Physics 1502Q:
4.2 Electric Potential Energy and Electric Potential I

Announcements & Reminders

- Pre-lab due **at the beginning of your lab this week**
- Reading Assignment due **Sunday at 11:59 PM**
- Homework due **Monday at 11:59 PM**
- **Office Hours:**
  - Posted on HuskyCT
- **No paper quiz this semester**
- **Midterm Exam 1:**
  - 4 pm, Friday, February 18th
  - Equation sheet will be provided
  - More information will be made available as soon as possible

Preview of this week and next week

Su	M	T	W	Th	F	Sa
6 Reading Assignment Due 11:59 PM	7 HW Due 11:59 PM	8 Gauss's Law II	9	10 Electric Potential Energy Electric Potential I	11 Lab 4: Electric Potential Pre-lab 4 Due before lab	12
13 Reading Assignment Due 11:59 PM	14 HW Due 11:59 PM	15 Electric Potential II	16	17 Capacitance	18 NO LAB NO PRELAB Exam 1 4 PM	19

LEARNING GOALS

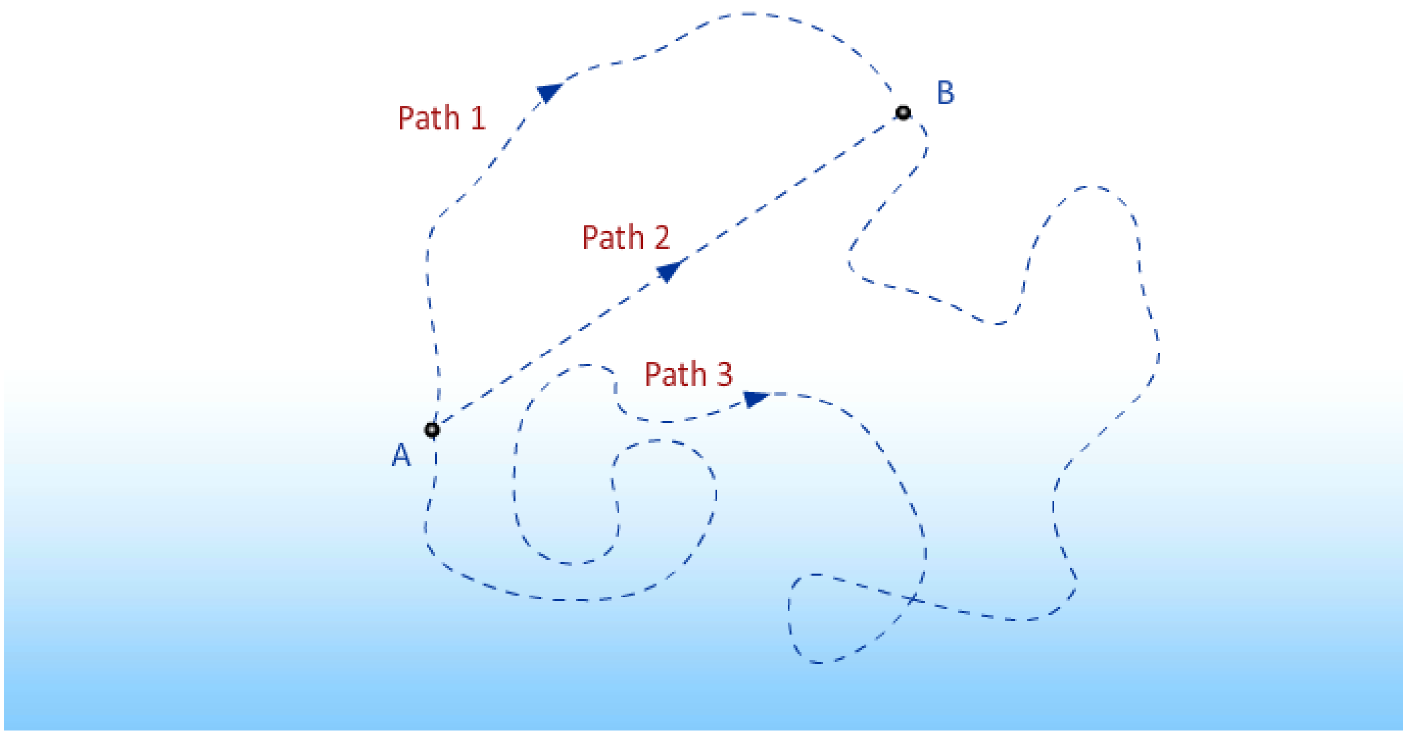
By the end of this unit, you should be able to:

- Explain how the electric force is a conservative force
- Relate potential energy to the work done by a conservative force
- Calculate the potential energy of a charge configuration
- Calculate the energy necessary to rearrange a system of charges
- Define electric potential
- Calculate the electric potential from the electric field
- Understand conceptually equipotentials and how they relate to electric field lines

