Note: Click **File** then Make a copy to create your own copy you can work on. Be sure to save it!

Math 2110Q Final Portfolio

Fall 2022

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Instructions:

1. Answer the questions directly in this document. For any written parts, you should type in complete sentences. For the parts asking for mathematical work, you may scan your work as a clear PDF or image and insert it in the document. A sample is provided on the next page.
2. When you submit your work, it will be scanned using plagiarism detection software. This means that your typed explanations should be yours alone and show not look similar to other students or resources.
3. Don't use Chegg/Bartleby/other resources. This is your (take-home) final exam. You cannot post these questions online, search for these exact questions or slight variations of these questions, or view solutions to these questions on Chegg or other similar websites.
4. You may work with other students currently taking Math 2110Q to complete the portfolio. However, your examples and explanations should be unique and your own. Any examples used should vary significantly from those presented in lecture notes, the textbook, and from anyone that you may work with (and changing a 4 to a 5, for example, does not count as a significant change). If a question requires you to provide different examples of something, those examples should be significantly different from each other as well.
5. If we discover any overwhelming similarity between your work and that of another student or resource on any portion, then you may receive a 0 on the entire Final Portfolio or an F in the course, regardless of other work that was completed. You will also be reported with evidence to the UConn Office of Community Standards.
6. The portfolio is due on

Monday, December 12th by 11:59 PM.

Portfolio Submission on Gradescope

Instructions:

1. If you use pdf format, you will need to select the pages for each question. **NOT DOING SO COULD RESULT IN A SCORE OF ZERO FOR EACH QUESTION NOT SELECTED PROPERLY.**
2. If you use image format for your submission instead, then make sure to upload only the appropriate image(s) for each question. If you need any assistance with uploading multiple image files, Professor Hall made a video that will hopefully help you: https://youtu.be/lYaQtN9QJvE
3. Gradescope is unable to accept a mixture of formats; you need to use image files for every question or a single pdf document where you will select the page(s) for each question after it is uploaded.
4. The portfolio is divided into 6 chapters, and each chapter will have 2 questions (e.g., 4.1 and 4.2). That means there will be a total of 12 questions that you will need to upload work for on Gradescope.
5. Your work **must** be in order and labeled clearly to receive credit. If we have to search for your answers, re-select any pages, or anything is out of order, you will not receive full credit.
6. The first question in each chapter has multiple parts. Answer each part and upload them all together for the first question. **PLEASE MAKE SURE TO CLEARLY LABEL EACH PART.**
7. For the second question in each chapter, you will choose a question to complete. Regardless of which you choose, upload your work and explanation as the answer for the second question and **CLEARLY LABEL YOUR WORK "A" OR "B."**
8. After the deadline has passed, you will not be able to re-select any pages or change your submission, so please plan accordingly. It is especially important that you check to make sure everything is selected and placed correctly.

Note: We strongly recommend getting everything into one Google Doc and then exporting it as a pdf file in order to upload to Gradescope. Then, make sure to select the page(s) for each question, and you are all set!

Sample Answer Format

Question requiring symbols and/or calculations:

Ex: Give an example of a situation where we need to change the order of integration and another where we need to change to polar coordinates.

We need to change to order of integration in

$$\int_0^1 \int_x^1 \cos(y^2)\, dy\, dx$$

since we can't integrate $\cos(y^2)$ with respect to y.

We need to change to polar coordinates in

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} e^{x^2+y^2}\, dx\, dy$$

since we can't integrate $e^{x^2+y^2}$ with respect to x.

Question requiring a written answer or explanation:

Ex: How do we think about changing the order of integration? Write out the steps you should take and how you should think about it.

When changing the order of integration, it's easiest for me to draw a picture of the domain. Using the above example (first one) and working inside out, I see that y goes from y=x to y=1. So we can draw y=x and y=1 on a graph. We also note that x goes from 0 to 1, so this gives the entire shaded triangle. Now, to change the bounds, we first want to find the bounds on x in terms of y. We move in the direction of the x-axis from left to right. We first hit x=0 then the line y=x which as an "x=" expression is x=y. So the bounds on x are 0 to y. Now, we can just look at the constant bounds on y, those are 0 to 1.

