

# Large Scale Structure Reconstruction with Short-Wavelength Modes: Light Cone Formalism and Halo Bias

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This is the second paper in a series where we propose a method of indirectly measuring large scale structure using information of small scale perturbations. The idea is to use two-point off-diagonal terms of density contrast modes to build a quadratic estimator for long-wavelength modes. We demonstrate in the first paper that our quadratic estimator works well on a dark-matter-only N-body simulation of the snapshot  $z = 0$ . Here we generalize our theory to the case of a light-cone with halo bias taken into consideration. We successfully apply our generalized version of quadratic estimator to a light cone halo catalog of a N-body simulation of size  $\sim 5.6 (h^{-1} \text{ Gpc})^3$ . The most distant point in the light cone is at a redshift of 1.4, which indicates that we might be able to apply our method in the next generation galaxy surveys.

## I. INTRODUCTION

We also need to account for halo bias [1][2] if eventually we want to apply the estimator to real cosmological surveys. Typically

We begin with our treatment of matter density contrast in a light cone and then build the quadratic estimator. We then construct the approximated form of matter perturbation field using information of halo masses and positions. We assess the detectability of the quadratic estimator. Finally we apply the estimator to a light cone halo catalog and successfully extract large scale modes. We use a flat  $\Lambda$ CDM model with cosmological parameters  $\Omega_m = 0.25$ ,  $\sigma_8 = 0.8$ ,  $n_s = 0.95$ ,  $\Omega_b = 0.044$ ,  $\Omega_\Lambda = 0.75$ ,  $h = 0.7$  (to match the parameters of the N-body simulation).

## II. LIGHT CONE FORMALISM

Here we consider the Fourier transform of the whole light cone, instead of cutting the light cone into thin redshift slices:

$$\delta_m^{\text{LC}}(\vec{k}) := \int_V d^3\vec{r} \delta_m(\vec{r}) \frac{D_{\text{ini}}}{D_1(a(r))} e^{-i\vec{k}\cdot\vec{r}} \quad (1)$$

Here LC stands for "light cone", and  $V$  is the whole volume of the light cone.  $a(r)$  is the cosmological scale factor. Here we set  $a(\vec{r}=0)$  to be the origin of the light cone thus  $a(\vec{r}) = a(r)$  will not depend of the direction of position  $\vec{r}$ .  $\delta_m(\vec{r})$  is the matter density perturbation at location  $\vec{r}$  with redshift  $1/(1+a(r))$ .  $D_1$  is the linear growth factor defined and  $D_{\text{ini}} = D_1(a = a_{\text{ini}})$  is the value of  $D_1$  at some initial time  $a_{\text{ini}}$ . [need to define  \$P^{\text{LC}}\$  and show  \$\langle\(1\)\(1\)\rangle = 0\$  somewhere.](#)

We can write the real space matter inhomogeneity field  $\delta_m(\vec{r})$  as:

$$\delta_m(\vec{r}) = \delta_m(\vec{r}; a(r)) = \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \delta_m(\vec{k}; a(r)) \quad (2)$$

$\delta_m(\vec{r})$  is the density contrast at location  $\vec{r}$  of the **light cone**; while  $\delta_m(\vec{r}; a(r))$  is the density contrast at location

$\vec{r}$  of the **snapshot** at time  $a(r)$ . Eq. (2) is the inverse Fourier transform of modes  $\delta_m(\vec{k}; a(r))$  at a **fixed** time  $a(r)$ . If we further transform it back we would get:

$$\delta_m(\vec{k}; a(r)) = \int \frac{d^3\vec{r}}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{r}} \underbrace{\delta_m(\vec{r}; a(r))}_{\text{fixed}} \quad (3)$$

Here the range of the integration is the whole space of that snapshot. And this equation still holds because we perform the calculation at a fixed time  $a(r)$ . We have the usual perturbative expansion of modes  $\delta_m(\vec{k}; a(r))$  and the time evolution of each order [3]:

$$\begin{aligned} \delta_m(\vec{k}; a(r)) &= \delta_m^{(1)}(\vec{k}; a(r)) + \delta_m^{(2)}(\vec{k}; a(r)) + \dots \\ &= \frac{D_1(a(r))}{D_{\text{ini}}} \delta_{m,\text{ini}}^{(1)}(\vec{k}) + \left[ \frac{D_1(a(r))}{D_{\text{ini}}} \right]^2 \delta_{m,\text{ini}}^{(2)}(\vec{k}) + \dots \end{aligned} \quad (4)$$

