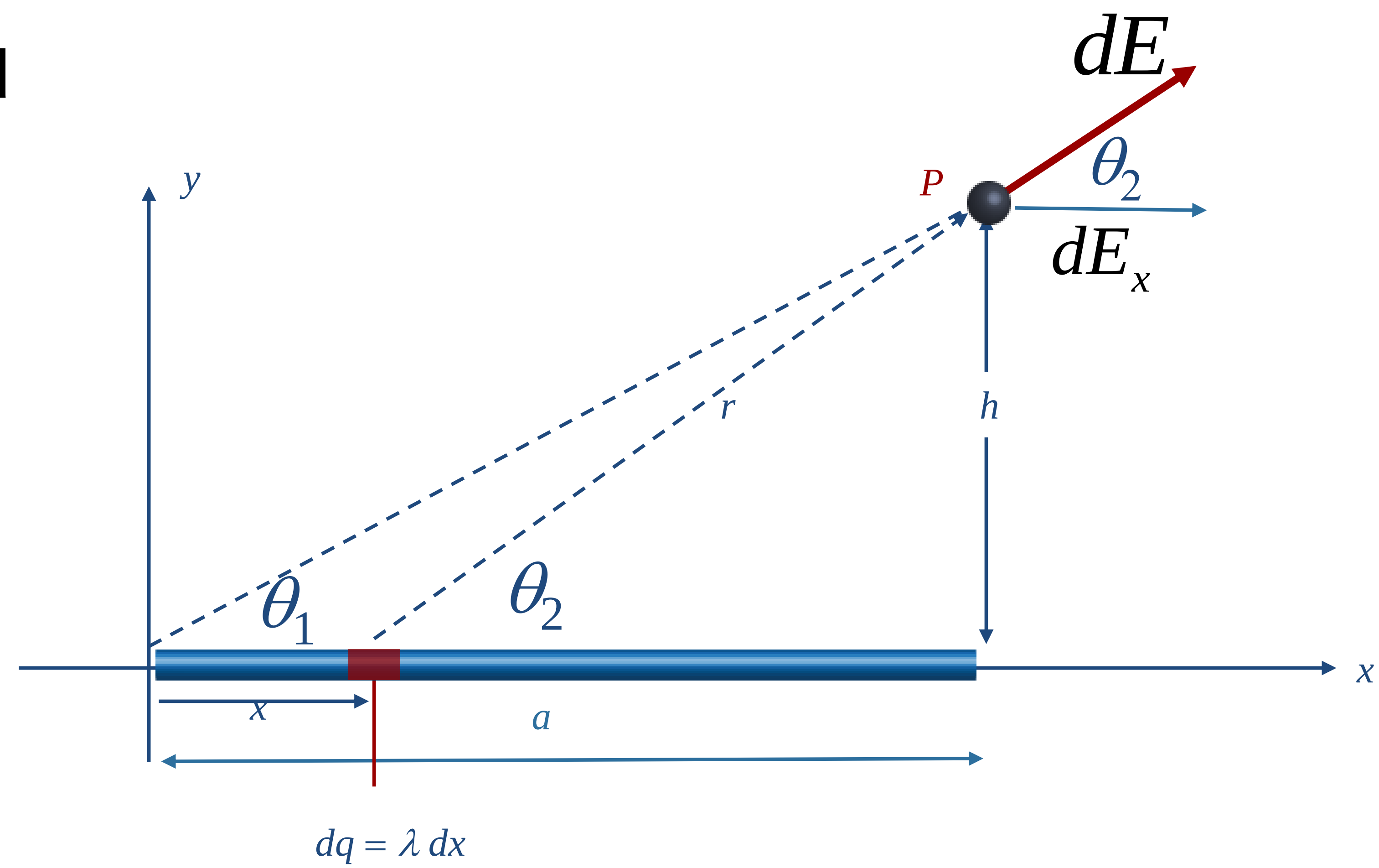
$$dE_x = dE\cos\theta_2$$

$$dE_y = dE\sin\theta_2$$



Example: Line of Charge - 4

A charge Q is uniformly distributed along the x-axis from the origin to $\boldsymbol{x} = \boldsymbol{a}$. The charge density is $+\boldsymbol{\lambda}$. What is the x-component of the electric field at point $\boldsymbol{P}$ located at $(\boldsymbol{a}, \boldsymbol{h})$?