Chapter 1: Definitions, Notation, & Coordinate Systems

1.1 Respond to each of the following:

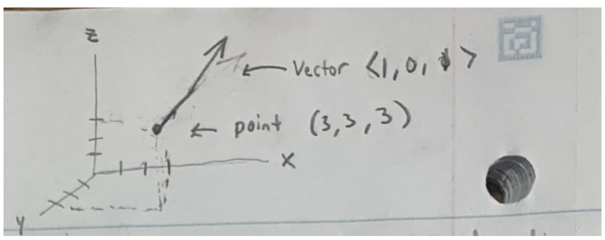
a) Give a brief definition or description of each of the following and give at least one specific example for each. If you are typing your responses, then do your best to make it clear what you are writing and describing. You may find it easier to write these by hand and insert an image or try out the equation editor in Google Docs, for example.

i. Notation of Point vs. Vector

A point is a position in space while a vector has a direction and magnitude but no fixed position.

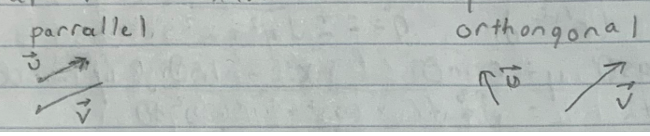
point $(1, 2, 3)$ (coordinates of (x, y, z))

vector $\langle 0, -1, 3 \rangle$ (direction & strength of pointer $\langle x, y, z \rangle$)



ii. Parallel and Orthogonal Vectors

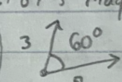
Parrell vectors have a 0° angle between them (or 180°) while orthongal vectors have a 90° angle between them



iii. Dot Product

The operation that takes two vectors magnitudes and aligns them in terms of direction.

$|a| \cdot |b| = a \times b \times \cos(\theta)$



$3 \cdot 5 \cdot \cos(60) = 7.5$

iv. Cross Product

The operation that takes two vectors and determines another vector perpendicular to the plane the original two create.

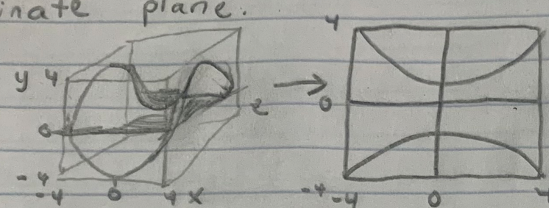
$a = \langle 0, 1, 2 \rangle$

$b = \langle 2, 4, -1 \rangle$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 2 \\ 2 & 4 & -1 \end{vmatrix} = \left.\begin{array}{l} (1\cdot -1 - (2\cdot 4))\hat{i} - \\ -(-1\cdot 0 - (2\cdot 2))\hat{j} \\ +(0\cdot 4 - (1\cdot 2))\hat{k} \end{array}\right\} \boxed{-7\hat{i} + 4\hat{j} - 2\hat{k}}$$

v. Trace/Level Curve

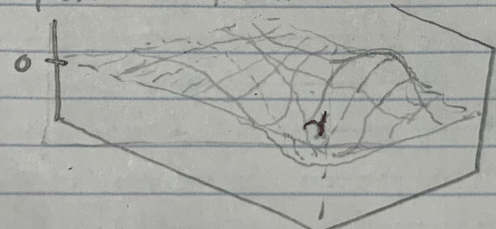
Trace is the intersection of the surface with the coordinate plane.



$z = f(x, c)$

$y = c = 0$

Level curve $f(x,y) = k$ is a set of all points in a domain. It is a trace of the graph in the horizontal plane $z = k$ projected down the xy-plane useful for Contour Maps.



vi. Partial Derivatives

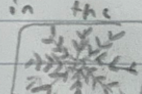
The derivative of a multivariable function where one or more variables is treated as a constant.

$f(x,y) = 2x^2 + y \quad f_x = 4x \quad f_y = 1$

vii. Gradient Vector of a Function

The maximum rate of change of a function. It's vector that points in the direction of a largest slope

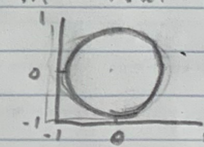
$df = \nabla f \cdot dr$



← ~~Center is the critical point min/max~~

viii. Vector-Valued Function

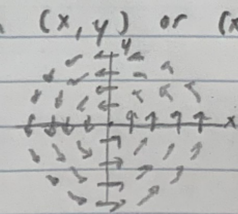
A function where the domain is in real numbers while the range is a vector.



