The Art of Political Manipulation: Unveiling Math's Mastery

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Abstract. This essay explores the application of McKelvey's theory in leveraging transitivity vulnerabilities within spatial voting, shedding light on the strategic tactics employed to manipulate democratic outcomes. It investigates the implications of these tactics on decision-making processes and offers insights into the dynamics at play in social choice theory. By analyzing real-world scenarios through programming and simulations, this essay delves into the practical applications of the theory and its impact on democratic decision-making.

I. Introduction to Social Choice Theory and Manipulation

In the realm of social choice theory, manipulation arises when individuals or groups strategically exploit the vulnerabilities of preference aggregation to influence outcomes in their favor. It is like a carefully orchestrated dance, where actors position themselves to achieve their desired results. One aspect of manipulation involves the violation of transitivity in preferences, which can have significant implications on decision-making. Additionally, spatial voting and McKelvey's theory offer insights into how preferences can be manipulated to shape collective choices. Furthermore, the application of programming and simulations allows us to explore the real-world implications of social choice theory and examine the potential for manipulation on a larger scale. Through this exploration, we unravel the captivating world where mathematics and human behavior intersect, shedding light on the intricate dynamics and power dynamics that drive decision-making in our society.

II. Transitivity, Spatial Voting and the Quest for Control

In decision-making, transitivity refers to the property that if one alternative is preferred to a second alternative, and the second alternative is preferred to a third alternative, then the first alternative should also be preferred to the third alternative. Symbolically, if A > B and B > C, then it follows that A > C. Transitivity is a fundamental principle that ensures consistency and logical coherence in preference relations, and they are true for numbers.

However, transitivity violations create opportunities for manipulation in decision-making processes. When preferences are not transitive, strategic actors can exploit these inconsistencies to manipulate the outcome. By selectively presenting alternatives and exploiting the inconsistencies in individual preferences, manipulators can engineer situations where certain alternatives are preferred over others, leading to desired outcomes.

As we live in a democratic society, citizens participate in voting to determine which policies become law. Each voter has their own preferences and rankings of the policies. Let's consider three voters: Voter 1 prefers Policy X, then Policy Y, and dislikes Policy Z. Voter 2 prefers Policy Z, then Policy X, and dislikes Policy Y. Voter 3 prefers Policy Y, then Policy Z, and dislikes Policy X. Mathematically, we can represent their preferences as follows: Voter 1: X > Y > Z, Voter 2: Z > X > Y, Voter 3: Y > Z > X, shown in Figure [1].

If there were a vote between Policy X and Policy Y, Policy X would win because two out of the three voters prefer it (Voters 1 and 2), resulting in X > Y. Similarly, if there were a vote between Policy Y and Policy Z, Policy Y would win (Voters 1 and 3), indicating Y > Z. However, the transitivity property does not hold in this case. So, does that mean X > Z? At first glance, one might expect that Policy X should be preferred over Policy Z based on transitivity. However, when directly comparing Policy X and Policy Z, Policy Z would win (Voters 2 and 3), leading to Z > X. Preferences, in general, do not always follow the rules of transitivity, as demonstrated by the preference order: Z > X > Y > Z.

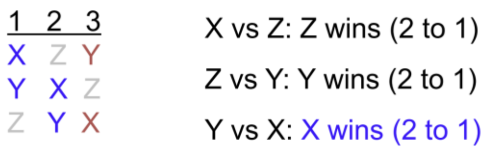
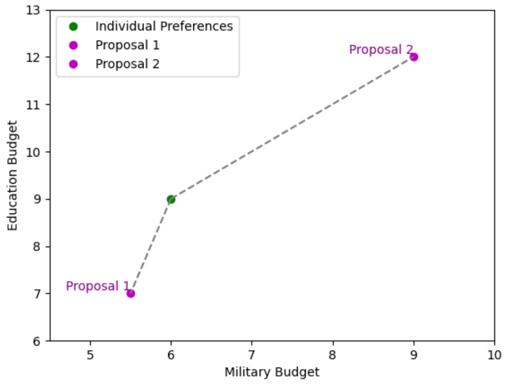


Figure [1]: examples of exploiting transitivity violation by arranging into a particular order of voting to get the desired outcome.

This creates an interesting dilemma when the current policy in our society is Policy Z. If we held a vote on whether to switch to Policy X based on voter preferences, we wouldn't switch because our society believes that Policy Z is better than Policy X. However, if we first vote on switching to Policy Y and then vote on switching to Policy X afterwards, our society would first switch from Policy Z to Policy Y and then switch from Policy Y to Policy X. This highlights the key problem in our democracy: manipulation of the agenda. The choice and order of the policies we vote on can completely change the policy outcome, showcasing the potential for manipulation.