6) Integrate each component of $d\vec{E}$ separately

$$dE = k\frac{dq}{r^2}$$

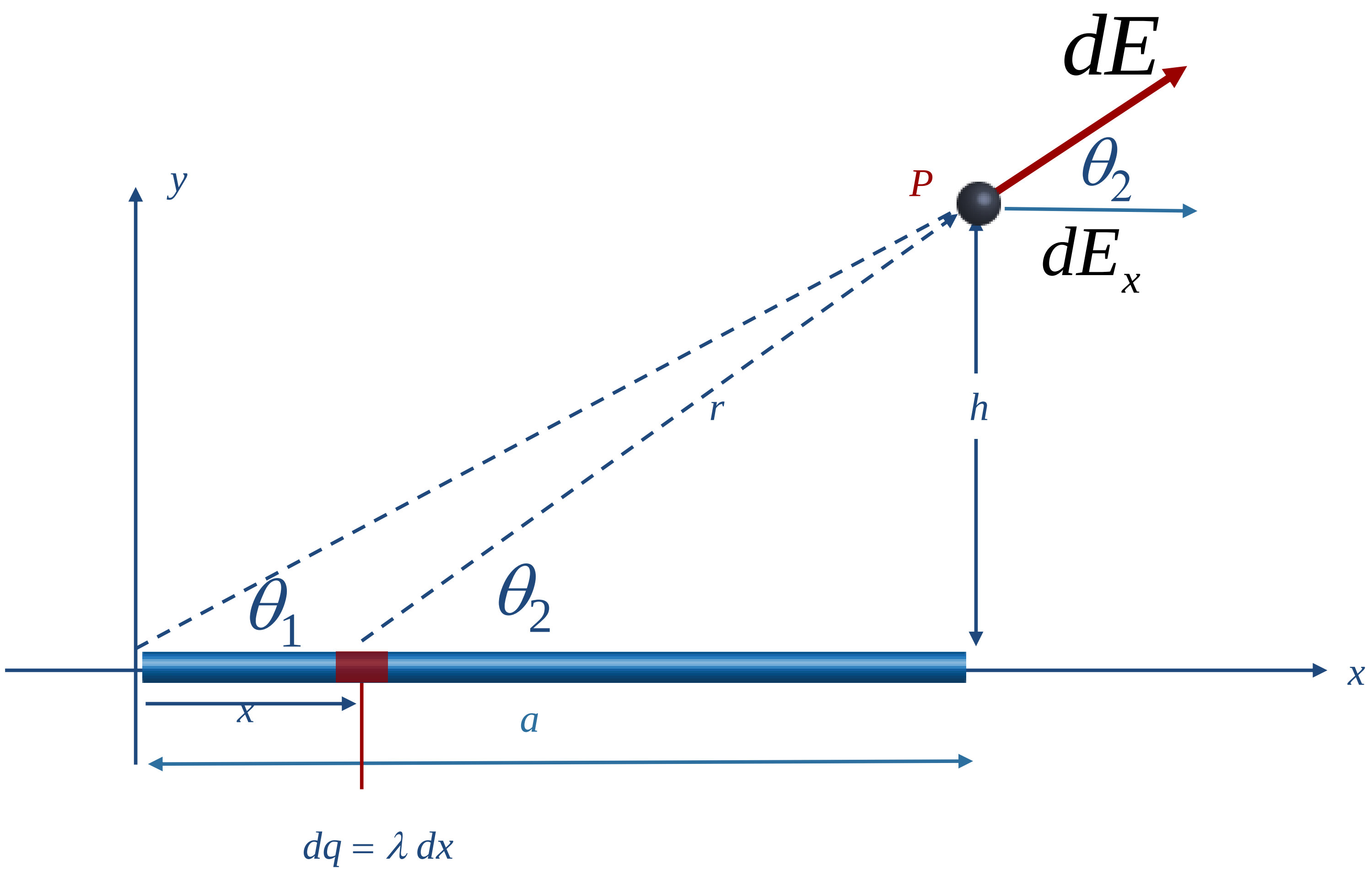
$$Ex = \frac{\lambda}{4\pi\epsilon_0}\int_0^a \frac{\cos\theta_2 \lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE_x = \int dE\cos\theta_2$$

Need to get theta 2 in terms of x.

Clicker Question: Geometry Review

A charge Q is uniformly distributed along the x-axis from the origin to $\boldsymbol{x = a}$. The charge density is $+\boldsymbol{\lambda}$. What is the x-component of the electric field at point $\boldsymbol{P}$ located at $(\boldsymbol{a}, \boldsymbol{h})$?



What is $\cos\theta_2$?