Similar to our previous work, we still want to compute the off-diagonal term of  $\delta_m^{\text{LC}}(\vec{k})$  up to second order. And the first and second order term of  $\delta_m^{\text{LC}}(\vec{k})$  will be:

$$\delta_m^{\text{LC},(i)}(\vec{k}) = \int_V d^3\vec{r} \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i(\vec{k}-\vec{k}')\cdot\vec{r}} \delta_m^{(i)}(\vec{k}; a(r)) \frac{D_{\text{ini}}}{D_1(a(r))} \quad (5)$$

with  $i = 1, 2$ . It's easy to prove that the first order term of  $\delta_m^{\text{LC}}$  satisfies the following relation:

$$\langle \delta_m^{\text{LC},(1)}(\vec{k}) \delta_m^{\text{LC},(1)}(\vec{k}') \rangle \simeq (2\pi)^3 \delta_{\text{D}}(\vec{k} + \vec{k}') P_{m,\text{ini}}(k) \quad (6)$$

with  $\delta_{\text{D}}$  being the Dirac delta function. This relation Eq. (6) tells us that the first order term still characterizes the linear evolution information of the light cone, since redshift difference in the light cone has been cancelled by the extra factor  $D_{\text{ini}}/D_1(a(r))$  of the integrand.

## III. QUADRATIC ESTIMATOR

Using expressions of Eq. (5), we can compute the two-point correlation of two short-wavelength modes  $\delta_m^{\text{LC}}(\vec{k}_s)$

and  $\delta_m^{\text{LC}}(\vec{k}_s')$ . We take the squeezed limit  $\vec{k}_l = \vec{k}_s + \vec{k}_s'$  with  $\vec{k}_s, \vec{k}_s' \gg \vec{k}_l$  and  $\vec{k}_l$  is a long-wavelength mode:

$$\begin{aligned} & \langle \delta_m^{\text{LC}}(\vec{k}_s) \delta_m^{\text{LC}}(\vec{k}_s') \rangle|_{\vec{k}_s + \vec{k}_s' = \vec{k}_l} \\ &= \langle \delta_m^{\text{LC},(1)}(\vec{k}_s) \delta_m^{\text{LC},(2)}(\vec{k}_s') \rangle + \langle \delta_m^{\text{LC},(2)}(\vec{k}_s) \delta_m^{\text{LC},(1)}(\vec{k}_s') \rangle \end{aligned} \quad (7)$$

Substituting Eq. (5) into Eq. (7) and evaluate the first bracket as an example:

$$\begin{aligned} & \langle \delta_m^{\text{LC},(1)}(\vec{k}_s) \delta_m^{\text{LC},(2)}(\vec{k}_s') \rangle \\ &= \int_V d^3\vec{r} \int_V d^3\vec{r}' \int \frac{d^3\vec{k}}{(2\pi)^3} \int \frac{d^3\vec{k}'}{(2\pi)^3} \\ & \quad e^{i(\vec{k} \cdot \vec{r} + \vec{k}' \cdot \vec{r}') - i(\vec{k}_s \cdot \vec{r} + \vec{k}_s' \cdot \vec{r}')} \frac{D_{\text{ini}}}{D_1(a(r))} \frac{D_{\text{ini}}}{D_1(a(r'))} \\ & \quad \times \langle \delta_m^{(1)}(\vec{k}; a(r)) \delta_m^{(2)}(\vec{k}'; a(r')) \rangle \end{aligned} \quad (8)$$