$r_1(t) = \cos(t)i + \sin(t)j$

ix. Vector Field

A function $\vec{F}$ that assigns to each (x, y) or (x, y, z) point a vector given by the function



x. Curl and Divergence of a Vector Field

Measurements of vector field, curl measures the tendency of the vectors to swirl around a point, Divergence measure the tendency to collect or disperse at a point.

Curl F: $\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)i + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)j + \left(\frac{\partial Q}{x} - \frac{\partial P}{\partial y}\right)k$ Divergence: $\text{div } F = \nabla \cdot F$

b) Explain how to convert a point from polar coordinates to Cartesian coordinates by stating the coordinate transfer equations (i.e., “x=…" and “y=…") and any other equations/relationships among the variables that can be helpful for the transformation.

$x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$, $\tan\theta = \frac{y}{x}$

Polar coordinates are given in (r, θ) and the above equations are used to help alter the coordinates

c) Explain how to convert a point from cylindrical coordinates or spherical coordinates to Cartesian coordinates (do both!) by stating the coordinate transfer equations (i.e., “x=…," “y=…," and “z=…") and any other equations/relationships among the variables that can be helpful for each transformation.

$x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$, $z = z$, $\tan\theta = \frac{y}{z}$

Similar to polar coordinate system the same helpful equations are used however cylindrical coordinate values are (r, θ, z) with the extra z-value it designates a height.

$p^2 = x^2 + y^2 + z^2$, $\tan\theta = \frac{y}{x}$, $\cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) = \phi$.

$x = p\sin\phi\cos\theta$, $y = p\sin\phi\sin\theta$, $z = p\cos\phi$

Spherical coordinates once again have 3 values (p, θ, ϕ) the above equations are used to convert the Cartesian values (x, y, z) to spherical.

1.2 Choose **one** of the following two problems to complete:

a) Choose two vectors $\vec{a}$ and $\vec{b}$ with all components nonzero. Determine the (approximate) angle between them using the dot product. Are your vectors orthogonal? How can you quickly tell using the dot product?

Vectors: $a = \langle 2, 4, 6 \rangle$ $\sqrt{2^2 + 4^2 + 6^2} = \sqrt{56}$

$b = \langle 8, 9, -3 \rangle$ $\sqrt{8^2 + 9^2 + (-3)^2} = \sqrt{154}$

$a \cdot b = a \times b \times \cos\theta = (2 \cdot 8) + (9 \cdot 4) + (6 \cdot -3) = 34$

$\cos^{-1}\left(\frac{a \cdot b}{|a| |b|}\right) = \theta \rightarrow \cos^{-1}\left(\frac{34}{\sqrt{56} \cdot \sqrt{154}}\right) = \cos^{-1}(0.36) = 68.5^\circ$

b) Choose two vectors $\vec{a}$ and $\vec{b}$ with all components nonzero. Determine the cross product of your two vectors. How does that vector relate to the vectors $\vec{a}$ and $\vec{b}$ that you chose?

Chapter 2: Common Curves, Surfaces, & Parameterization

2.1 Respond to each of the following:

a) Give examples of equations for the following common surfaces: plane, sphere, (elliptic) paraboloid, hyperbolic paraboloid, (circular) cylinder, half cone. For each, which coordinate system(s) are easiest to express the equations?

1. plane:
   a. Equation: $3x + 7y - 9z + 2 = 0$
   b. Best Coordinate system: Cartesian
2. sphere:
   a. Equation: $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = r^2$ (centered at (1,2,3))
   b. Best Coordinate system: Spherical
3. (elliptic) paraboloid:
   a. Equation: ${\frac{x}{25}}^2 + {\frac{y}{16}}^2 = z$
   b. Best Coordinate system: Cartesian
4. hyperbolic paraboloid:
   a. Equation: ${\frac{x}{4}}^2 - {\frac{y}{9}}^2 = z$
   b. Best Coordinate system: Cartesian
5. (circular) cylinder:
   a. Equation: $x^2 + y^2 = r^2$
   b. Best Coordinate system: Cylindrical
6. half cone:
   a. Equation: $(x - a)^2 + (y - b)^2 = r^2$
   b. Best Coordinate system: Cylindrical

b) Determine a parameterization for both of the following curves: a line segment connecting two points, and half of a circle centered at the origin.