A) $\dfrac{x}{\sqrt{a^2 + h^2}}$

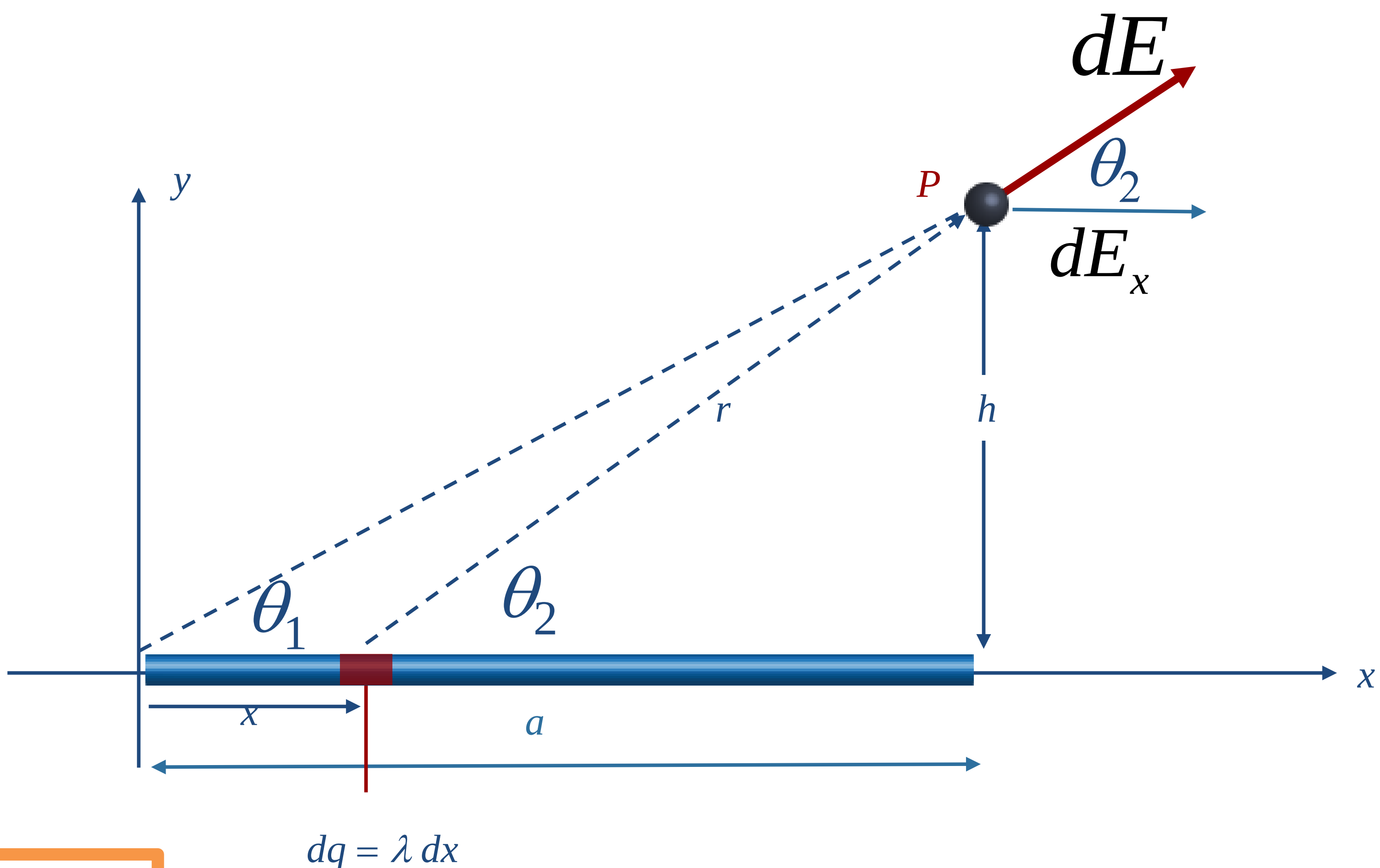
B) $\dfrac{a - x}{\sqrt{(a - x)^2 + h^2}}$

C) $\dfrac{a}{\sqrt{a^2 + h^2}}$

D) $\dfrac{a}{\sqrt{(a - x)^2 + h^2}}$

Clicker Question: Geometry Review Answer

A charge Q is uniformly distributed along the x-axis from the origin to $\boldsymbol{x} = \boldsymbol{a}$. The charge density is $+\boldsymbol{\lambda}$. What is the x-component of the electric field at point $\boldsymbol{P}$ located at $(\boldsymbol{a}, \boldsymbol{h})$?



What is $\cos\theta_2$?

