Equestrian Schedule

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Introduction

All sports demand a unique combination of athleticism, coordination, mental acuity, and strategic finesse, with each discipline requiring a specialized blend of physical ability and intellectual agility. Success in equestrian disciplines, such as show jumping, is intricately tied not only to the innate skill of the rider but also to the meticulous crafting of a training regimen. Balancing the physical demands placed on both athlete and horse, coupled with the imperative for recovery and skill enhancement, introduces a complex scheduling challenge. This optimization endeavor aims to forge an ideal training schedule that not only maximizes performance but also safeguards the well-being of both the equestrian athlete and their equine partner. Leveraging mathematical modeling and data-driven insights, we embark on a mission at Lindman-Piedrahita (LP) Sport Consulting to provide a solution for equestrian's schedule. Our client, the accomplished show jumping athlete, Henry Edwards, seeks our expertise as he prepares to make his debut at the Winter Equestrian Festival in Florida. Our mission is to elevate the performance of Henry and his top five equine partners by meticulously tailoring a competition schedule that maximizes success while prioritizing their well-being.

Problem

Statement

Our client, Henry Edwards, is a highly ranked European rider who is embarking on a significant venture to the Winter Equestrian Festival in Florida, marking his debut in this prestigious event. Recognizing the importance of optimizing his performance during the festival, Henry has sought the expertise of Lindman-Piedrahita (LP) Sport Consulting. The Winter Equestrian Festival spans 12 weeks and features a total of 19 events, each categorized at different levels, ranging from 2* to 5*. With aspirations to participate in every event offered, Henry faces the complex challenge of managing his five horses effectively throughout the festival. Additionally, Henry has expressed a preference not to have more than three horses competing in any given week, and he wishes to ensure the well-being of his equine partners by limiting each horse's participation to a maximum of six weeks over the course of the festival and no more than two consecutive weeks. This case presents a multi-dimensional optimization problem that demands meticulous scheduling to balance Henry's competitive ambitions with the welfare of his horses, ultimately enhancing his chances of success at the Winter Equestrian Festival.

In our comprehensive analysis of Henry Edwards' performance optimization for the Winter Equestrian Festival, we will evaluate success in terms of prize money earned. To calculate this metric accurately, we have computed the average earnings for each event. This financial aspect will provide a tangible measure of Henry's success and guide our decision-making process as we strive to create an ideal training and event schedule tailored to his unique goals and constraints. In further discussion we will refer to these potential earnings as "Prize Money", but it does not mean this is the guaranteed amount but the average of all of events.

Optimization Model

We have chosen to address this problem through a binary optimization model, where the decision variable x_{ij} is binary. This variable takes a value of 1 if horse j is participating in event i, and 0 otherwise. The mathematical expression for the decision variable is represented in Equation 1 below. Here, i ranges from 1 to 19, representing the events, and j ranges from 1 to 5, representing the horses.

$$x_{ij} = \begin{cases} 1, & if\ horse\ j\ is\ participating\ in\ event\ i \\ 0, & otherwise \end{cases}$$

Equation 1: Decision Variables

The objective function, represented below in Equation 2, of this model was defined to be the sum of products between the prize money of each event (c_i) and the decision variable of each horse-event combination (x_{ij}).

$$max\ prize\ money = \sum_{i=1}^{19} . \sum_{j=1}^{5} c_i x_{ij}$$

Equation 2: Objective Function

Considering the well-being of the horses, our client has stipulated two essential constraints. Firstly, there should be a maximum of 6 weeks of showing per horse, as expressed in Equation 3.

$$\sum_{i=1}^{19} x_{ij} \leq 6$$

Equation 3: Events per Horse Constraint

Additionally, to ensure the horses have adequate rest and to avoid overexertion, our client requests that no horse participates in more than two consecutive weeks, as formulated in Equation 4. Notably, Equation 4 is calculated for every week (w) from 1 to 10, where each week is the sum of events (e) occurring in that week. It is important to highlight that for weeks with only one event, denoted as e_{i2}, equal to 0, this term is excluded from the model. This simplification, implemented to maintain a model size suitable for Excel Solver, may require inclusion for a solution in Gurobi.

$$\sum_{i=w}^{w+2} x_{ij} \leq 2$$

$$where\ w = e_{i1} + e_{i2}$$

Equation 4: Consecutive Weeks per Horse Constraint

In accordance with the client's preferences, an additional set of constraints has been established. The client desires participation in every event, but with the condition that no more than 3 horses are involved in each event or week. Equation 5, outlined below, articulates the limitation on the number of horses (j) in each event (i). This constraint ensures a balanced representation of horses across all 19 events, aligning with the client's objective of diversified participation while maintaining a manageable level of involvement for each horse.

$$1 \leq \sum_{j=1}^{5} x_{ij} \leq 3$$

Equation 5: Horse Per Event Constraint

E Equation 6 captures the total number of horses participating each week (w), and this calculation is performed for every 12 weeks. This constraint ensures a balanced distribution of horse participation over the duration of the event, promoting diversity and avoiding potential strain on specific weeks.

$$w_i \leq 3$$

$$where\ w = e_{i1} + e_{i2}\ and\ e_i = \sum_{j=1}^{5} x_{ij}\ for\ every\ event$$

Equation 6: Horses per Week Constraint

Solution

This model was optimized using Excel Solver analysis due to its size and simplicity. Since some constraints, particularly the weekly constraints, were easier to calculate manually, employing Python programming to solve this model might have introduced unnecessary complexity. Excel Solver's user-friendly interface and compatibility with this specific problem facilitated a more efficient and straightforward optimization process. Table 1 represents the values for vector c_i used to calculate the maximum prize money in Equation 2. Values on the prize money column represent the average earning of the top 12 placing for the three most important classes of each event, considered the most common event schedule for a top rider.