1. Line Segment connecting two points

Equations: (x_1,y_1), (x_2,y_2)

Let: $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = t$

Thus

$x = x_1 + t(x_2 - x_1) \quad (1), y = y_1 + t(y_2 - y_1) \quad (2)$

2. Half a Circle at a origin

Equation: $x^2 + y^2 = r^2$

The parametric equation is given by:

$x = acos(\theta) \quad (1), y = asin(\theta),\ 0 \leq \theta \leq \pi \quad (2)$

c) Determine a parameterization for **two** of the following common surfaces: plane, sphere, (circular) paraboloid, (circular) cylinder, and half cone (**choose only 2!**).

1. Plane

Equation: $ax + by + cz + d = 0$

The parametric equation is given by:

$x = t \quad (1),\ y = s \quad (2),\ z = d - \frac{a}{t} - \frac{b}{s} \quad (3)$

2. Sphere

Equation: $r = \sqrt{a^2 + b^2 + c^2 - d}$

The parametric equation is given by:

$x =- a + rcos(\theta)sin(\phi) \quad (1),\ y =- b + rsin(\theta)sin(\phi) \quad (2),\ z =- c + rcos(\phi) \quad (3)$

2.2 Choose **one** of the following two problems to complete:

a) Choose 3, three-dimensional points (x, y, z). Determine if these three points all lie on the same line and clearly state your conclusion. Then, determine an equation of the plane that contains all three points.

Three points: A $(0,0,2)$ B $(0,2,0)$ C$(2,0,0)$

$\vec{AB} = \vec{B} - \vec{A} = (0,2,0) - (0,0,2) = (0,2,-2)$

$\vec{AC} = \vec{C} - \vec{A} = (2,0,0) - (0,0,2) = (2,0,-2)$

$\vec{AB}$ are not parrellel with $\vec{AC}$ & $\vec{AB} \neq \lambda \vec{AC}$ for real λ

a (They do not lie on the same line.)

So the equation of the plane becomes

$\vec{AB} \times \vec{AC} = (a, b, c)$

$a = (B_y - A_y)(C_z - A_z) - (C_y - A_y)(B_z - A_z)$

$a = (2 - 0)(0 - 2) - (0 - 0)(0 - 2) = -4$

$b = (B_z - A_z)(C_x - A_x) - (C_z - A_z)(B_x - A_x) =$

$b = (0 - 2)(2 - 0) - (- 2)(0 - 0) = -4$

$c = (B_x - A_x)(C_y - A_y) - (C_x - A_x)(B_y - A_y)$

$c = (0 - 0)(0 - 0) - (2 - 0)(2 - 0) = -4$

$d = -(aA_x + bA_y + cA_z) = 8$

Thus

$\boxed{-4x - 4y - 4z + 8 = 0}$

b) Determine a parameterization for the curve of intersection of the surfaces $z = x^2 + y^2$ and the plane $x + y = 1$. What shape does this curve have? It may be helpful to include a sketch and/or image from GeoGebra.

Chapter 3: Differential Calculus with Multivariable Functions

3.1 Respond to each of the following:

a) Choose a nonconstant function $f(x, y)$ and verify Clairaut's Theorem.

Let $f(x, y) = x^2 + y^2$

To verify Clairaut's Theorem, we choose a nonconstant function and compute its' partial derivates.

$f_x'(x, y) = 2x$

$f_y'(x, y) = 2y$

If the partial derivates are equal at any point in its domain the Theorem is verified.

$2x = 2y$ is true for all x & y. $\boxed{\therefore \text{Clairauts is verified}}$

b) State the definition of the directional derivative using the gradient vector of a function $f(x, y)$. Using the definition, show why the maximum rate of change always occurs in the direction of the gradient.

Definition of the directional derivative using the gradient function of a function $f(x,y)$ is $D_u f = \nabla f \cdot u$ for a function f at (x_0, y_0) in the direction of a unit vector $u = \langle a, b \rangle$.

$$D_u f(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0 + ha, y_0 + hb) - f(x_0, y_0)}{h}$$

Since the numerator is always positive in the direction of the gradiant vector, The maximum rate of change must always occur in the direction of the gradiant vector

c) Briefly explain the solution method of Lagrange multipliers to determine the absolute maximum and/or minimum values of a function $f(x, y)$ over a curve $g(x, y) = k$.