A) $\dfrac{x}{\sqrt{a^2+h^2}}$

B) $\boxed{\dfrac{a-x}{\sqrt{(a-x)^2+h^2}}}$

C) $\dfrac{a}{\sqrt{a^2+h^2}}$

D) $\dfrac{a}{\sqrt{(a-x)^2+h^2}}$

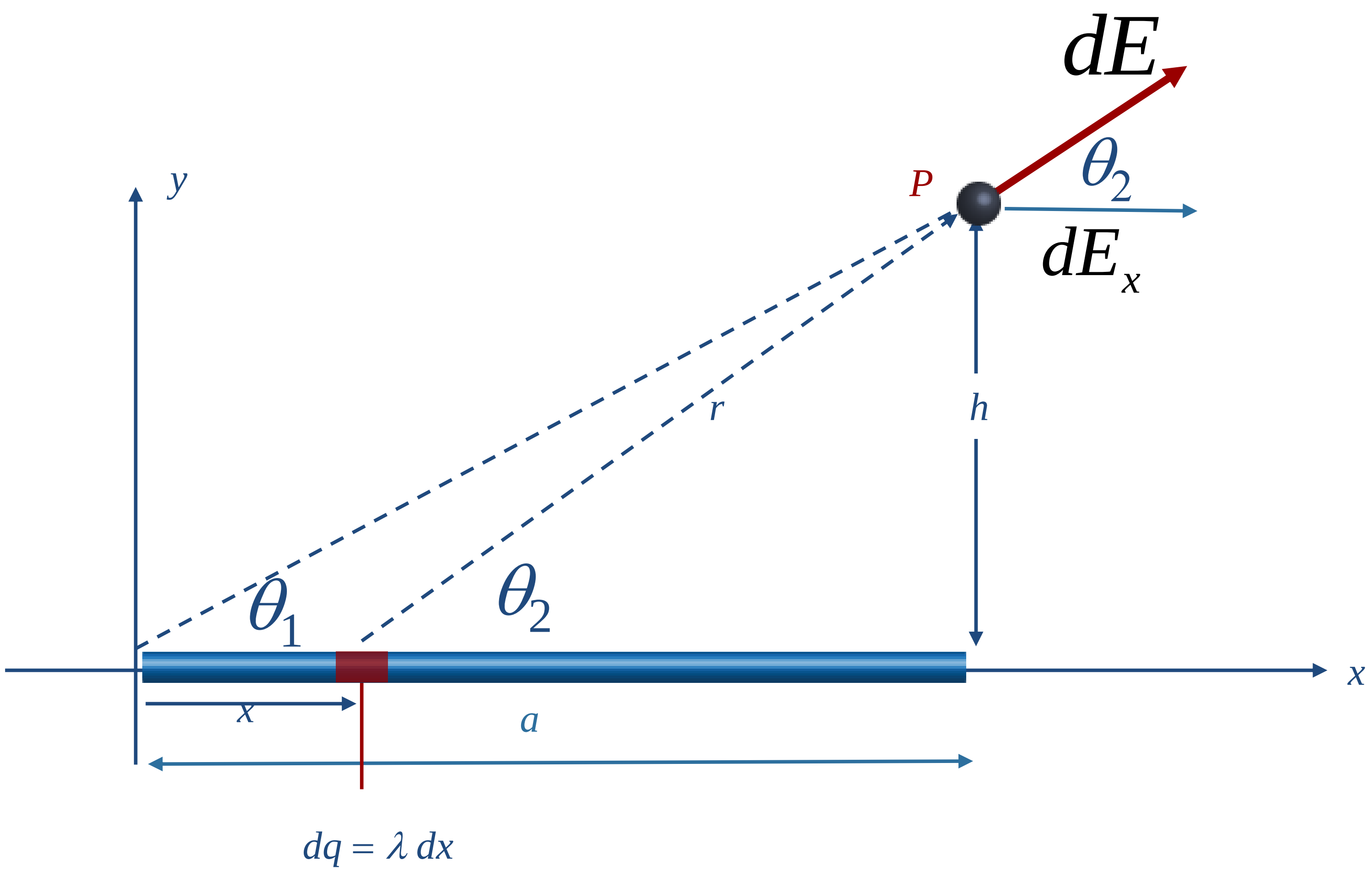
$$\tan\theta_2 = \frac{h}{(a-x)}$$

$$\cos\left(\arctan\left(\frac{w}{v}\right)\right) = \frac{1}{\sqrt{\left(\frac{w}{v}\right)^2+1}}$$

$$\cos\left(\arctan\left(\frac{w}{v}\right)\right) = \frac{v}{\sqrt{v^2+w^2}}$$

Example: Line of Charge - 5

A charge Q is uniformly distributed along the x-axis from the origin to $\boldsymbol{x} = \boldsymbol{a}$. The charge density is $+\boldsymbol{\lambda}$. What is the x-component of the electric field at point $\boldsymbol{P}$ located at $(\boldsymbol{a}, \boldsymbol{h})$?



What is $E_x(p)$?

$$Ex(P) = \frac{\lambda}{4\pi\epsilon_0}\int_0^a \frac{a-x}{\left((a-x)^2+h^2\right)^{\frac{3}{2}}}dx$$

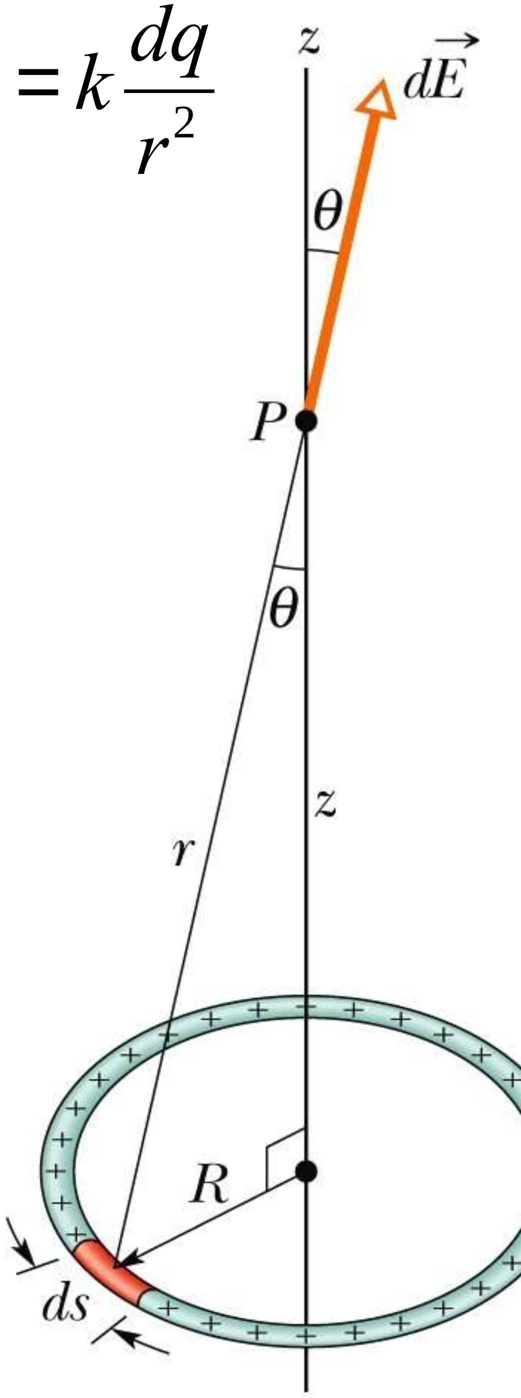
From a table of integrals: $$\int \frac{x\,dx}{(x^2+c^2)^{3/2}} = -\frac{1}{\sqrt{(x^2+c^2)}}$$

$$\boxed{E_x(P) = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{h} - \frac{1}{\sqrt{a^2+h^2}}\right)}$$

The Electric Field Due to a Charged Ring

Find the electric field due to a ring of total charge, Q, at a point, P, along the axis.

