

Trinomial model for pricing Options

In the lecture we mentioned that a binomial model can be generalized to a trinomial model. In a trinomial model we need to consider three stock price developments: in one period the price increases by a factor of u with the probability p_u , decreases by a factor of d with the probability p_d , or remains unchanged with the probability $1 - p_u - p_d$. The trinomial model can be built in a similar way to the binomial model, e.g., by matching the expectation and the variance and using $ud = 1$ we obtain:

$$u = e^{\sigma\sqrt{2\Delta t}}, \quad d = e^{-\sigma\sqrt{2\Delta t}},$$
$$p_d = \left(\frac{e^{\sigma\sqrt{\frac{\Delta t}{2}}} - e^{r\frac{\Delta t}{2}}}{e^{\sigma\sqrt{\frac{\Delta t}{2}}} - e^{-\sigma\sqrt{\frac{\Delta t}{2}}}} \right)^2 \quad \text{and} \quad p_u = \left(\frac{e^{r\frac{\Delta t}{2}} - e^{-\sigma\sqrt{\frac{\Delta t}{2}}}}{e^{\sigma\sqrt{\frac{\Delta t}{2}}} - e^{-\sigma\sqrt{\frac{\Delta t}{2}}}} \right)^2.$$

For the programming task define firstly all the necessary parameters with some assumed values, and write the corresponding Python functions as follows:

- 1) A function to calculate the up and down factors.
- 2) A function to calculate the probabilities in the trinomial model.
- 3) A function to calculate the stock prices in the trinomial model.
- 4) A function to calculate the European option price (call and put) using the trinomial model.
- 5) A function to calculate the American option price (call and put) using the trinomial model.

You must comment your code. You can use the Black-Scholes formula to check your output in the 4.step.