Lagrange mutipler method determines the absolute min/max of a function. The method requires the finding of critical points of the function subject to a constraint. Depending on where the critical points are in relation to each other they are determined to be the absolute min/max of the function of local min/max.

3.2 Choose **one** of the following two problems to complete:

a) Choose a function and pick a point where it is defined. Determine an equation of the tangent plane at that point. Note: your tangent plane may not be flat, i.e., it may not have the simplified form "$z = k$."

b) The function $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$ has at least one of each type of critical point. Determine the critical points and use the Second Derivative Test to classify each of them.

$f(x,y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$

$\boxed{f_x = 6xy - 6x}$ $\boxed{f_y = 3x^2 + 3y^2 - 6y}$

To find the critical points:

$f_x = 0 \Rightarrow 6xy - 6x = 0 \rightarrow 6x(y-1)$ $x=0$ or $y=1$

$f_y = 0 \Rightarrow 3x^2 + 3y^2 - 6y = 0 \rightarrow 3(x^2+y^2-2y) \rightarrow$

$3(0)^2 + 3y^2 - 6y = 0 \rightarrow 3y^2 - 6y = 0 \rightarrow 3y(y-2) = 0$

$3x^2 + 3(1)^2 - 6(1) = 0 \rightarrow 3x^2 + 3 - 6 = 0$ $\quad y = 2$ or 0

$\rightarrow 3x^2 - 3 = 0$ $\boxed{x = 1 \text{ or } -1}$

Thus critical points are $(0,0), (0,2), (1,1), (-1,1)$

Second derivative test $A = f_{xx} = 6y - 6$

$B = f_{xy} = 6x$

$C = f_{yy} = 6y - 6$

At $(0,0)$

$f_{xx} = -6$ $\quad f_{xy} = 0$ $\quad f_{yy} = -6 \Rightarrow AC - B^2 = 36 - 0 > 0$

$(0,0)$ is a local max

At $(0,2)$

$f_{xx} = 6$ $\quad f_{xy} = 0$ $\quad f_{yy} = 6 \rightarrow AC - B^2 = 36 - 0 < 0$

$(0,2)$ is a local min

At $(1,1)$

$f_{xx} = 0$ $\quad f_{xy} = 6$ $\quad f_{yy} = 0 \rightarrow AC - B^2 = 0 - 36 < 0$

$(1,1)$ is a saddle point

At $(-1,1)$

$f_{xx} = 0$ $\quad f_{xy} = -6$ $\quad f_{yy} = 0 \rightarrow AC - B^2 = 0 - 36 < 0$

$(-1,1)$ is a saddle point

Since $\lim_{y\to\infty,\, x=0} f(x,y) = -\infty$ no global min

Since $\lim_{x\to\infty,\, y=0} f(x,y) = -\infty$ no global max

Chapter 4: Double and Triple Integrals

4.1 Respond to each of the following:

a) State how to use a double integral to determine the area of a 2-D region D.

The integral of the function $f(x,y)$ over a region D can be the volume under the surface $z = f(x,y)$. By integrating the function $f(x,y) = 1$ over the region D. it would give the volume under the function $A: \int_b^a \int_{f_2(x)}^{f_1(x)} D \, dx dy$

b) State how to use a triple integral to determine the volume of a 3-D region E.

The integral of the function $f(x,y)$ over a region E. The

$V = \iiint D \, dV$

The iterated integral

$V = \int_b^a \int_{g_2(x)}^{g_1(x)} \left(\int_{F_2(x,y)}^{F_1(x,y)} dz \right) dy \, dx$

c) What type of region must we have if all six bounds (limits of integration) are constant (numerical values, no variables) in cylindrical coordinates? What about in spherical coordinates? Is it possible to have a region represented with only constants in BOTH cylindrical and spherical simultaneously?

Cylindrical coordinates focus on a rotational symetry around the z - axis.

Spherical coordinates focus on a region with spherical symetry.

A Region cannot be expressed in both cylindrical & spherical symetry

4.2 Choose **one** of the following two problems to complete:

a) Say that you need to compute a double integral of the function f(x,y)=xy over the region D bounded by the x-axis, $y = x$, $x^2 + y^2 = 1$, and $x^2 + y^2 = 16$. Explain in words and/or show in a picture why this would be (unnecessarily) complicated in Cartesian coordinates. Then, setup and evaluate the integral using polar coordinates.