We have computed  $\langle \delta_m^{(1)}(\vec{k}) \delta_m^{(2)}(\vec{k}') \rangle$  in our previous work. We can use the result to further determine the value of the bracket in Eq. (8):

$$\begin{aligned} & \langle \delta_m^{(1)}(\vec{k}; a(r)) \delta_m^{(2)}(\vec{k}'; a(r')) \rangle \\ &= 2 \frac{D_1(a(r))}{D_{\text{ini}}} \left[ \frac{D_1(a(r'))}{D_{\text{ini}}} \right]^2 \langle \delta_{m,\text{ini}}^{(1)}(\vec{k}) \delta_{m,\text{ini}}^{(2)}(\vec{k}') \rangle \\ &= 2 \left[ \frac{D_1(a(r'))}{D_{\text{ini}}} \right]^2 F_2(-\vec{k}, \vec{k} + \vec{k}') P_{m,\text{ini}}(\vec{k}) \delta_m^{(1)}(\vec{k} + \vec{k}'; a(r)) \end{aligned} \quad (9)$$

where we take advantage of the the definition of the linear growth factor  $D_1(a(r))$ .  $P_{m,\text{ini}}$  is the linear matter power spectrum at some initial time  $a_{\text{ini}}$ . Plugging Eq. (9) into Eq. (8), we can see that the only  $\vec{r}'$  dependent integral can be written as (here we simply choose a cube volume):

$$\int_V d^3\vec{r}' e^{-i(\vec{k}' - \vec{k}_s') \cdot \vec{r}'} \frac{D_1(a(r'))}{D_{\text{ini}}} \simeq C (2\pi)^3 \delta_D(\vec{k}' - \vec{k}_s') \quad (10)$$

since  $D_1(a(r'))$  is a slowly varying function.  $C$  is a constant and can be further determined via integrating over  $\vec{k}$  on both sides of Eq. (10). Thus:

$$\begin{aligned} & \langle \delta_m^{\text{LC},(1)}(\vec{k}_s) \delta_m^{\text{LC},(2)}(\vec{k}_s') \rangle \\ &= 2C \int_V d^3\vec{r} \int \frac{d^3\vec{k}}{(2\pi)^3} F_2(\vec{k}, -\vec{k} + \vec{k}_s') P_{m,\text{ini}}(|\vec{k} - \vec{k}_s'|) \\ & \quad \times \frac{D_{\text{ini}}}{D_1(a(r))} e^{-i(\vec{k} - \vec{k}_s - \vec{k}_s') \cdot \vec{r}} \delta_m^{(1)}(\vec{k}; a(r)) \\ &\simeq 2C F_2(-\vec{k}_s, \vec{k}_s + \vec{k}_s') P_{m,\text{ini}}(k_s) \\ & \quad \times \int_V d^3\vec{r} \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{D_{\text{ini}}}{D_1(a(r))} e^{-i(\vec{k} - \vec{k}_s - \vec{k}_s') \cdot \vec{r}} \delta_m^{(1)}(\vec{k}; a(r)) \\ &= 2C F_2(-\vec{k}_s, \vec{k}_s + \vec{k}_s') P_{m,\text{ini}}(k_s) \delta_m^{\text{LC},(1)}(\vec{k}_s + \vec{k}_s') \end{aligned} \quad (11)$$

where in the first step, we perform a redefinition of integration dummy variable. And we use an approximation in the second step in order to get the first order term  $\delta_m^{\text{LC},(1)}(\vec{k}_l)$  from the integral.

Again, we have proved that with this construction in Eq. (1), we can get long-wavelength modes from off-diagonal terms of short-wavelength modes:

$$\langle \delta_m^{\text{LC}}(\vec{k}_s) \delta_m^{\text{LC}}(\vec{k}_s') \rangle|_{\vec{k}_s + \vec{k}_s' = \vec{k}_l} = f(\vec{k}_s, \vec{k}_s') \delta_m^{\text{LC},(1)}(\vec{k}_l) \quad (12)$$

with

$$\begin{aligned} f(\vec{k}_s, \vec{k}_s') &= 2C F_2(-\vec{k}_s, \vec{k}_s + \vec{k}_s') P_{m,\text{ini}}(k_s) \\ & \quad + 2C F_2(-\vec{k}_s', \vec{k}_s + \vec{k}_s') P_{m,\text{ini}}(k_s') \end{aligned} \quad (13)$$

The quadratic estimator can be formed as:

$$\delta_m^{\text{LC},(1)}(\vec{k}_l) = A(\vec{k}_l) \int \frac{d^3\vec{k}_s}{(2\pi)^3} g(\vec{k}_s, \vec{k}_s') \delta_m^{\text{LC}}(\vec{k}_s) \delta_m^{\text{LC}}(\vec{k}_s') \quad (14)$$

with  $\vec{k}_s' = \vec{k}_l - \vec{k}_s$  and  $g$  being a weighting function.  $A$  is the normalization factor determined by requiring that  $\langle \delta_m^{\text{LC},(1)}(\vec{k}_l) \rangle = \delta_m^{\text{LC},(1)}(\vec{k}_l)$ :

$$A(\vec{k}_l) = \left[ \int \frac{d^3\vec{k}_s}{(2\pi)^3} g(\vec{k}_s, \vec{k}_s') f(\vec{k}_s, \vec{k}_s') \right]^{-1} \quad (15)$$