Coulomb force is conservative

Conservative Forces

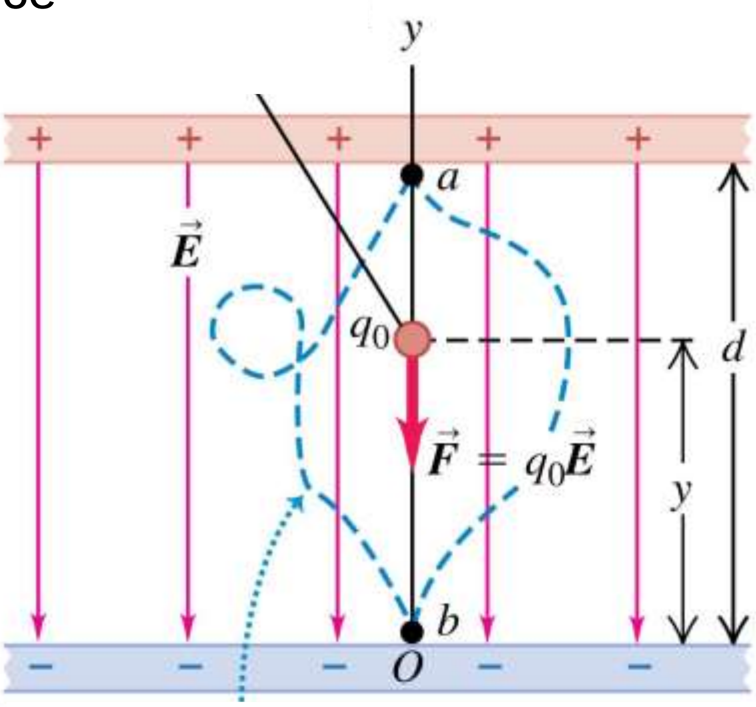
$$W_{Path\,1} = W_{Path\,2} = W_{Path\,3} = U_A - U_B$$



Conservative and Non-conservative Forces

A force is **conservative** if the work it does on the object is independent of the path the object takes going from y_i to y_f. The work done by that force is 0 J if $y_i = y_f$.
Example: gravitational force, spring force, **electric force**

A force is **non-conservative** if the work it does on the object does depend on the path the object takes going from x_i to x_f.
Example: frictional force



Review of Work and Energy Principles

- **Definition of work:** $W = \int_i^f \vec{F} \cdot d\vec{r}$
  - Work done by constant force: $W = \vec{F} \cdot \vec{d} = Fd\cos\theta$
- **Kinetic energy (*K*):** Energy associated with motion of particles

$$K = \frac{1}{2}mv^2$$

- **Potential energy (*U*):** Energy associated with position of particles
  - Gravitational potential energy (near surface of planet/star/etc.):

$$U_{grav} = mgy$$

  - Gravitational potential energy (general formula):

$$U_{grav} = -G\frac{m_1 m_2}{r}$$

Work and Potential Energy

Because the electric force is conservative, a system of charged particles has an electric potential energy *U* associated with its configuration.

$$\Delta U \equiv -W_{conservative}$$

If electric force does **negative work**, potential energy **increases**.

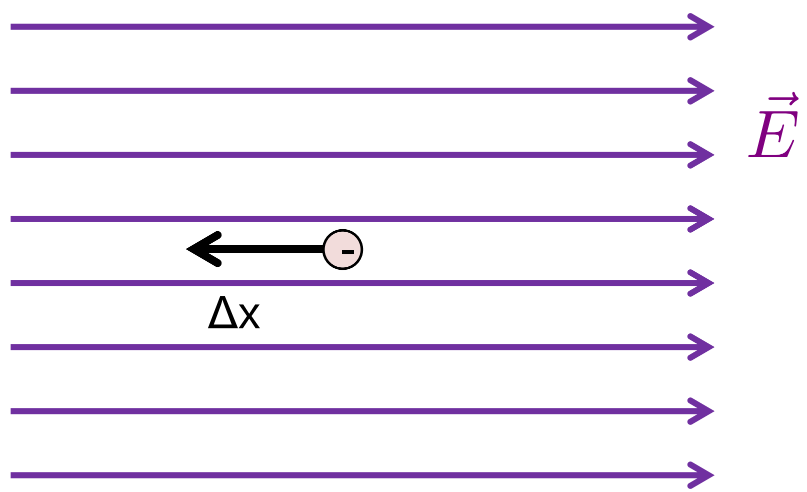
→ Analogous to moving uphill…

If electric force does **positive work**, potential energy **decreases**.

→ Analogous to moving downhill… (this is what nature “wants”)

Clicker Question: Work

An electron moves to the left in a uniform electric field, as shown below.



Which of the following is true?

A) The electric field does positive work on the electron.

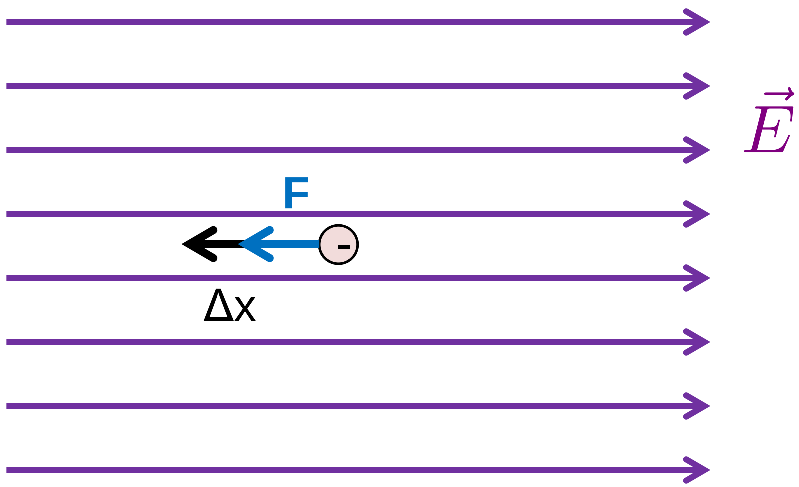
B) The electric field does negative work on the electron.

C) The electric field does no work on the electron.

D) The electric field does work on the electron, but we cannot determine if it is positive or negative.

Clicker Question: Work

An electron moves to the left in a uniform electric field, as shown below.



The displacement and force are pointing in the same direction, so the work is positive (dot product gives a positive number)

Which of the following is true?

A) The electric field does positive work on the electron.