$$dE = k\frac{dq}{r^2}$$

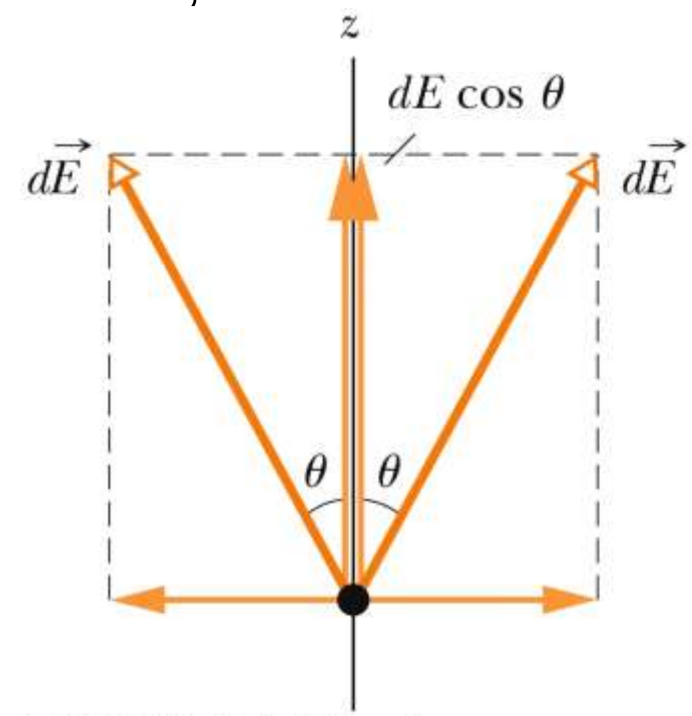


Step 1: Arbitrary element of charge is in orange and r is the vector from dq to point P.

Step 2: Draw electric field at point P due to dq.

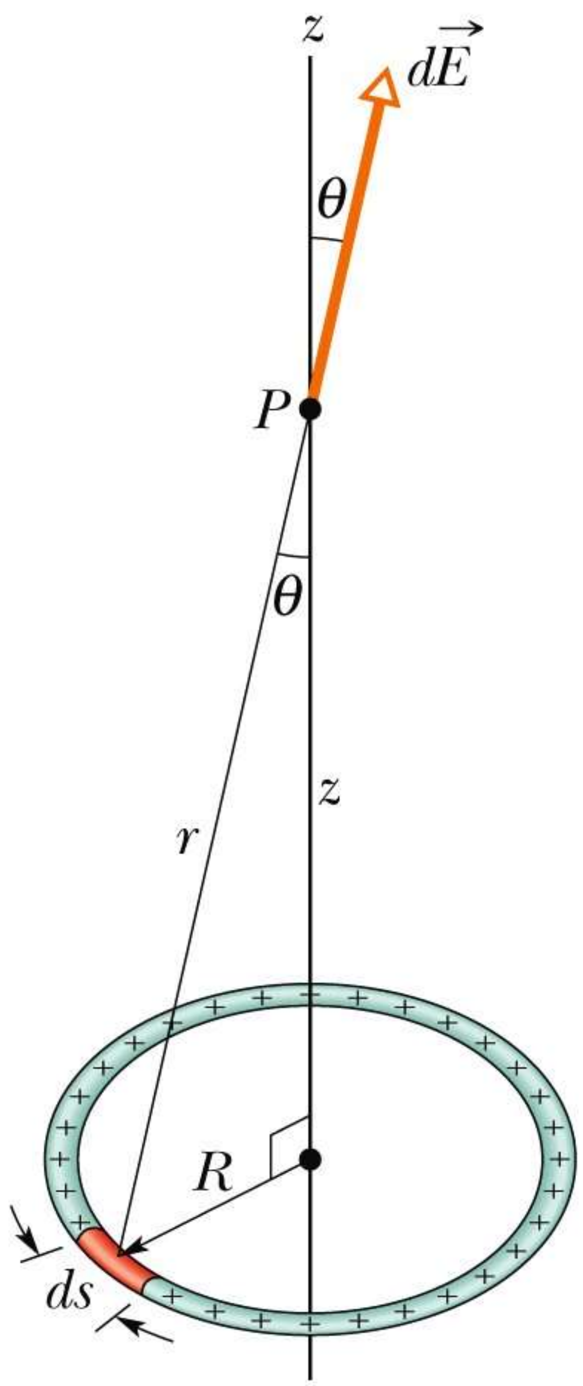
Consider the charge element on two opposite sides of the ring. They both create a field of magnitude *d**E***, but the vectors are in different directions.

Only need to solve for dE_z



The two components perpendicular to the z axis cancel. All around the ring, this cancelation occurs, so we can neglect all the perpendicular components.

The Electric Field Due to a Charged Ring



Find the electric field due to a ring of total charge Q, at a point P, along its central axis.

Step 3: Write dq in terms of geometry of the problem.

$$dq = \lambda ds \qquad \lambda = \frac{Q}{L}$$

Step 4: Express r in terms of variables.