Table 1: Events Description and Prize Money

Event (i)	Event Description	Prize Money (c)
1	1 - 3*	$ 32,923.90
2	2 -3*	$ 32,923.90
3	3-2*	$ 14,308.05
4	3-4*	$ 41,847.20
5	4-2*	$ 14,308.05
6	4-4*	$ 41,847.20
7	5-2*	$ 14,308.05
8	5-5*	$ 79,694.30
9	6 -3*	$ 28,154.55
10	7-2*	$ 14,615.75
11	7-5*	$ 83,386.70
12	8 - 4*	$ 41,539.50
13	9-2*	$ 15,231.15
14	9-5*	$ 83,386.70
15	10 - 4*	$ 46,770.40
16	11-2*	$ 14,615.75
17	11-4*	$ 44,001.10
18	12-2*	$ 15,538.85
19	12-5*	$ 105,387.25

Figure 1 below illustrates the binary optimization matrix, where decision variables are shaded in grey, constraints are highlighted in yellow, and the objective function, representing prize money, is highlighted in blue. This visual representation allows for a clear understanding of the solution to our problem, revealing a maximum average prize money of $1,420,958.60.

Event (i)	Horse 1	Horse 2	Horse 3	Horse 4	Horse 5	Horse per Event
1	1	0	0	0	0	1
2	0	1	0	0	0	1
3	0	0	1	0	0	1
4	1	0	0	0	1	2
5	0	1	0	0	0	1
6	0	0	0	1	1	2
7	0	0	0	1	0	1
8	1	1	0	0	0	2
9	0	0	1	0	0	1
10	0	0	0	0	1	1
11	1	0	1	0	0	2
12	0	1	0	1	1	3
13	0	1	0	0	0	1
14	0	0	1	1	0	2
15	1	0	1	0	1	3
16	0	0	0	1	0	1
17	0	1	0	0	1	2
18	1	0	0	0	0	1
19	0	0	1	1	0	2
	6	6	6	6	6	
Prize Money	$ 1,420,958.60					

Weeks in a row per horse					
	Horse 1	Horse 2	Horse 3	Horse 4	Horse 5
Weeks 1,2,3	2	1	1	0	1
Weeks 2,3,4	1	2	1	1	2
Weeks 3,4,5	2	2	1	2	2
Weeks 4,5,6	1	2	1	2	1
Weeks 5,6,7	2	1	2	1	1
Weeks 6,7,8	1	1	2	1	2
Weeks 7,8,9	1	2	2	2	2
Weeks 8,9,10	1	2	2	2	2
Weeks 9,10,11	1	2	2	2	2
Weeks 10,11,12	2	1	2	2	2

Week	Number of Horses
1	1
2	1
3	3
4	3
5	3
6	1
7	3
8	3
9	3
10	3
11	3
12	3

Figure 1: Optimization Model in Excel

Based on the optimization model described above, the ideal schedule for Henry Edwards in the Winter Equestrian Festival, designed to maximize his potential prize earnings while considering the welfare of his horses and adhering to his preferences, is as follows:

Weekly Schedule:

- Week 1 (Event 1 – 3*): Horse 1
- Week 2 (Event 2 – 3*): Horse 2
- Week 3 (Event 3 – 2*): Horse 3
- Week 3 (Event 4 – 4*): Horses 1 and 5
- Week 4 (Event 5 – 2*): Horse 2
- Week 4 (Event 6 – 4*): Horses 4 and 5
- Week 5 (Event 7 – 2*): Horse 4
- Week 5 (Event 8 – 5*): Horses 1 and 2
- Week 6 (Event 9 – 3*): Horse 3
- Week 7 (Event 10 – 2*): Horse 5
- Week 7 (Event 11 – 5*): Horses 1 and 3
- Week 8 (Event 12 – 4*): Horses 2, 4 and 5
- Week 9 (Event 13 – 2*): Horse 2
- Week 9 (Event 14 – 5*): Horses 3 and 4
- Week 10 (Event 15 – 4*): Horses 1, 3 and 5
- Week 11 (Event 16 – 2*): Horse 4
- Week 11 (Event 17 – 4*): Horses 2 and 5
- Week 12 (Event 18 – 2*): Horse 1
- Week 12 (Event 19 – 5*): Horse 3 and 4

Horse-Specific Schedule:

- Horse 1 – Events 1, 4, 8, 11, 15, 18
- Horse 2 – Events 2, 5, 8, 12, 13, 17
- Horse 3 – Events 3, 9, 11, 14, 15, 19
- Horse 4 – Events 6, 7, 12, 14, 16, 19
- Horse 5 – Events 4, 6, 1o, 12, 15, 17

This optimized schedule ensures a strategic distribution of events for each horse throughout the festival, aligning with Henry Edwards' performance goals and the well-being of his equine partners.

Conclusion

In conclusion, Lindman-Piedrahita (LP) Sport Consulting has crafted an effective optimization model that tailors the training and event schedule for our client, Henry Edwards, at the Winter Equestrian Festival. Utilizing mathematical modeling and data-driven insights, the objective was to maximize prize money earnings while prioritizing the welfare of Henry's horses and adhering to his personal preferences. Looking forward, the model holds potential for further development through the adoption of Gurobi optimization in Python programming. This transition would enhance the model's scalability and adaptability, enabling its application across various equestrian circuits and events. The Gurobi/Python implementation not only optimizes computational efficiency but also allows for customization, accommodating diverse circuit structures, event formats, and individual rider preferences.