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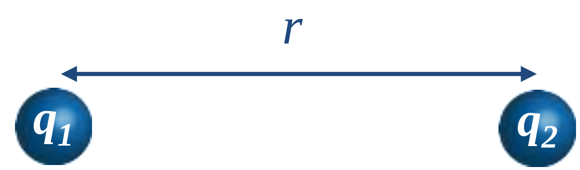
Electric Potential Energy

For a pair of charges:



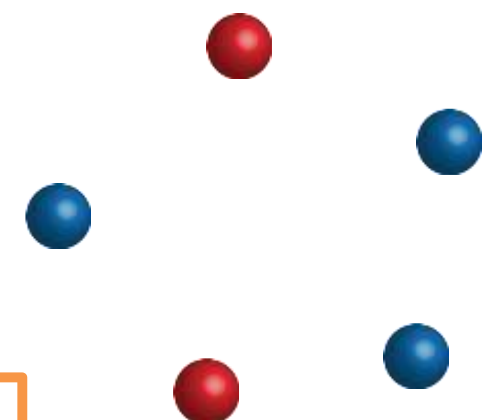
Potential energy $U = k\dfrac{q_1 q_2}{r}$ SI unit: Joule (J)

(We usually choose U = 0 to be where the charges are far apart)

For a collection of charges:

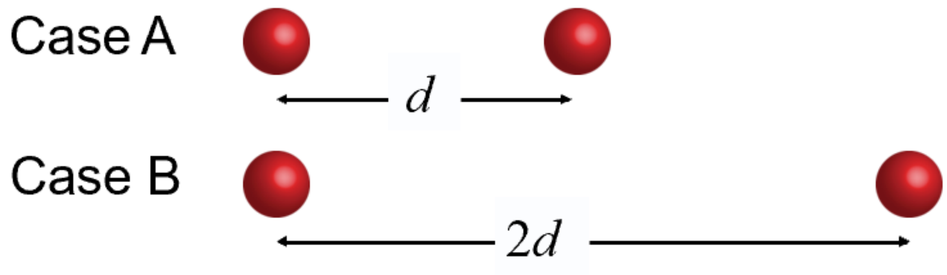
Charges can be positive or negative

Sum up $U = \sum_{i \neq j} k\dfrac{q_i q_j}{r_{ij}}$ for all pairs



Recall that energy is a scalar quantity, so no “direction” is necessary in the calculation of the energy

Clicker Question: Potential Energy

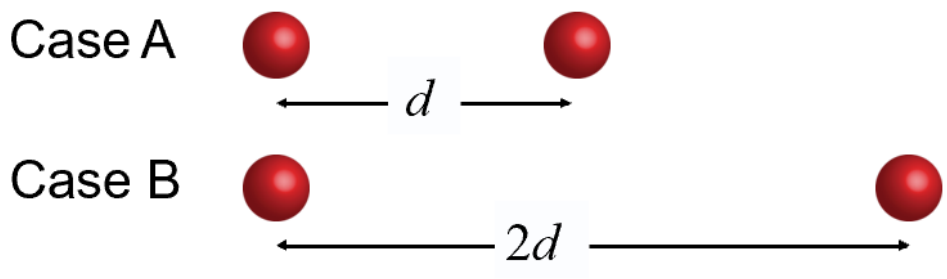


In Case A two negative charges which are equal in magnitude are separated by a distance d. In Case B the same charges are separated by a distance 2d. Which configuration has the higher potential energy?

A) Case A

B) Case B

Clicker Question: Potential Energy



In Case A two negative charges which are equal in magnitude are separated by a distance d. In Case B the same charges are separated by a distance 2d. Which configuration has the higher potential energy?