Similar to our last work,  $g$  can be calculated by minimizing the noise term and the result is:

$$\begin{aligned} g(\vec{k}_s, \vec{k}_s') &= \frac{f(\vec{k}_s, \vec{k}_s')}{2P_m^{\text{LC}}(k_s)P_m^{\text{LC}}(k_s')} \\ &= C \frac{F_2(-\vec{k}_s, \vec{k}_s + \vec{k}_s') P_{m,\text{ini}}(k_s) + F_2(-\vec{k}_s', \vec{k}_s + \vec{k}_s') P_{m,\text{ini}}(k_s')}{P_m^{\text{LC}}(k_s)P_m^{\text{LC}}(k_s')} \end{aligned} \quad (16)$$

#### IV. HALO BIAS

Density contrast of a halo catalog can be written as:

$$\delta_h(\vec{r}) = \frac{\sum_i M_i \delta_D(\vec{r} - \vec{r}_i) - \sum_i M_i/V}{\sum_i M_i/V} \quad (17)$$

Here  $i$  is the index for halos.  $M_i$  and  $\vec{r}_i$  is the array for halo mass and position, respectively. We can construct a matter density contrast out of this halo catalog using Tinker bias function  $b_1(M, z)$ :

$$\delta_m(\vec{r}) = \frac{\sum_i \frac{M_i}{b_1(M_i, z(r_i))} \delta_D(\vec{r} - \vec{r}_i) - \sum_i \frac{M_i}{b_1(M_i, z(r_i))}/V}{\sum_i M_i/V} \quad (18)$$

such that  $\delta_h/\delta_m \approx b_1$ .

#### V. DEMONSTRATION WITH N-BODY SIMULATION

We use the MICE Grand Challenge light-cone N-body simulation [4][5] to demonstrate the power of the estimator. The catalog contains one octant of the full sky up to  $z = 1.4$  (comoving distance  $3072 h^{-1} \text{ Mpc}$ ) without

simulation box repetition. We use the largest cube that can be fit into the octant as the region  $V$  of integration in Eq. (1), as shown in Fig. 1. With the boxsize being  $1774 h^{-1} \text{ Mpc}$  we can numerically compute the constant  $C$  in Eq. (10) and the result gives 12345.

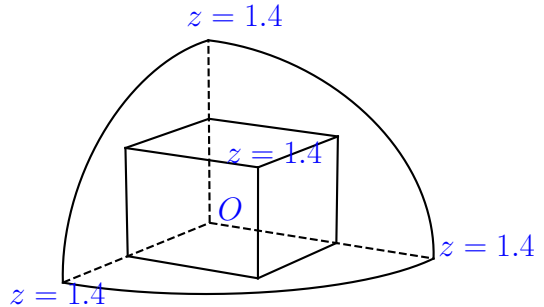


FIG. 1. Boxsize of the cube is  $L = 3072/\sqrt{3} h^{-1} \text{ Mpc} \sim 1774 h^{-1} \text{ Mpc}$ . Volume is  $V = L^3 \sim 5.6 (h^{-1} \text{ Mpc})^3$ . Only one point in the cube can reach redshift 1.4, and  $z = 0$  is at the origin  $O$  of the octant.

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## Appendix A: Non-cube Volume

For a more generic case, we might have a non-cube volume. Thus:

$$\delta_{\text{m}}^{\text{LC}}(\vec{k}) := \int_V d^3\vec{r} \delta_{\text{m}}(\vec{r}; a(r)) \left[ \frac{D_{\text{ini}}}{D_1(a(r))} \right]^2 f_V(\vec{r}) e^{-i\vec{k} \cdot \vec{r}} \quad (\text{A1})$$

$f_V$  is a position-dependent function and its form will be fully determined by the light cone's shape. For a cube volume, simply we have  $f_V = 1$ .