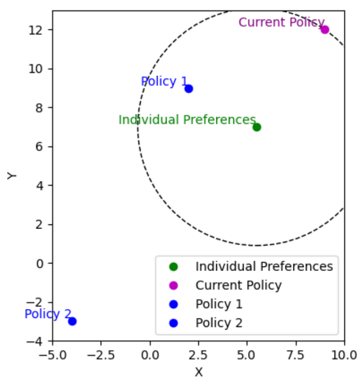


Diagram [1] : The more similar to their preference, the closer the distance   Diagram [2] : Once there is a policy, new policy will be voted for if its inside the circle/set

Consider the following scenario depicted in Diagram [1]. Imagine an individual who belongs to the opposition party and seeks to challenge the government's extra budget proposals. While their party generally opposes excessive government spending, this particular MP has a unique perspective. Perhaps driven by genuine concerns or a desire to uphold their campaign promises, they prioritize education reform and advocate for allocating more funds to education rather than the military.

Now, if I introduce two proposal options, Proposal 1 and Proposal 2, as illustrated in the diagram. In this case, the individual would theoretically vote in favour of Proposal 1 because it aligns more closely with their ideal preferences compared to Proposal 2. This preference is determined by the Euclidean distance, or the geometric length, between the individual's preferences and each proposal. The shorter the distance, the greater the similarity between the individual's preferences and the proposal, indicating a higher likelihood of their support.

This example highlights how individual preferences, represented as points on the diagram, can influence voting decisions. By examining the distances between preference points and policy points, we gain insights into the alignment or divergence of preferences and how these factors contribute to voting outcomes.

What if there is already an existing policy in place? Then we want to determine whether the individual will vote in favour or against a proposed change. In this case, we examine the distance between the new policy and the current policy. As we discussed earlier, individuals tend to prefer options that are closer to their ideal preferences as they perceive them to be more beneficial. To visually illustrate this concept, we can observe Diagram [2]. The circle, with a radius equal to the Euclidean distance from the individual's preference point to the current policy point, represents the range of policies that will be voted in favour (YES). In this instance, Policy 1 falls within the circle and will likely receive a YES vote, while Policy 2 lies outside the circle and will likely be voted against (NO). Therefore, we can conclude that any new policies that lie within the circle where the individual preference is the centre and with a radius equal to Euclidean distance from the point to the current policy, will be voted YES.

This is the idea of spatial voting, in this model we analyze their individual preferences and identify the alternatives they prefer over the current policy, which are anything within the set/circle, using the principles we just discussed above. Now, when multiple amounts of those sets intersect, it represents a region where multiple people would agree to vote for, as those new policy points are within the circle, which makes the new policy they are voting for closer and more beneficial towards them.

By comparing the preferences of all voters, we can determine the minimum number of votes needed to defeat the status quo. This minimum threshold, often referred to as "k" in social choice theory, is obtained by taking the number of voters and dividing it by 2, rounding up to the nearest integer. Or in simple terms, “k” stands for how many people are needed to win a vote (at least half of the total number of voters).

The majority rule win set (The set/region where policy can be changed to as it can gain at least half of the population’s vote) of x is the set of alternatives that a majority prefers to x: $W(x) = \{y \mid yPx\}$. It is created by drawing indifference curves through x and shading points that a majority of voters prefer to x. Diagram [3] illustrates the combination of overlapping preferred-to-sets that constitute the collective preferences surpassing the status quo.

Mathematically we can say that:

Define the set $S = \{A, B, C, D, ..., Z\}$ as the circle sets of individual preferences.

Let $k = \lceil n/2 \rceil$, where $n \in \mathbb{Z}^{+}$ represents the number of total elements in the set.

Then $\mathbf{W(x) = \Sigma\ C(n, k) \cdot \cap(Si)}$

Here, C(n, k) represents the binomial coefficient for choosing k elements from a set of n elements. The symbol ∩(Si) denotes the intersection of sets Si for each selected element. Annotations beside Diagram [4] and Diagram [5] will expand out in detail.