$\boxed{\text{A) Case A}}$

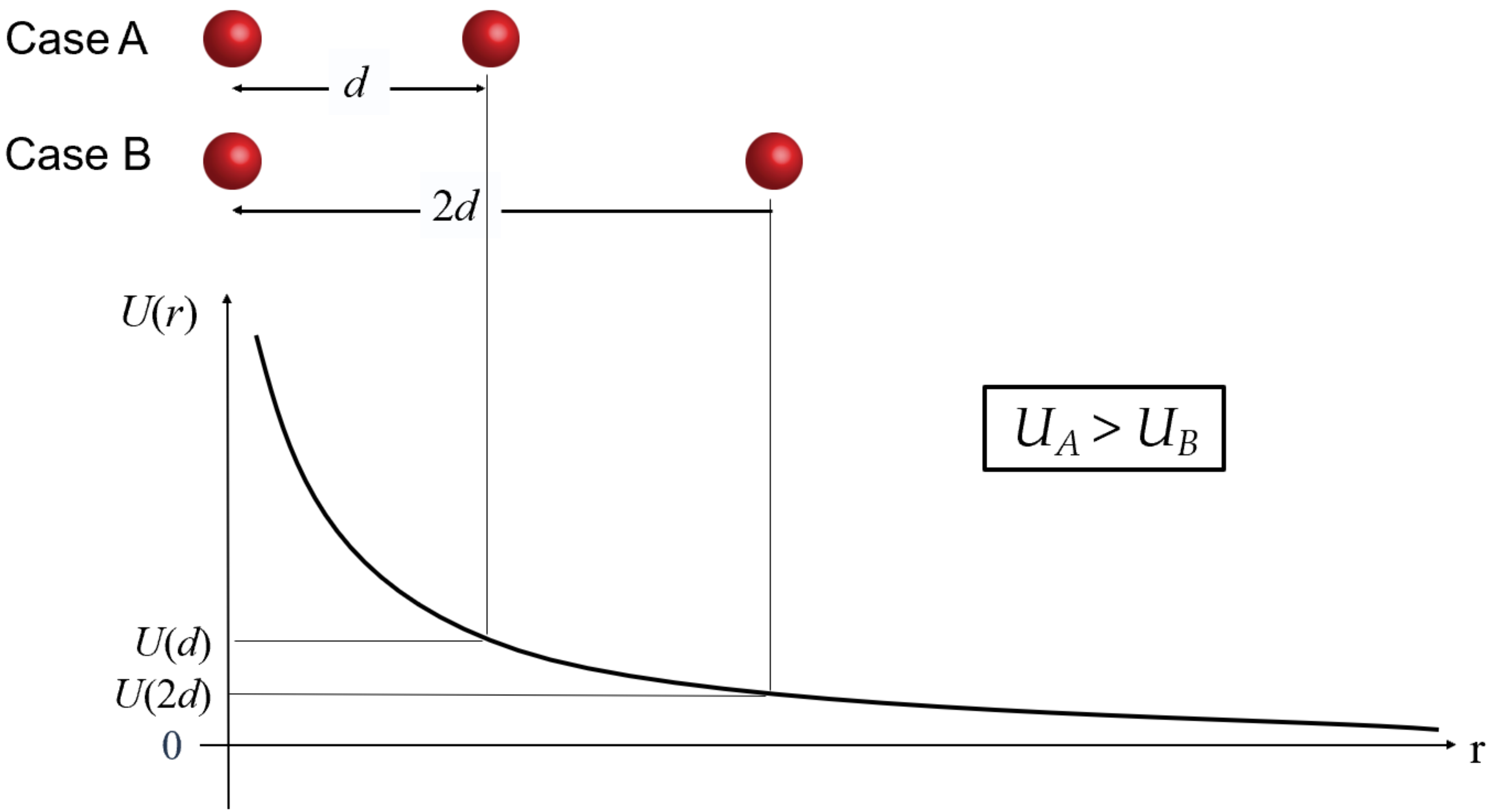
B) Case B

Think about which system is in its more unnatural / stressed state…

Clicker Question: Potential Energy - Discussion

We define $U = 0$ to be at infinity:

$$U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}$$



$$\boxed{U_A > U_B}$$

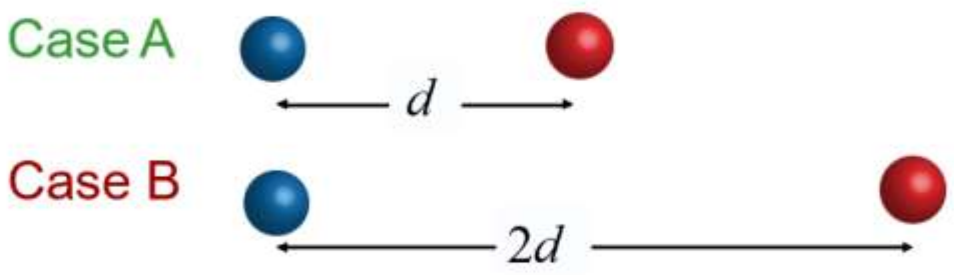
How to reduce electric potential energy…

In order to reduce their electric potential energy:

+ve charges move in the direction of E-field

-ve charges move in opposite direction to E-field

Clicker Question: Potential Energy

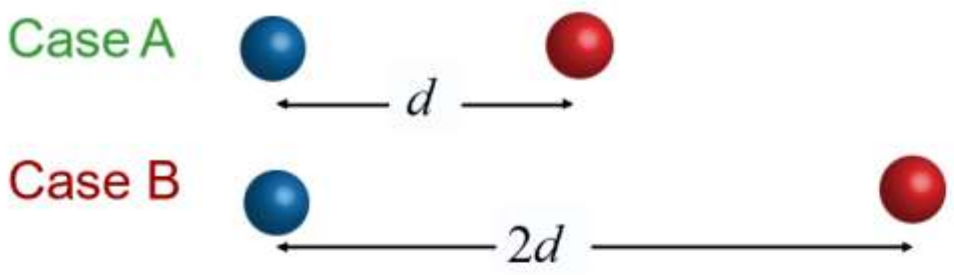


In case A two charges which are equal in magnitude but opposite in sign are separated by a distance d. In case B the same charges are separated by a distance $2d$. Which configuration has the higher potential energy?

A) Case A

B) Case B

Clicker Question: Potential Energy



In case A two charges which are equal in magnitude but opposite in sign are separated by a distance *d*. In case B the same charges are separated by a distance *2d*. Which configuration has the higher potential energy?

