The Profit in Being Unbalanced

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Sample Scenario

Flight to Detroit, where bottle water is scarce and expensive.

Price in Oregon	$ 1.00
Price in Detroit	$ 2.50

Profit $= \$1.50$ per water bottle

Arbitrage

A riskless transaction that results in an economic profit.

Theorem (Balanced Matrix)

A balanced matrix is defined as a square matrix with positive entries such that $a_{ik}a_{kj} = a_{ij}$ *for all i, j, and k.*

A given a_{ij} *represents the exchange rate of currency* i *for currency* j*.*

Proof.

For a given balanced matrix,

$$M^2 = \sum_{k=1}^{n} a_{ik} a_{kj}$$

$$M^2 = \begin{pmatrix} a_{11} & \cdots & a_{1k} \\ \vdots & \ddots & \\ \vdots & & \ddots \end{pmatrix} \begin{pmatrix} a_{11} & \cdots & \cdots \\ \vdots & \ddots & \\ a_{k1} & & \ddots \end{pmatrix}$$

It follows that,

$$\sum_{k=1}^{n} a_{ik} a_{kj} = \sum_{k=1}^{n} a_{ij} = n a_{ij} \qquad \text{since } a_{ij} a_{jk} = a_{ik}$$

Therefore,

$$M^2 - nM = 0$$

Example of a balanced matrix

Set up, properties of balanced matrices, and transaction paths.

$$\mathsf{M} = \begin{array}{c} \\ \mathsf{USD} \\ \mathsf{EUR} \\ \mathsf{CAD} \end{array} \begin{array}{c} \begin{array}{ccc} \mathsf{USD} & \mathsf{EUR} & \mathsf{CAD} \end{array} \\ \begin{pmatrix} 1 & 2 & 4 \\ 0.5 & 1 & 2 \\ 0.25 & 0.5 & 1 \end{pmatrix} \end{array}$$

$$M^2 - nM = 0$$

Unbalanced Matrix

Definition and example.

$$\mathsf{M} = \begin{array}{c} \\ \text{USD} \\ \text{EUR} \\ \text{CAD} \end{array} \begin{array}{c} \begin{array}{ccc} \text{USD} & \text{EUR} & \text{CAD} \end{array} \\ \begin{pmatrix} 1 & 2 & 4 \\ 0.5 & 1 & 3 \\ 0.25 & 0.5 & 1 \end{pmatrix} \end{array}$$

$$M^2 - nM = \begin{pmatrix} 0 & 0 & 2 \\ \frac{1}{4} & \frac{1}{2} & -1 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Unbalanced Matrix Cont.

Definition and example.

$$M = \begin{array}{c} \\ \text{USD} \\ \text{EUR} \\ \text{CAD} \end{array} \begin{array}{c} \begin{array}{ccc} \text{USD} & \text{EUR} & \text{CAD} \end{array} \\ \begin{pmatrix} 1 & 2 & 4 \\ 0.5 & 1 & 3 \\ 0.25 & 0.5 & 1 \end{pmatrix} \end{array}$$

1 USD $\rightarrow$ 2 EUR 2 EUR $\rightarrow$ 6 CAD 6 CAD $\rightarrow$ 1.5 USD

Comparison

$$\text{Balanced M} = \begin{array}{c} \\ \text{USD} \\ \text{EUR} \\ \text{CAD} \end{array} \begin{array}{c} \begin{array}{ccc} \text{USD} & \text{EUR} & \text{CAD} \end{array} \\ \begin{pmatrix} 1 & 2 & 4 \\ 0.5 & 1 & 2 \\ 0.25 & 0.5 & 1 \end{pmatrix} \end{array}$$

$$\text{Unbalanced M} = \begin{array}{c} \\ \text{USD} \\ \text{EUR} \\ \text{CAD} \end{array} \begin{array}{c} \begin{array}{ccc} \text{USD} & \text{EUR} & \text{CAD} \end{array} \\ \begin{pmatrix} 1 & 2 & 4 \\ 0.5 & 1 & 3 \\ 0.25 & 0.5 & 1 \end{pmatrix} \end{array}$$