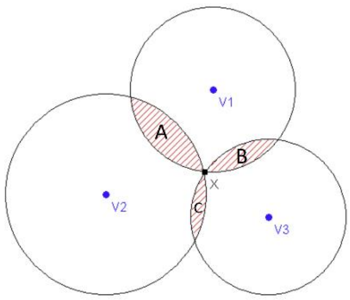


Diagram [3]

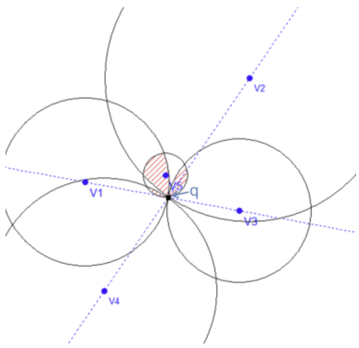


Diagram [4]

In diagram [4]: n = 3 and k = 2: W(x) = (A ∩ B) + (A ∩ B) + (B ∩ C) In diagram [5]: n = 5 and k = 3: W(x) = (A ∩ B ∩ C) + (A ∩ B ∩ D) + (A ∩ B ∩ E) + (A ∩ C ∩ D) + (A ∩ C ∩ E) + (A ∩ D ∩ E) + (B ∩ C ∩ D) + (B ∩ C ∩ E) + (B ∩ D ∩ E) + (C ∩ D ∩ E)

In Diagram [3], we observe a scenario involving three voters and a minimum threshold of k = 2 for the win set. The win set of the policy point x is represented by the shaded region. In this particular case, the win set consists of overlapping regions; Region A formed by the intersection of preferences from V1 and V2, Region B formed by the intersection of preferences from V1 and V3, and Region C formed by the intersection of preferences from V2 and V3.

To illustrate the impact of the win set, consider any new policy point that falls within Region A. In this scenario, the policy would pass through the democratic process, as it aligns with the preferences of the majority vote, comprising V1 and V2. The same principle applies to the other regions within the win set. If a new policy falls within Region B or Region C, it would also gain majority support and pass through the democratic voting process.

Put yourself in the shoes of V3 in Diagram [4]. Your goal is to strategically vote for a change in policy (q) that will benefit you the most. By securing at least half of the votes, you can influence the policy to shift closer to your preferred position. But how? It seems impossible right? Why would anybody agree to do that? The application of this demonstrates how the spatial voting model allows individuals to strategically position the policy point and manipulate the outcome in their favor. Additionally, the impact of the voting order will be explored to understand how it can influence the effectiveness of manipulation strategies.

By identifying and understanding these win set regions, we can assess the likelihood of policy approval based on the preferences and intersections of voters' choices. The win set serves as a valuable tool for analyzing democratic decision-making processes and determining the range of policy options that have the potential to achieve majority support.

III. The Chaos Theorem and Manipulative Strategies

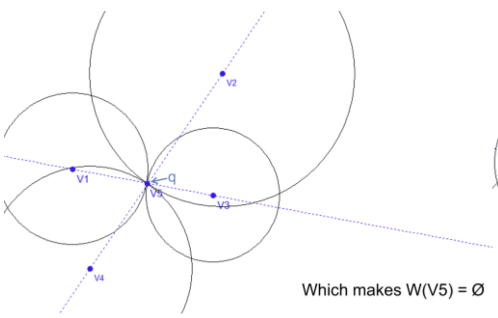


Diagram [5] : When there are no better policy for at least half of the people

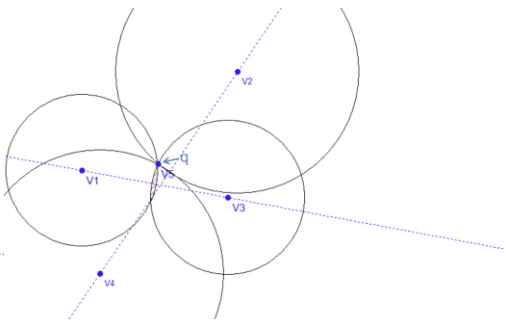


Diagram [6] : In reality, its really unlikely that Diagram [5] will happen

In the realm of multidimensional spatial voting, McKelvey's theorem provides valuable insights into the dynamics of decision-making processes. This theorem asserts that in almost all multidimensional spatial settings, the majority rule will not lead to an empty-winset point like in Diagram [5]. Because there is more sample size in the real world or even the slightest difference from that point in terms of preferences (shown by the shift of V3 from the position in Diagram [5] to Diagram [6]), there will be no Condorcet winner—a candidate who would win in a head-to-head competition against every other candidate. This absence of a clear victory opens up opportunities for strategic manipulation. In this scenario, there is at least a region that will satisfy 3 people out of 5.

McKelvey's theorem further highlights that any point within the alternative space can be reached by the individual who controls the order of voting. This implies that a clever agenda-setter wields significant influence in determining the final outcome. The strategic manipulation of the voting order enables the agenda setter to shape the preferences and alliances among the voters, ultimately influencing the collective decision. This revelation exposes a vulnerability in the decision-making process, leaving room for manipulation and the potential distortion of outcomes.