A) Case A

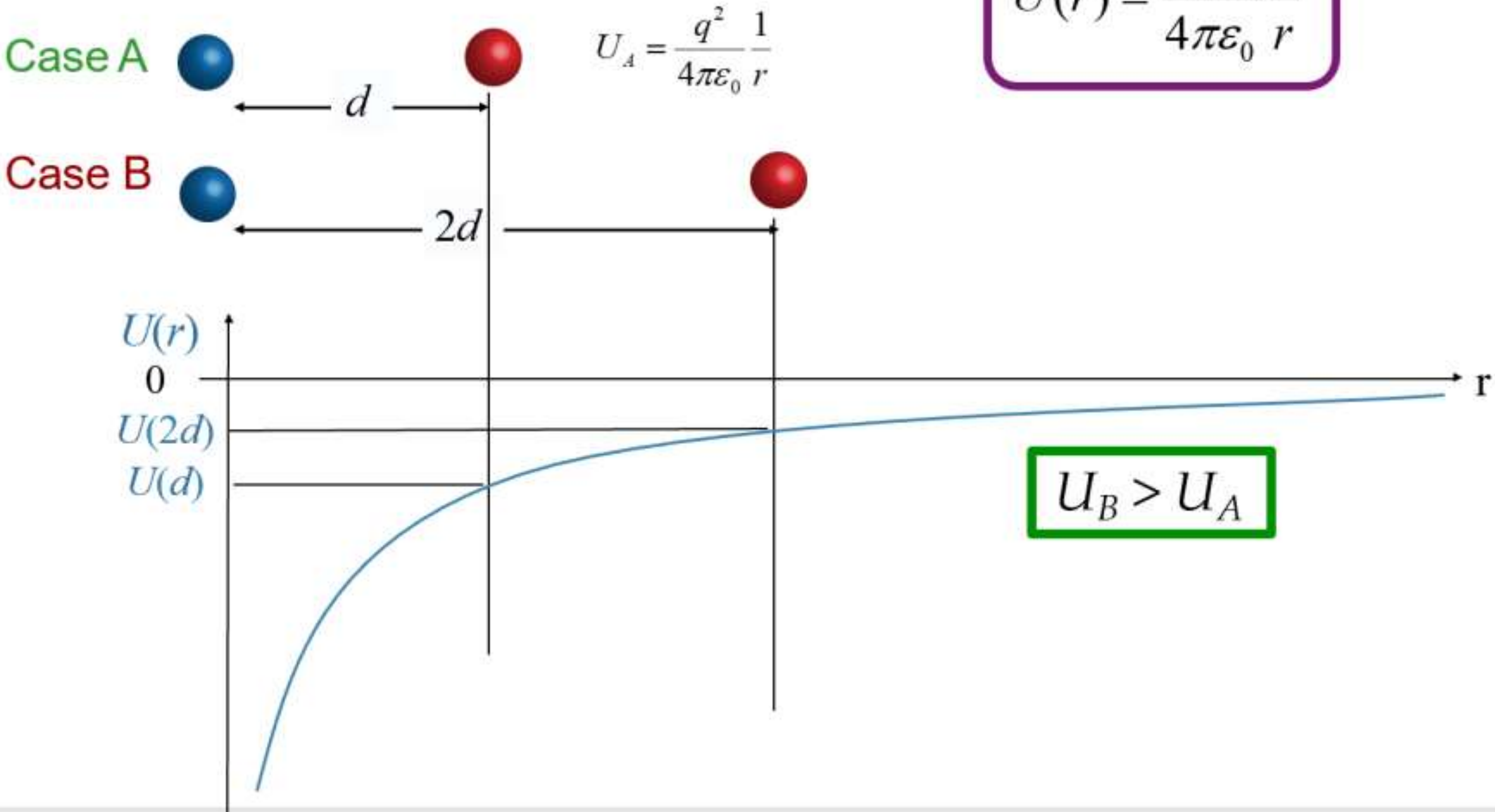
B) Case B

Think about which system is in its more unnatural / stressed state…

Clicker Question Discussion

As usual, choose $U = 0$ to be at infinity:

$$\boxed{U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}}$$



$$\boxed{U_B > U_A}$$

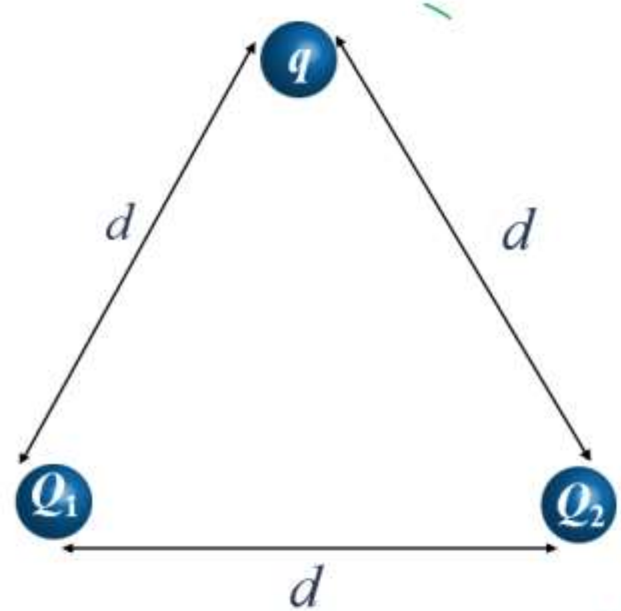
Potential Energy for System of Charges

Two charges are separated by a distance d. What is the change in potential energy when a third charge q is brought from far away to a distance d from the original two charges?

$$\Delta U = \frac{qQ_1}{4\pi\varepsilon_0}\frac{1}{d} + \frac{qQ_2}{4\pi\varepsilon_0}\frac{1}{d}$$

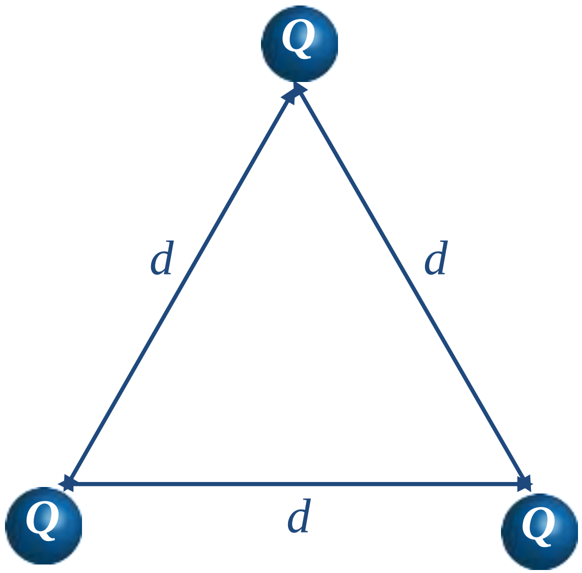
Work done by the Coulomb interaction (force) between q and Q_1

Work done by the Coulomb interaction (force) between q and Q_2



Example problem: Energy required to build array of point charges

What is the total energy required to bring in three identical charges, from infinitely far away to the points on an equilateral triangle shown?