Profit

Transaction path in an unbalanced matrix:

$$\begin{array}{c} \\ \text{i} \\ \text{p} \\ \text{j} \end{array} \begin{array}{c} \begin{array}{ccc} \text{i} & \text{p} & \text{j} \end{array} \\ \begin{pmatrix} 1 & a_{ip} & a_{ij}+d \\ a_{pi} & 1 & a_{pj} \\ a_{ji} & a_{jp} & 1 \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ \text{i} \\ \text{p} \\ \text{j} \end{array} \begin{array}{c} \begin{array}{ccc} \text{i} & \text{p} & \text{j} \end{array} \\ \begin{pmatrix} k & & \\ & & \\ & & \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ \text{i} \\ \text{p} \\ \text{j} \end{array} \begin{array}{c} \begin{array}{ccc} \text{i} & \text{p} & \text{j} \end{array} \\ \begin{pmatrix} & k(a_{ip}) & \\ & & \\ & & \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ i \\ p \\ j \end{array} \begin{array}{c} \begin{array}{ccc} i & p & j \end{array} \\ \begin{pmatrix} & & \\ & & \\ & & k(a_{ip})(a_{pj}+d) \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ i \\ p \\ j \end{array} \begin{array}{c} \begin{array}{ccc} i & p & j \end{array} \\ \begin{pmatrix} & & \\ & & \\ k(a_{ip})(a_{pj}+d)(a_{ji}) & & \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ i \\ p \\ j \end{array} \begin{array}{c} \begin{array}{ccc} i & p & j \end{array} \\ \begin{pmatrix} k+kda_{jp} & & \\ & & \\ & & \end{pmatrix} \end{array}$$

$$\text{Profit} = kda_{pj}$$

Fees, commissions, and brokerage fees

Let all the negative effects on exchange rate resulting from fees, commissions, and brokerage fees to be encapsulated by the following:

$$\text{Corrected value for a transaction} = xa_{pi} \qquad \text{where } x < 1$$

$$\text{Losses due to fees} = k - x^n k = k(1 - x^n)$$

Fees, commissions, and brokerage fees cont.

$$\begin{array}{c} \\ \text{i} \\ \text{p} \\ \text{j} \end{array} \begin{array}{c} \begin{array}{ccc} \text{i} & \text{p} & \text{j} \end{array} \\ \begin{pmatrix} 1 & a_{ip} & a_{ij}+d \\ a_{pi} & 1 & a_{pj} \\ a_{ji} & a_{jp} & 1 \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ \text{i} \\ \text{p} \\ \text{j} \end{array} \begin{array}{c} \begin{array}{ccc} \text{i} & \text{p} & \text{j} \end{array} \\ \begin{pmatrix} k & & \\ & & \\ & & \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ \text{i} \\ \text{p} \\ \text{j} \end{array} \begin{array}{c} \begin{array}{ccc} \text{i} & \text{p} & \text{j} \end{array} \\ \begin{pmatrix} & kx(a_{ip}) & \\ & & \\ & & \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ i \\ p \\ j \end{array} \begin{array}{c} \begin{array}{ccc} i & p & j \end{array} \\ \begin{pmatrix} & & \\ & & \\ & & kx^2(a_{ip})(a_{pj}+d) \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ i \\ p \\ j \end{array} \begin{array}{c} \begin{array}{ccc} i & p & j \end{array} \\ \begin{pmatrix} & & \\ & & \\ kx^3(a_{ip})(a_{pj}+d)(a_{ji}) & & \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ i \\ p \\ j \end{array} \begin{array}{c} \begin{array}{ccc} i & p & j \end{array} \\ \begin{pmatrix} kx^3 + kdx^3 a_{jp} & & \\ & & \\ & & \end{pmatrix} \end{array}$$

Revenue $= kx^3 d a_{jp}$ Costs $= k(1 - x^n)$

Fees, commissions, and brokerage fees cont.

New conditions for obtaining a profit.

$$\text{Profit} = \text{Revenue} - \text{Costs}$$

$$\text{Profit} = kdx^3 a_{jp} - k(1 - x^3) > 0$$

$$kx^3 a_{jp} > k(1 - x^3)$$

$$dx^3 a_{jp} a_{pj} > x^3 a_{pj}$$

$$dx^3 > a_{pj}(1 - x^3)$$

The Dissapointing Stuff

Data for international currency exchange obtained at 3:57 pm on Monday, April 15th.

	GBP	USD	EUR	CHF	JPY	DKK	CAD	AUD
GBP	1	1.305	1.155	1.314	146.113	8.624	1.744	1.82
USD	0.766	1	0.885	1.007	111.93	6.606	1.336	1.394
EUR	0.866	1.13	1	1.137	126.48	7.465	1.51	1.575
CHF	0.761	0.993	0.879	1	111.202	6.563	1.327	1.385
JPY	0.007	0.009	0.008	0.009	1	0.059	0.012	0.012
DKK	0.116	0.151	0.134	0.152	16.943	1	0.202	0.211
CAD	0.573	0.749	0.662	0.753	83.78	4.945	1	1.043
AUD	0.55	0.717	0.635	0.722	80.304	4.74	0.959	1

The DS Cont.