Function $f(x,y) = xy$ bounded by $x=y$, $x^2+y^2=1$, $x^2+y^2=16$
this is unnessary in cartesian coordinade cause two of the boundary equations are given for circles with radiuses Making polar much more paactical.

$x = r\cos\theta$

$y = r\sin\theta$ $\qquad \sqrt{1} \quad \sqrt{16}$

$f(r,\theta) = r^2\cos\theta\sin\theta \qquad r = 1, 4$

$dydx = r\,dr\,d\theta$

$$\therefore \int_0^{\frac{\pi}{4}}\int_1^4 r^3 \sin\theta\cos\theta \, dr\, d\theta$$

$$\frac{1}{2}\int_0^{\pi/4}\int_1^4 r^3 \sin 2\theta \, dr\, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}} \sin 2\theta \left[\frac{r^4}{4}\right]_1^4 d\theta$$

$$\frac{1}{2}\cdot\frac{255}{4}\left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/4} = \frac{255}{8}\left(\frac{1}{2}\right) = \boxed{\frac{255}{16}}$$

b) Choose a region E and set up a triple integral using Cartesian coordinates that represents its volume. Include a sketch or image of your chosen region. Then, repeat this for a region in cylindrical and spherical coordinates. The regions for all 3 do NOT need to be the same, and you are encouraged to think about what type of region works best for each example. Do NOT evaluate any of the integrals.

4.3 BONUS QUESTION

This question is entirely optional and is worth extra credit for correct answers with supporting work and images.

Set up a triple integral to calculate the volume of “the orange slice” between $y = \cos(x)$, $z = y$ and $z = 0$ using four (of the six) different orders of integration. See https://www.geogebra.org/3d/qtqcww4j.

Chapter 5: Line Integrals

5.1 Respond to each of the following:

a) State how to use a line integral to determine the arc length of a curve C.

The line integral $\int_C f(x,y,z)\,ds$ can be described as $x(t)$, $y(t)$, $z(t)$ on the same integral t, $t\,[a,b]$.

Thus, $ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}$

$$\int_a^b f(x,y,z)\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}$$

Please not $f(x,y,z)$ is not a curve it is the function being integrated over.

b) Use a line integral to verify that the circumference of a circle of radius R is given by $2\pi R$.

$L = 4\int_0^r \sqrt{1+(r')^2}\,dr \qquad r^2 = \pm\sqrt{y^2+x^2}$

Let, $x = r\cos\theta$ $y = r\sin\theta$ & $x' = -r\sin\theta$, $y' = r\cos\theta$

Combined with $L = \int_a^b \sqrt{[x'(\theta)]^2 + y'(\theta)^2}\,d\theta$

$L = 4\int_0^{\pi/2} \sqrt{r^2\sin^2\theta + r^2\cos^2\theta}\,d\theta$

$= 4\int_0^{\pi/2} \sqrt{r^2(\sin^2\theta + \cos^2\theta}\,d\theta$

$= 4\int_0^{\pi/2} r\,d\theta$

$= 4\,[r\theta]_0^{\pi/2}$

$\boxed{L = 2\pi r}$

c) If the value of a line integral of a scalar function $f(x,y)$ over some curve C is negative, then what conclusion(s) can you make about the function $f(x,y)$ and its graph (i.e., how do you interpret the value of the integral)? What about if the line integral was taken of a vector field $\vec{F}(x,y)$ instead?

If the value of a line integral of a scalar function is negative then the function and its graph have conflicting vectors the function and vector field are opposite one another and thus negative.

If the line integral was taken out of a vector field $\vec{F}(x,y)$ then the function & the field would match up exactly being a gradient.

5.2 Choose **one** of the following two problems to complete:

a) What type of geometric shape's area does the following line integral represent? Compute the area using the line integral and then verify your work using the classical area formula for this shape.

$\int_C f(x,y)ds$ where $f(x,y) = 6 - x - 2y$ and C is the line segment from (0,0) to (1,1).