Work to bring in first charge:

$$W_1 = 0\,, \qquad \Delta U_1 = 0$$

Work to bring in second charge :

$$\Delta U_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

Work to bring in third charge :

$$\Delta U_3 = \frac{1}{4pe_0}\frac{Q^2}{d} + \frac{1}{4pe_0}\frac{Q^2}{d} = \frac{2}{4pe_0}\frac{Q^2}{d}$$

$$\Delta U = \Delta U_1 + \Delta U_2 + \Delta U_3 = \frac{3}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

$$\boxed{\Delta U = +\frac{3}{4\pi\varepsilon_0}\frac{Q^2}{d}}$$

Electric Potential

U always depends on the magnitude of *q*, but the potential energy per unit charge at any point in an electric field does not depend on *q*.

We call the potential energy per unit charge the electric potential.

electric potential

$$\boxed{\Delta V = \frac{\Delta U}{q}}$$

electric potential
=
Electric potential difference
=
Potential
=
voltage

unit of electric potential: Volts (1 V = 1 J/C)

- Electric potential is a scalar.
- Electric potential can be positive or negative

Please note that:

Electric Potential Energy $\Delta U \neq$ Electric Potential ΔV

$$\Delta V = \frac{\Delta U}{q}$$

Similar relationship to Coulomb force F and Electric Field E

Charges moving in region with electric potential

As a charged particle moves through a changing electric potential, energy is conserved:

	Electric potential	
	Increasing ($\Delta V > 0$)	Decreasing ($\Delta V < 0$)
+ charge	Slows down	Speeds up
− charge	Speeds up	Slows down

Conservation of Energy

When no nonconservative forces do work on a system, the total mechanical energy is constant:

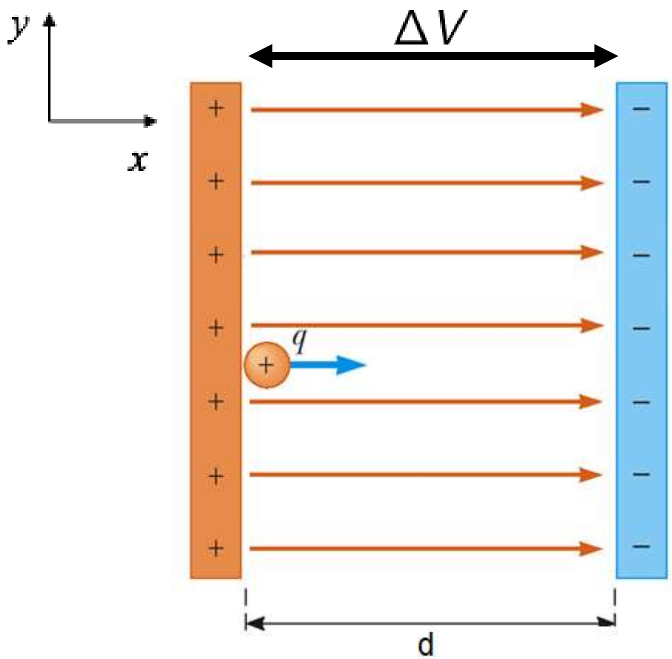
$$K_f + U_f = K_i + U_i$$

Sub in $U = qV$

$$K_f + qV_f = K_i + qV_i$$

Example Problem

A parallel plate of separation d = 2cm, has a potential difference of 5V across the plates. A proton starts from rest at the positive plate and accelerates toward the negative plate.



What is the speed of the proton the moment it reaches the negative plate?

Use conservation of energy

$$\cancel{K_i} + U_i = K_f + U_f$$

$$K_f = U_i - U_f$$

$$K_f = q(V_i - V_f)$$

$$\frac{1}{2}mv^2 = q(|\Delta V|)$$

Potential decreases toward the right, so

$$V_i > V_f$$

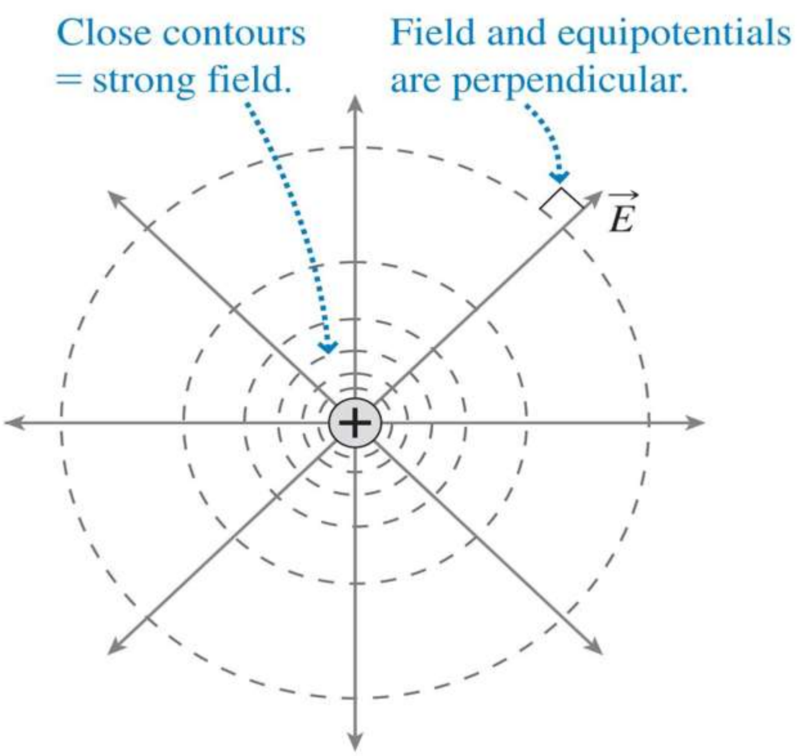
$$v = \left(\frac{2q|\Delta V|}{m}\right)^{1/2}$$