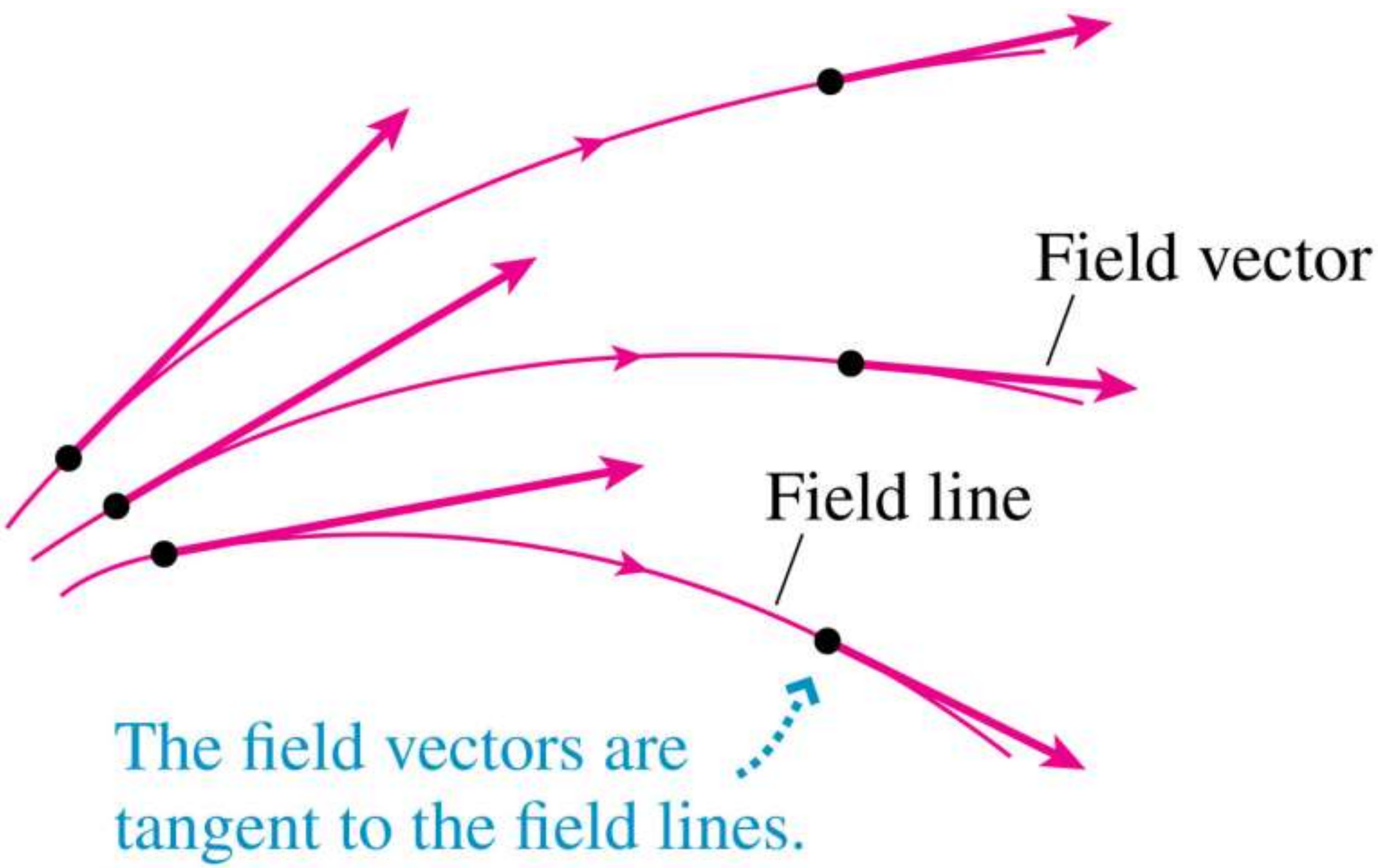
$$r^2 = R^2 + z^2$$

Step 5: Break dE into components.

From the figure we see that the parallel components (dE_z) each have magnitude $dE\cos\theta$.