$\int_C f(x,y)\,ds$ The geometric is a ellipsoid

$\hookrightarrow \int_C (6-x-2y)\,ds \quad C: \vec{r}\,t = \langle 0,0\rangle + t\langle 1-0, 1-0\rangle$

$r'(t) = \langle 1,1\rangle = \sqrt{1^2+1^2} = \sqrt{2} \quad \langle t, t\rangle, \quad 0 \le t \le 1$

$= \sqrt{2}\int_0^1 [6 - t - 2(t)]\,dt$

$= \sqrt{2}\left(6t - 3\frac{t^2}{2}\right)\Big|_0^1$

$= \sqrt{2}\left(6 - \frac{3}{2}\right) = \sqrt{2}\left(\frac{12}{2} - \frac{3}{2}\right) = \boxed{\frac{9}{2}\sqrt{2}}$

$z = 6 - x - 2y \quad ax^2 + y^2 + z^2 = r^2 \quad a \neq 1$

$z + x + 2y = 6$ Ellipsoid formula

Reasons its an Ellipsoid (1) all the coordinate variable have the same sign on the same side. (2) $a \neq 1$ y has a scalar of 2. (3) while $r = \sqrt{6}$ the other square roots are easy given $\sqrt{1} = 1 = 1^2$

b) Choose an example of a nonzero, conservative vector field $\vec{F}(x,y)$ with continuous first-order partial derivatives and a positively oriented, simple, closed curve C of your choice. Show that you get 0 for the value of the line integral of $\vec{F}(x,y)$ over C in three ways: directly (evaluate the line integral by definition), using the Fundamental Theorem for Line Integrals, and using Green's Theorem.

Chapter 6: Surface Integrals

6.1 Respond to each of the following:

a) State how to use a surface integral to determine the surface area of a surface S.

The surface integral consits of $\vec{r}(u,v)$ & a scalar function. The function of the points are mutiplied with the cross product of the partial derivatives $\vec{r}_u$ & $\vec{r}_v$ resulting in the equation:

$$\iint_D f(\vec{r}(u,v))\,|\vec{r}_u \times \vec{r}_v|\, dA$$

b) Explain briefly how you can use the curl of a vector field to determine if it is conservative.

To determine if a vector field is conservative using the curl, the cross product of del & F either is equal to 0 and thus conservative or not $\emptyset$ and thus not conservative.

c) Let S be the surface given by $x^2 + y^2 + z^2 = 1$ with $z \geq 0$ and upward orientation. Explain why we cannot use the Divergence Theorem and how we could adapt the problem slightly so that we can. Explain the process and how you would compute the flux, but you do not need to actually set up or evaluate any integrals.

The Divergent Theorem ($\oiint \vec{F}\cdot d\vec{S} = \iiint \operatorname{div}\vec{F}\, dV$) cannot be used because $x^2+y^2+z^2=1$ is not a closed surface. To alter the problem different surface areas can be formed together creating a closed surface. As a result Divergent Theorem can now be used in combination with algebra to isolate the specfic area asked for.

6.2 Choose **one** of the following two problems to complete:

a) Let S be the portion of the plane $3x + 4y + 5z = 60$ in the first octant. Use a surface integral to determine the surface area of S.

If $z=0$, $y = -\frac{3}{4}x + 15$ & $0 = -\frac{3}{4}x + 15$ $x = 20$

$D = \{ (x,y) : 0 < y < -\frac{3}{4}x + 15,\ 0 < x < 20 \}$

$f_x = -\frac{3}{5}$

$f_y = -\frac{4}{5}$

$S = \iint_D \sqrt{f_x^2 + f_y^2 + 1}\, dA$

$= \sqrt{2} \int_0^{20} \int_0^{-\frac{3}{4}x+15} dy\,dx$

$= \sqrt{2} \int_0^{20} \left(-\frac{3}{4}x + 15\right) dx$

$= \sqrt{2} \left[-\frac{3}{4} \cdot \frac{x^2}{2} + 15x \right]_0^{20} = \sqrt{2} \left(-\frac{3}{4} \cdot \frac{20^2}{2} + 15 \cdot 20 \right)$

$= \sqrt{2}\,(150) = 150\sqrt{2}$

b) Let $\vec{F}(x, y, z) = \langle x + z,\ y + z,\ z \rangle$ and let S be the surface given by $x^2 + y^2 + z^2 = 1$ with $z \geq 0$ and upward orientation. Compute $\iint_S curl\,\vec{F} \cdot d\vec{S}$ using Stokes' theorem.