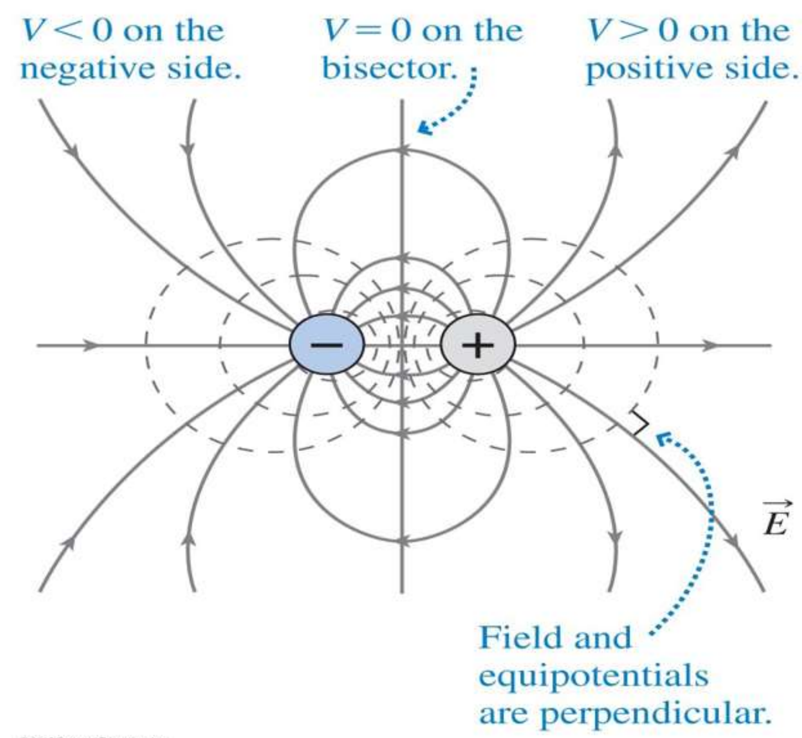
$$= 3.1 \times 10^4 \text{ m/s}$$

Equipotential Surfaces

An **equipotential surface** is a surface (real or imaginary) on which **all points have the same electric potential.**

No work is required to move a charge along the surface
- The potential energy is the same everywhere





Electric Potentials and Fields

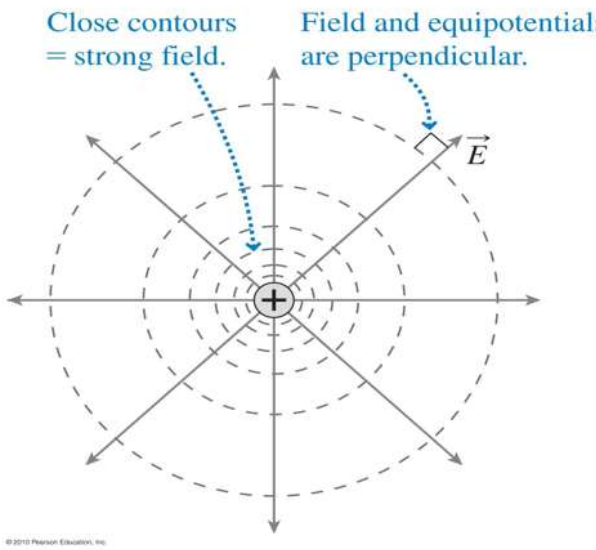
How to get the Electric Field from the Potential:

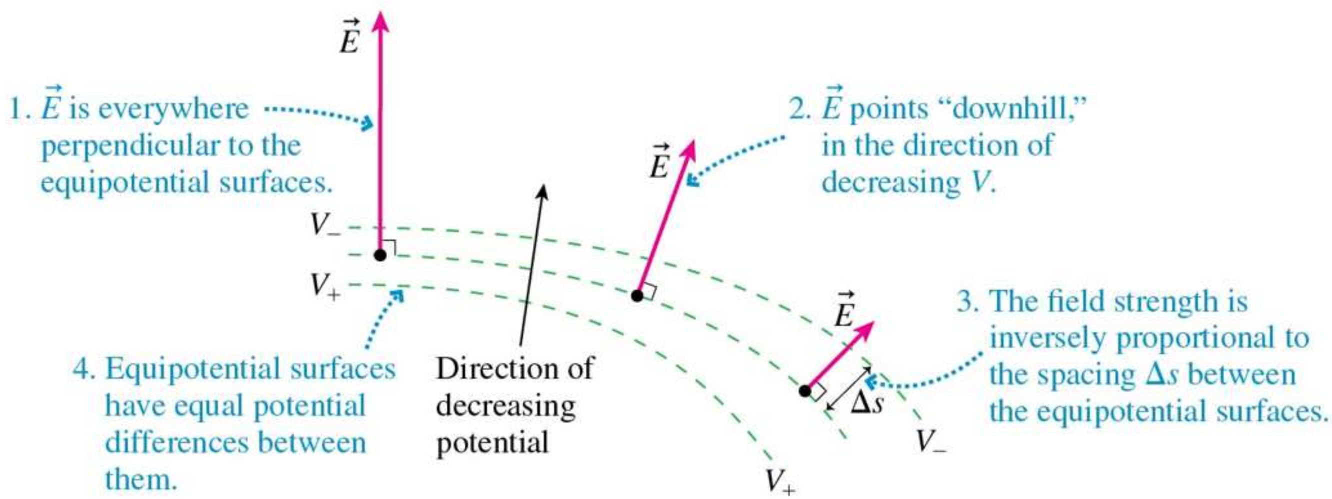
Qualitatively:

1. Equipotential surfaces are always **perpendicular** to electric field lines.

2. The electric field always points in the direction of **decreasing** potential

3. The electric field is stronger when the equipotential lines are closer together

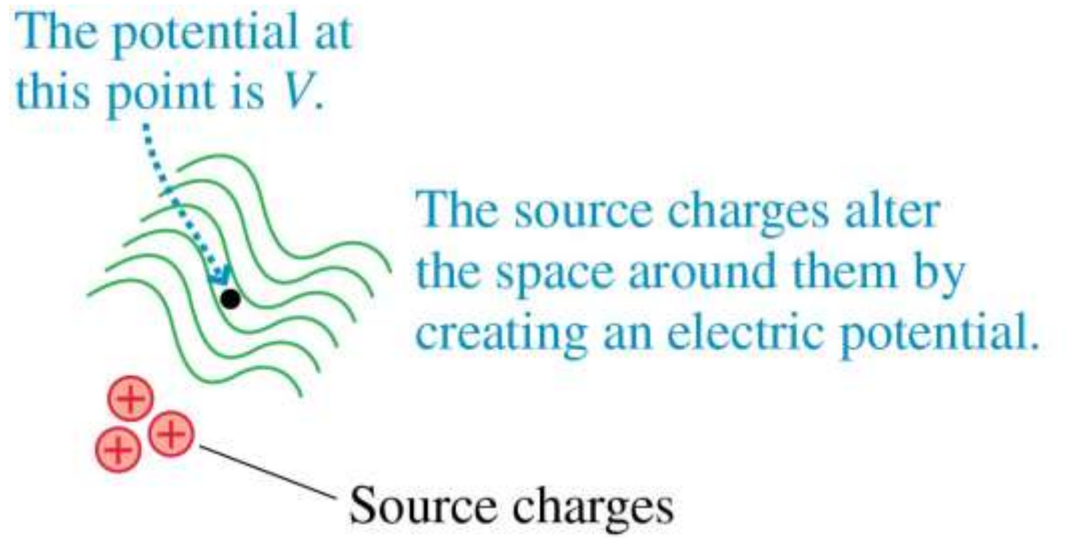


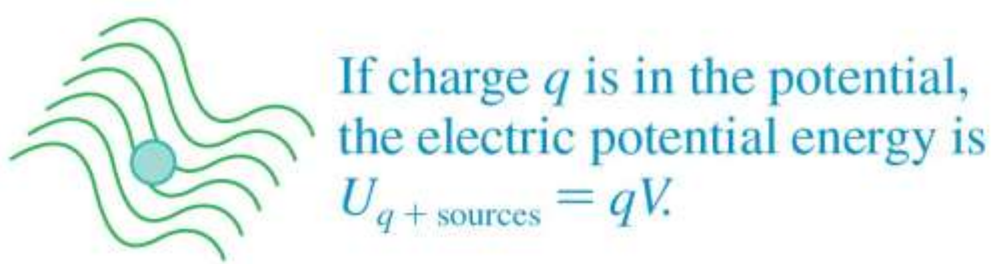


Electric Potential of a Point Charge

- The electric potential, like the electric field, is a property of the source charges.

- A test charge q can be used as a probe to determine the electric potential, but the value of V is *independent of* q.





The Potential of a Point Charge (when V=0 at ∞)

Potential of a point charge:

$$V = k\frac{q}{r}$$

Property of **"the space"** just like E-field

Potential of many point charges:

$$V = k\sum_i \frac{q_i}{r_i}$$

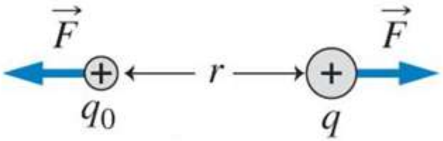
distance of i^th charge from point where *V* is calculated

Potential of a continuous distribution of charge:

$$V = k\int \frac{dq}{r}$$

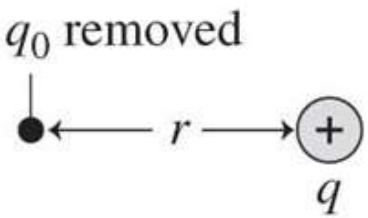
Case of the Point Charge

- Electric Force: $F = \dfrac{kqq_0}{r^2}$
  - Force the charges exert on each other



- Potential Energy: $U = \dfrac{kqq_0}{r}$
  - Potential energy associated with the pair of charges

- Electric Field: $E = \dfrac{F}{q_0} = \dfrac{kq}{r^2}$
  - Field at a point



- Potential: $V = \dfrac{U}{q_0} = \dfrac{kq}{r}$
  - Potential at a point

Clicker Question: Electric Dipole

At the midpoint between these two equal but opposite charges,

A. $E = 0$; $V = 0$

B. $E = 0$; $V > 0$

C. $E = 0$; $V < 0$

D. E points right; $V = 0$

E. E points left; $V = 0$

Clicker Question: Electric Dipole

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