$$M^2$$

	GBP	USD	EUR	CHF	JPY	DKK	CAD	AUD
GBP	8.0233	10.4484	9.25468	10.5085	1168.88	68.9862	13.96	14.4902
USD	6.14677	8.00467	7.09015	8.05073	895.497	52.8514	10.695	11.1012
EUR	6.94562	9.04497	8.0116	9.09701	1011.88	59.7201	12.0849	12.5439
CHF	6.1057	7.95117	7.04277	7.99692	889.512	52.4982	10.6235	11.027
JPY	0.054991	0.071613	0.063431	0.072025	8.01146	0.472829	0.095681	0.099316
DKK	0.929779	1.21081	1.07248	1.21777	135.455	7.99444	1.61775	1.67919
CAD	4.59979	5.9901	5.30574	6.02457	670.123	39.5501	8.00331	8.30728
AUD	4.41005	5.74301	5.08688	5.77606	642.481	37.9187	7.67318	7.96462

The DS Cont.

$$M^2 - nM$$

	GBP	USD	EUR	CHF	JPY	DKK	CAD	AUD
GBP	-7	-8.73698	-7.90598	-8.7854	20180.1	5.38138	-10.9105	-11.2476
USD	-5.54124	-7	-6.29678	-7.04195	11632.9	-9.20876	-8.9031	-9.20876
EUR	-6.17804	-7.7631	-7	-7.80323	14985.4	-3.99377	-9.7999	-10.1194
CHF	-5.50888	-6.95795	-6.25936	-7	11476.3	-9.43103	-8.85507	-9.16178
JPY	-0.055951	-0.071919	-0.063936	-0.071919	-7	-0.468519	-0.095856	-0.095856
DKK	-0.914544	-1.1852	-1.05404	-1.1929	151.521	-7	-1.5752	-1.64348
CAD	-4.25567	-5.431	-4.85776	-5.45699	6348.85	-15.107	-7	-7.25615
AUD	-4.0975	-5.22191	-4.67678	-5.25472	5806.3	-15.4524	-6.75232	-7

The DS Cont.

```
In[84]:= 100 * 1.305 * 0.885 * 1.137 * 111.202 * 0.059 * 0.202 * 1.043 * 0.55
Out[84]= 99.8337
```

How to determine the best transaction path?

Paper only describes how to check if path exists.

The Fun Stuff

$$Luis = M * Transpose[M]$$

	GBP	USD	EUR	CHF	JPY	DKK	CAD	AUD
GBP	1	0.99963	1.00023	0.999954	1.02279	1.00038	0.999312	1.001
USD	0.99963	1	1.00005	0.999951	1.00737	0.997506	1.00066	0.999498
EUR	1.00023	1.00005	1	0.999423	1.01184	1.00031	0.99962	1.00013
CHF	0.999954	0.999951	0.999423	1	1.00082	0.997576	0.999231	0.99997
JPY	1.02279	1.00737	1.01184	1.00082	1	0.999637	1.00536	0.963648
DKK	1.00038	0.997506	1.00031	0.997576	0.999637	1	0.99889	1.00014
CAD	0.999312	1.00066	0.99962	0.999231	1.00536	0.99889	1	1.00024
AUD	1.001	0.999498	1.00013	0.99997	0.963648	1.00014	1.00024	1

Other Possible Applications

Cryptocurrency - easier, immediate exchanges, but lower probability of finding arbitrage opportunities