Now, we will delve into the realm of strategic manipulation within the framework we have discussed thus far. In our quest to manipulate voting outcomes, we will employ a strategic agenda-setting approach that involves constructing an agenda starting with point y and concluding with point x, see Diagram [7]. It is important to note that point x does not belong to the win set of point y ($x \notin W(y)$).

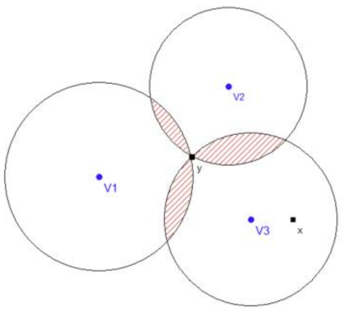


Diagram [7] : Start at y to end at x

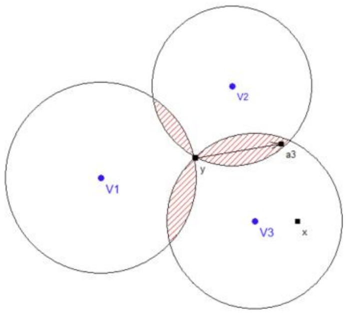
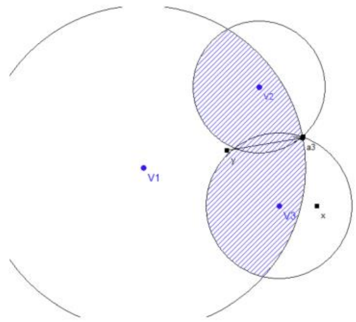


Diagram [8]



Diagram[9]

To execute this strategy, we initiate a series of votes that gradually shift the current policy from its initial position to a new point within the win set, see Diagram [8]. We then use that point (a3 in the diagram) as the “current” policy with the same fixed points for individual preferences, which creates a new win set, Diagram [9]. This is essentially, simulating the future of the conflict in the future after changing the policy to the new point (a3). We repeat this process iteratively (Diagram [10]), strategically selecting each intermediate point to ensure a progressive movement towards the desired outcome (Diagram [11]). Through this methodical manipulation of the agenda, we aim to influence the collective decision-making process, ultimately positioning point x within the win set of point y.

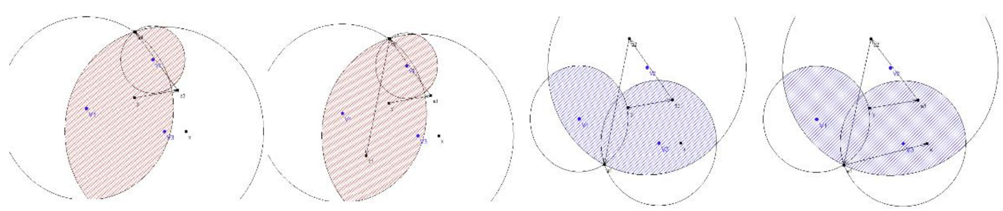


Diagram [10]

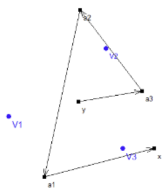


Diagram [11]

Although initially, it may seem impractical and unrealistic to achieve consensus in a society with significantly different opinions, the theory and analysis presented above demonstrate that manipulating the decision-making process to align collective preferences with one's own interests is indeed possible. This challenges the notion that democracy is impervious to manipulation and sheds light on the vulnerabilities within the system, which challenges the notion that democracy is immune to manipulation and highlights the potential vulnerabilities within the system.

IV. Strategically manipulate using computed code

Data collection is crucial to applying these concepts in real-life situations. Methods like questionnaires and interviews capture individual preferences and perspectives, providing a comprehensive understanding of the environment. Analyzing the vast number of combinations becomes challenging, but with advanced technologies, mathematical programming enables us to simulate real-world scenarios accurately. By leveraging these advancements, we can gain valuable insights and develop strategies to address the complexities and intricacies of decision-making. This enables us to navigate the challenges and implications effectively and shape outcomes in a desired manner.

As I embarked on my research journey, I discovered a lack of existing work in the field of simulating the theory I have presented. Motivated by this gap, I took it upon myself to develop my own code to simulate and explore the intricacies of this theory. The culmination of my efforts is a program that incorporates mathematical principles and data collected through extensive research. In the following sections, we will delve into the role of mathematics in constructing and utilizing this program, unveiling its potential applications and insights.

Due to the limited availability of functions for this specific task, I have created our own algorithm. We can start by calculating the distances between the preference points and the current policy point using the Pythagorean theorem as this model is represented in a two dimensional space. These distances are then used to determine the radii of the circles. By finding the intersection points and curve paths, we construct the region of intersection, which will consist of multiple polygons.

