MA2071 Lecture 1

Liam Godin

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Why is Linear Algebra important?

- It arises from the study of linear equations

Example in dimension 1

solve

$ax = b$

where a, x, b are in R

and x is the unknown

$x = \frac{b}{a}$, if a != 0

if a = 0 $0 * x = b$

if b = 0, 0 * x = 0

x is any number in R.

if b != 0, 0 * x = b

incompatible, there are no solutions

In Dimension 2

solve

$a_{11}x_1 + a_{12}x_2 = b_1$

$a_{21}x_1 + a_{22}x_2 = b_2$

The unknowns are x_1 and x_2 in R the unknown is $(x_1, x_2) in R^2$

Graphical interpretation

Assume that

$(a_{11}, a_{12})! = (0, 0)$ and $(a_{21}x_1 + a_{22}x_2 = b_2$

and the equations of 2

two lines $L_1 and L_2$ in the x_1x_2 plane

solving this system is the same as finding the intersection of L_1 and L_2

These 2 lines can be such that there is one intersection point, both lines are parallel, and both lines are the same ($L_1 = L_2$). In this case there are infinitely many solutions and the set of solutions is the set of points $(x_1, x_2) on L_1 = L_2$

For a systematic approach to all cases in all dimensions, we need linear algebra.

dimension n = 3 these equations with n = 3 unknowns may be related to the physical space R^3

n = 4 physical space and another dimension for time

cases where n may be >= 3 or 4 * an optimization

problem where n variables need to be optimized

- Find a numerical approximization to a differential equation for the unknown function x(t)

approximate x(t) at n points in time t_1, ... , t_n.

In some scientific computations, n could be very large n ~= $10^4 or 10^6$

A systematic approach to solving linear equations:

Example

$x_1 + 5x_2 = 7$ $-2x_1 - 7x_2 = -5$

eliminate x_1 in the second equation by adding 2* first equation

$x_1 + 5x_2 = 7$ $3x_2 = 9$

$x_2 = 3$

back substitution

$x_1 + 15 = 7$ $x_2 = 3$

A wrong way to solve this system

Starting from

$x_1 + 5x_2 = 7$ $-2x_1 - 7x_2 = -5$

=> $3x_2 = 9$ => $x_2 = 3$ => $x_1 + 15 = 7$ => $x_1 = 8$

Solve

$x_1 + 5x_2 = 7$ $-2x_1 - x_2 = -5$

Write this system in matrix form

1 5 7

-2 -7 -5

$R_2 + 2R_1 -> R_1$

1 5 7

0 3 9

$\frac{R_3}{3} -> R_3$

1 5 7

0 1 3

$R_1 - 5R_2 -> R_1$

1 0 | -8 0 1 | 3

$x_1 = -8$ $x_2 = 3$

We have used elementary operations on rows

1. Add to one row a multiple of another row (Replacement)
2. Multiply a row by a non zero number (Scaling)
3. Interchange (interchanging 2 rows)