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}$$

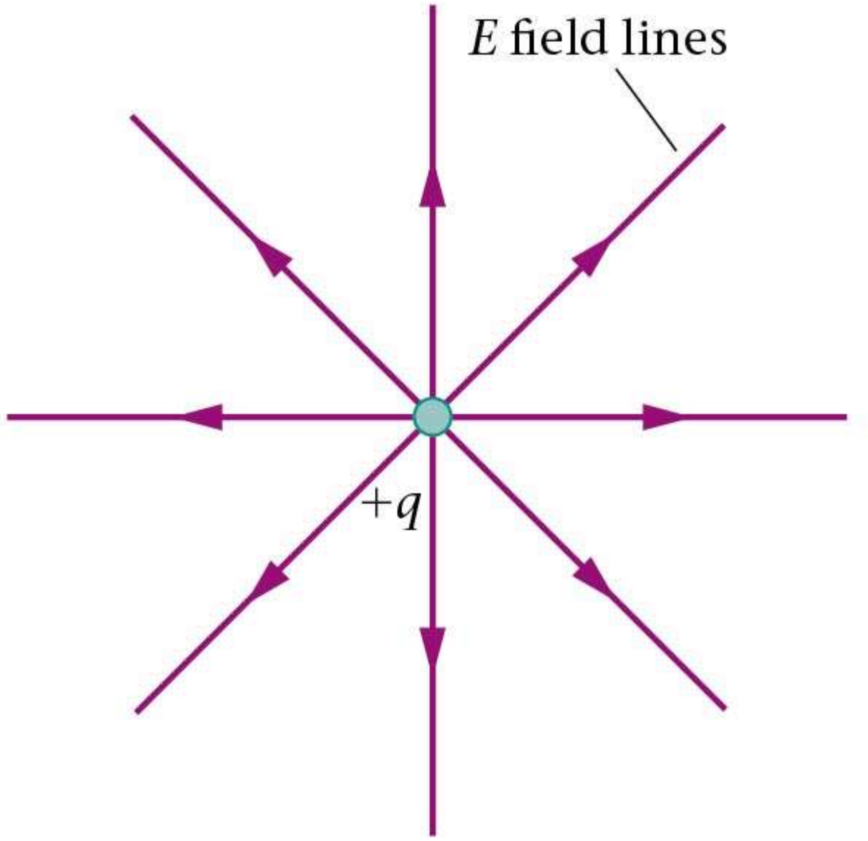
Electric Field Lines



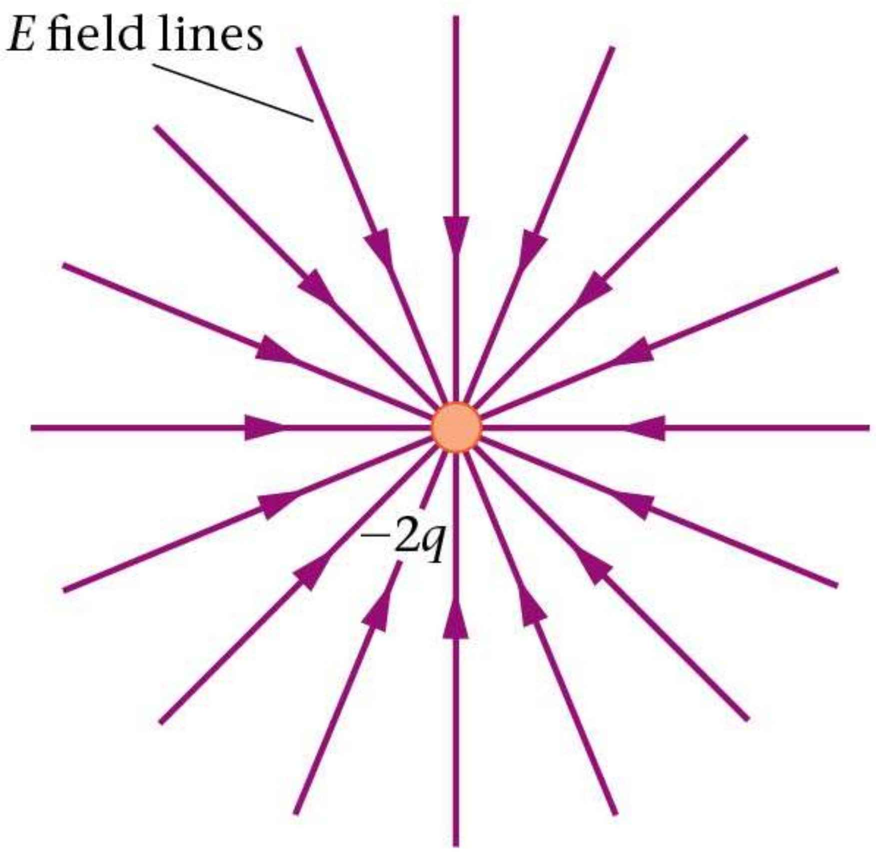
- Electric field lines are **continuous** curves tangent to the electric field vectors.
- Closely spaced field lines indicate a greater field strength.
- Electric field lines start on positive charges and end on negative charges.
- Electric field lines never cross.

Electric Field Lines of Point Charges

The charge on the right is twice the magnitude of the charge on the left (and opposite in sign), so there are twice as many field lines, and they point toward the charge rather than away from it.