To trace the intersection region, we employ a step-wise approach similar to the principles of integration. We divide the intersection region into smaller segments similar to how lim ($dx \to 0$), however not exactly 0, sacrificing some tracing accuracy for improved computational efficiency. Then we choose alternatives successively further away from the “middle”, thereby creating larger and larger win sets. For each combination of circles, we iteratively intersect the current intersection region to use it as the current Policy for the next circle in the combination. We lastly store these win sets in a tree-like data structure, allowing us to trace back the paths leading to each win set, to find the simplest way to reach the final policy.

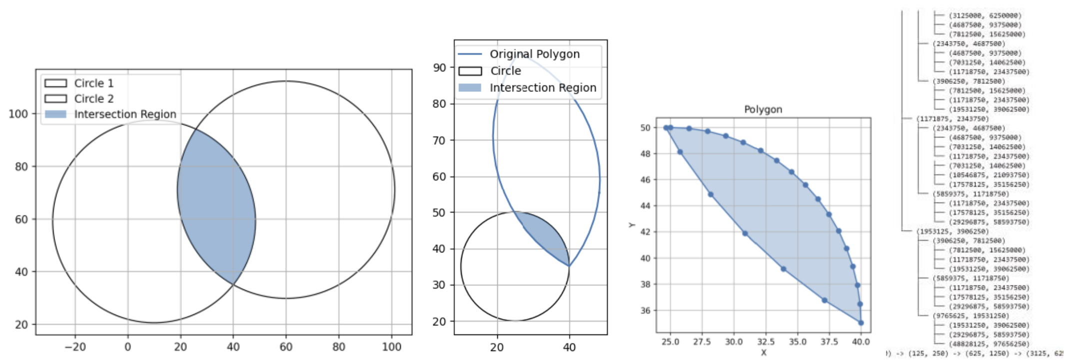


Figure [2] : A visualization of how the program works, finding the intersection region, selecting the points and storing them in a tree structure to allow traceback

By iterating through the combinations and generating win sets, we are able to explore various relocation possibilities. The algorithm captures the progression of policy relocation, providing insight into the decision-making process. Although the tracing accuracy is slightly compromised for computational efficiency, the algorithm's time efficiency makes it suitable for analyzing a large number of combinations.

V. Conclusion

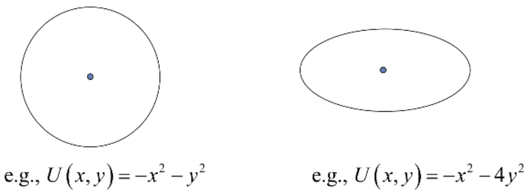


Diagram [12] : Example of how weigh can be applied

Figure [3] : Example of multi-dimensional spatial voting

In conclusion, the model presented in this essay provides valuable insights into the dynamics of social choice theory and the potential for manipulation in decision-making through McKelvey's theorem uses in spatial voting. However, it is important to acknowledge its limitations and areas for improvement. One criticism is the model's assumption of equal weights between options, which fails to capture the varying degrees of importance individuals may assign to different policies. Incorporating personalized weights would lead to a more accurate representation of individual preferences. Diagram [12] provides an example of how this could be modified.

Additionally, the current two-dimensional representation of the model restricts its ability to handle multiple policies simultaneously. Expanding the model to include additional dimensions like Figure [3] would provide a more comprehensive and realistic depiction of real-world decision-making scenarios.

To ensure successful manipulation in decision-making processes, it is crucial to continuously refine and improve the model by addressing these limitations and considerations. Leveraging data collection and mathematical programming enhances our understanding of social choice theory and the potential for manipulation. These advancements enable us to navigate decision-making complexities, develop strategies for influencing outcomes, and ethically assess our vulnerability to manipulation in real-life scenarios. Through these tools, we strive for comprehensive insights and the creation of fair and transparent systems as well.

References

Two-dimensional spatial voting models and the win set, Robi Ragan

https://www.youtube.com/watch?v=L4d5XKd4dvc&t=18s

Spatial voting, School of Public and International Affairs - University of Georgia

http://spia.uga.edu/faculty_pages/dougherk/svt_12_multi_dimensions1.pdf

The Mathematical Danger of Democratic Voting, Brian Yu

https://www.youtube.com/watch?v=goQ4ii-zBMw

Intransitivities in multidimensional voting models and some implications for agenda control,

Richard D McKelvey

https://doi.org/10.1016/0022-0531(76)90040-5

Interested in the algorithm?

The algorithm wrote by me can be publicly accessed via:

https://github.com/lplathip/chaosTheorem/