#	Cryptocurrency	Price in USD	Price in BTC	Market Cap	Exchange volume 24h
1	BTC Bitcoin	$ 5,206.53 -0.45% ($23.7) in 12h -3.52% ($190) in 7d	1 BTC +0% in 12 hours +0% in 7 days	$ 92,014,672,302 17,672,941 BTC	476,791 BTC 476,790.57 BTC 2,482,423,709.2 USD
2	ETH Ethereum	$ 155.02 +0.39% ($0.60) in 12h -9.59% ($17.2) in 7d	0.03 BTC +0.84% in 12 hours -6.71% in 7 days	$ 16,411,706,636 105,865,308 ETH	6,063,006 ETH 180,526.03 BTC 939,913,941.3 USD
3	EOS EOS	$ 4.48 -3.19% ($0.15) in 12h -14.92% ($0.79) in 7d	0.00086 BTC -2.75% in 12 hours -11.82% in 7 days	$ 4,678,543,440 1,043,723,698 EOS	95,391,534 EOS 82,127.14 BTC 427,597,301.62 USD
4	LTC Litecoin	$ 66.89 -1.77% ($1.2) in 12h -13.58% ($10.5) in 7d	0.013 BTC -1.32% in 12 hours -10.43% in 7 days	$ 4,119,160,659 61,578,271 LTC	5,444,385 LTC 69,949.05 BTC 364,191,731.9 USD
5	BCH Bitcoin Cash	$ 234.2 -3.53% ($9.58) in 12h -20.33% ($59.8) in 7d	0.045 BTC -3.49% in 12 hours -17.43% in 7 days	$ 4,155,228,868 17,742,464 BCH	1,418,651 BCH 63,812.87 BTC 332,243,534.61 USD
6	XRP XRP	$ 0.292 -1.09% in 12 hours -10.27% in 7 days	0.000056 BTC -0.64% in 12 hours -7% in 7 days	$ 12,254,379,761 42,004,966,728 XRP	647,380,880 XRP 36,274.57 BTC 188,864,598.06 USD
7	ETC Ethereum Classic	$ 5.57 -0.03% in 12 hours -6.19% in 7 days	0.0011 BTC +0.42% in 12 hours -2.77% in 7 days	$ 613,608,858 110,090,688 ETC	26,494,811 ETC 26,363.09 BTC 147,673,261.16 USD
8	TRX TRON	$ 0.022 -1.28% in 12 hours -11.26% in 7 days	0.0000043 BTC -0.84% in 12 hours -8.03% in 7 days	$ 2,208,677,898 99,270,633,156 TRX	5,159,334,049 TRX 22,047.38 BTC 114,790,313.32 USD
9	BNB Binance Coin	$ 21.61 -1.63% ($0.36) in 12h -8.44% ($1.99) in 7d	0.0042 BTC -1.18% in 12 hours -5.11% in 7 days	$ 3,050,679,426 141,175,651 BNB	4,750,618 BNB 19,716.9 BTC 102,656,606.53 USD
10	NEO Neo	$ 9.38 -1.84% ($0.18) in 12h -13.61% ($1.48) in 7d	0.0018 BTC -1.39% in 12 hours -10.46% in 7 days	$ 609,960,622 65,000,000 NEO	6,702,728 NEO 12,080.69 BTC 62,898,460.55 USD
11	IOT IOTA	$ 0.302 -0.5% in 12 hours -1.45% in 7 days	0.000058 BTC -0.04% in 12 hours +2.14% in 7 days	$ 839,693,300 2,779,530,283 IOT	167,979,518 IOT 9,746.69 BTC 50,746,443.32 USD
12	ZEC Zcash	$ 57.82 -2.15% ($1.27) in 12h -16.57% ($11.5) in 7d	0.011 BTC -1.7% in 12 hours -13.52% in 7 days	$ 371,514,713 6,425,463 ZEC	775,303 ZEC 8,609.83 BTC 44,827,350.57 USD
	ADA	$ 0.065	0.000012 BTC	$ 1,675,485,546	692,576,202 ADA

	Market	Last	Buy	Sell
EOS/USD	hitbtc	4.47 USD	4.47 USD	4.47 USD
EOS/USD	bitfinex	4.67 USD	4.67 USD	4.67 USD
EOS/USD	gdax	4.41 USD	4.4 USD	4.4 USD
EOS/USD	exmo	4.53 USD	4.53 USD	4.54 USD
EOS/USD	livecoin	4.5 USD	4.5 USD	4.81 USD
Buy EOS/USD on Changelly				
EOS/USDT	huobi	4.48 USDT	4.47 USDT	4.48 USDT
EOS/USDT	bitz	4.47 USDT	4.47 USDT	4.48 USDT
EOS/USDT	binance	4.47 USDT	4.47 USDT	4.47 USDT
EOS/USDT	poloniex	4.48 USDT	4.46 USDT	4.48 USDT
EOS/BTC	rightbtc	0.00086 BTC	0.00086 BTC	0.00086 BTC
EOS/BTC	hitbtc	0.00086 BTC	0.00086 BTC	0.00086 BTC
EOS/BTC	bitz	0.00086 BTC	0.00086 BTC	0.00086 BTC
EOS/BTC	binance	0.00086 BTC	0.00086 BTC	0.00086 BTC
EOS/BTC	huobi	0.00086 BTC	0.00086 BTC	0.00086 BTC
EOS/BTC	bitfinex	0.00086 BTC	0.00086 BTC	0.00086 BTC
EOS/BTC	poloniex	0.00086 BTC	0.00086 BTC	0.00086 BTC
EOS/BTC	exmo	0.00086 BTC	0.00086 BTC	0.00086 BTC
EOS/BTC	livecoin	0.00085 BTC	0.00084 BTC	0.00086 BTC
Buy EOS/BTC on Changelly				
EOS/ETH	rightbtc	0.029 ETH	0.029 ETH	0.029 ETH
EOS/ETH	huobi	0.029 ETH	0.029 ETH	0.029 ETH
EOS/ETH	hitbtc	0.029 ETH	0.029 ETH	0.029 ETH
EOS/ETH	binance	0.029 ETH	0.029 ETH	0.029 ETH

Conclusion

- Set up M
- Check if there is an opportunity for arbitrage $(M^2 - nM)$
- Choose a great transaction path
- Make Money, Be Great

References

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