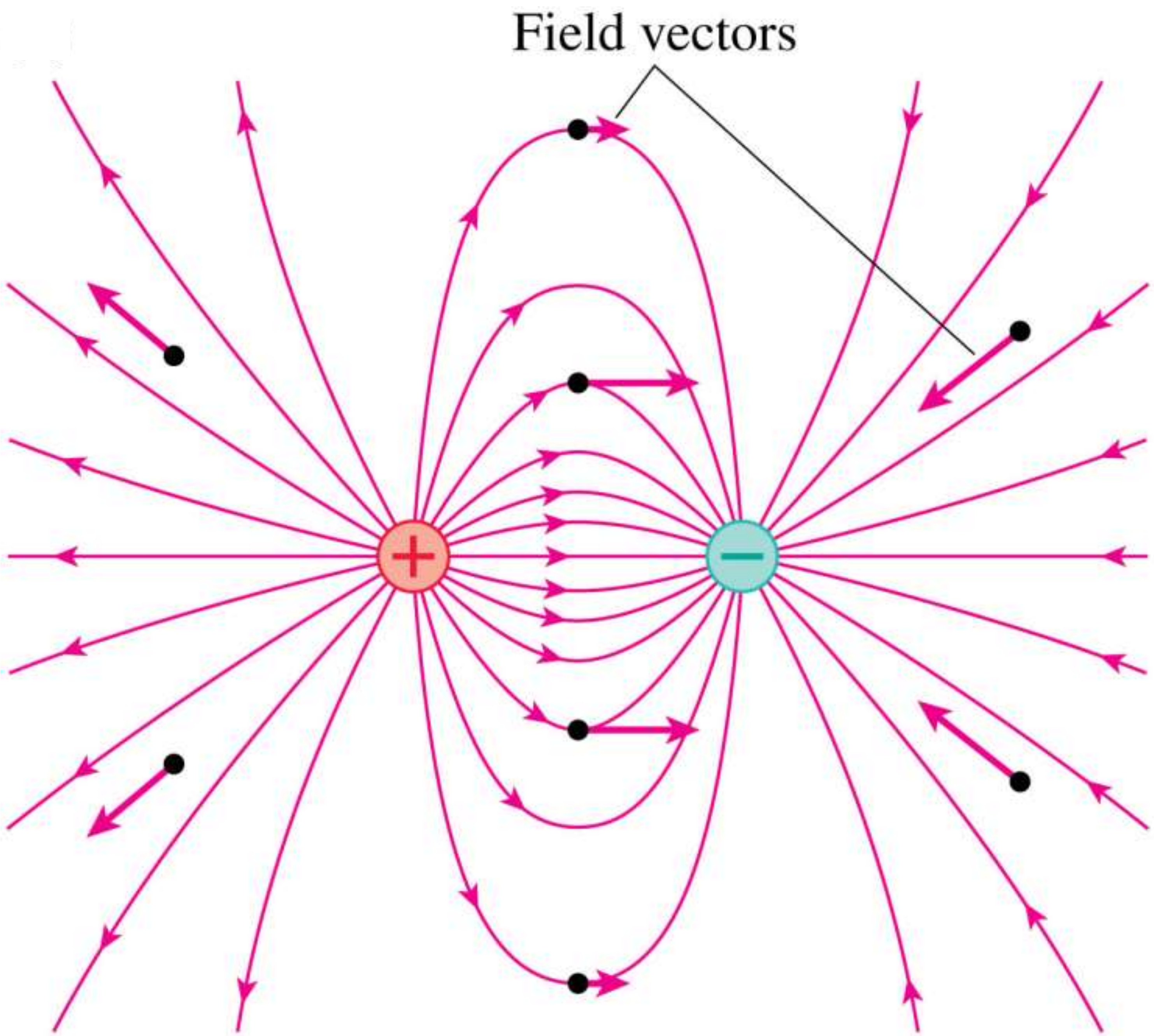


(a) *E* field lines point away from positive charges



(b) *E* field lines point toward negative charges

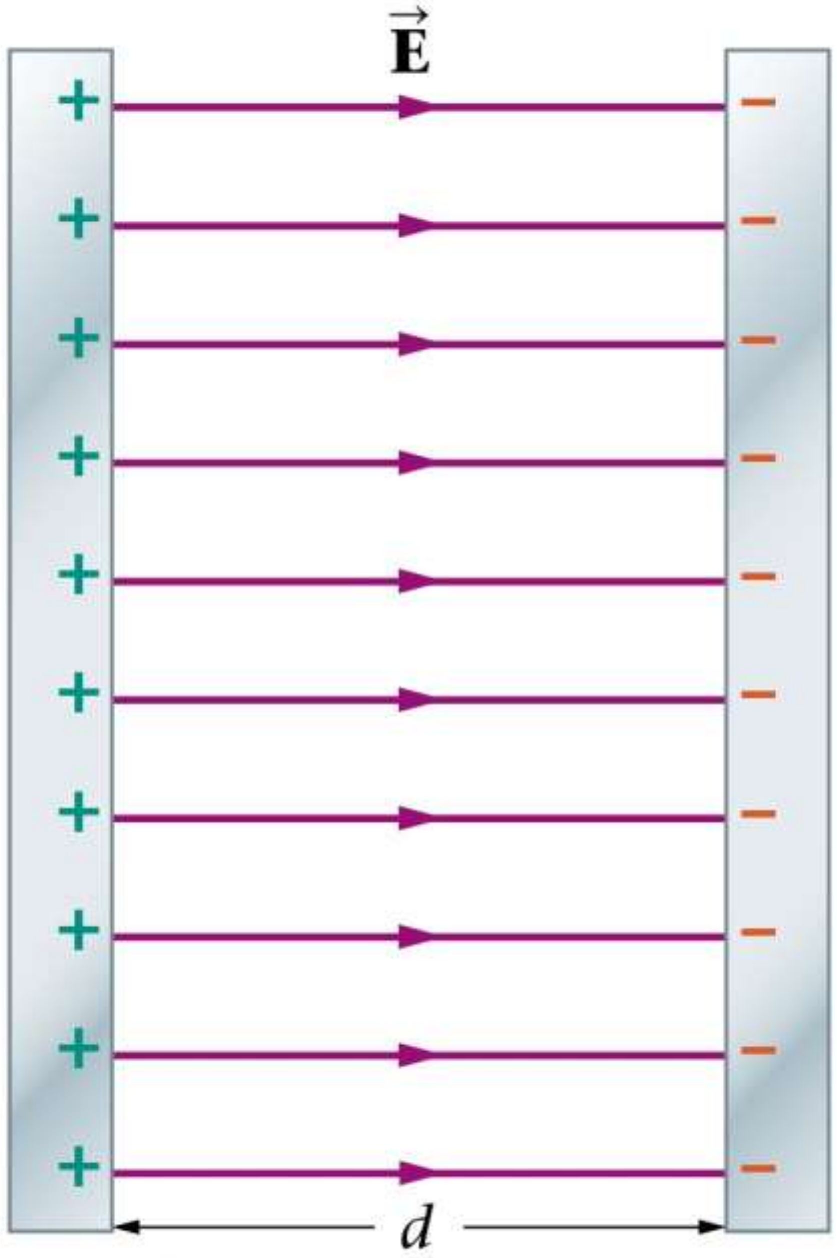
Electric Field Lines of a Dipole Configuration



This figure represents the electric field of a dipole using electric field lines.

Electric Field Lines of a Parallel Plate Configuration

- The device consists of plates of positive and negative charge
- The total electric field between the plates is constant
- The field outside the plates is zero
- This configuration is commonly used in capacitors



3.1 Problem-solving Tutorial Session

E-Fields II; E-Field Lines

Today's tutorial problems can be found at the following HuskyCT location:

Course Contents

>> 3 – E-Fields II; E-Field Lines; Electric Flux; Gauss's Law I

>> 3.1 - E-Fields II; E-Field Lines

>> Tutorial 3.1