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Models of Neural Systems, WS 2019/20
Project 5: Dynamics of the Izhikevich model

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Background

The aim of the project is to study the firing patterns of the different electrophysiological classes of neurons with the help of the simple model suggested by Eugene M. Izhikevich in 2003. The author introduced it with the following words:

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A model is presented that reproduces spiking and bursting behavior of known
types of cortical neurons. The model combines the biological plausibility of
Hodgkin-Huxley-type dynamics and the computational efficiency of integrate-
and-fire neurons. Using this model, one can simulate tens of thousands of
spiking cortical neurons in real time using a desktop PC.
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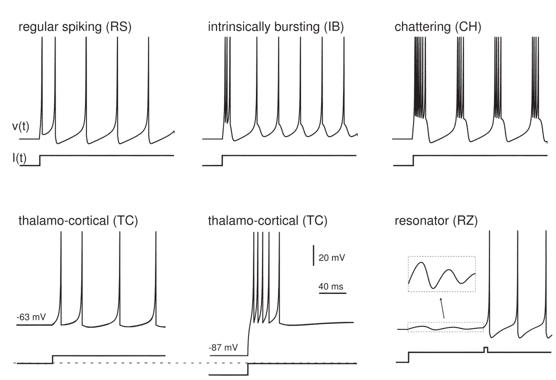
In the original paper, a neuron is modeled by just two dimensionless variables: membrane potential v and membrane recovery variable u, responsible for activation of K^+ and inactivation of Na^+. (In dimensional units v would be measured in mV and time units would be ms). The variables obey the differential equations:

$$\begin{aligned} dv/dt &= 0.04v^2 + 5v + 140 - u + I && (1) \\ du/dt &= a\,(b\,v - u) && (2) \end{aligned}$$

with the auxiliary condition of spike-resetting: if $v \geq 30$, then both variables are instantaneously reset: $\left\{ \begin{array}{ccc} v & \to & c \\ u & \to & u + d \end{array} \right.$. Here, I denotes the stimulating electric current, whereas a, b, c and d are the numerical coefficients (*parameters*).

Depending on the parameter values, the neuron reacts on the increase of I with single firing events (*spikes*), or with patches of spikes (*bursts*). Moreover, the parameter values regulate durations of time intervals between the spikes: short or long, so that the properly tuned model can imitate virtually all typical firing patterns of *in vitro* neurons.



Problems

All necessary theoretical notions – excitability classes, nullclines, resonators, integrators etc. – will be discussed in the first two January lectures.

Numerically solve the equations (1,2) for initial conditions v_0=–80, u_0=0 and different values of the parameters. You have to determine yourself, when the integration can be stopped (either because the solution periodically repeats itself, or because it converges to the equilibrium). Note that you will need to include your integration code with the project report!

1. Reproduce the so-called integrator variant of the model at parameter values a=0.1, b=0.05, c=–50, d=8. In order to fire, the value of I should exceed a certain threshold I_{thr}. Locate the threshold (by method of bisection of the interval) to the accuracy of 0.001. Plot the observed firing rate as function of I in the range $I_{\mathrm{thr}} \leq I < I_{\mathrm{thr}} + 1.5$.

   Further, let I be time-dependent with a constant component slightly below the threshold: $I(t) = I_{\mathrm{thr}} - 0.05 + 0.04 \sin(\omega\, t)$. You will observe that the voltage behaves as $v(t) \approx v_0 + A(\omega) \sin(\omega t - \varphi)$. Plot the *transfer function* $A(\omega)$ in the range $0 < \omega < 1$.

2. Proceed to the so-called resonator variant. The parameter values are a=0.1, b=0.26, c=–65, d=2. Start with the constant current I and, by increasing it, locate the firing threshold I_{thr}. Plot the observed firing rate in the appropriate range of I.

   For a subthreshold oscillatory stimulation $I(t) = I_{\mathrm{thr}} - 0.05 + 0.04 \sin(\omega\, t)$, estimate the transfer function $A(\omega)$ in the range $0 < \omega < 1$. Explain the difference to the previous case. In this way, the firing can be initiated by resonant currents, and even by the action of short depression of the stimulus *(postinhibitory rebound)*!

3. Fix a=0.003, b=0, c=–65, d=0.2. Consider the adaptation of the firing rate at the stepwise increase of the stimulus: start at t=0 with I=16 and switch at t=100 to I=18. Integrate until T=500. Explain, how evolution of the interspike interval is related to dynamics in the phase plane of the model.

4. Start at the parameter values a=0.02, b=0.2, c=–50, d=8 corresponding to the regular spiking (RS) neuron. Fix the value of stimulating current at I=5 and decrease in several steps the parameter d to the value d=2, that corresponds to the chattering (CH) neuron. Explain the transformation of the firing pattern in terms of the position of the nullclines on the phase plane.

References

Paper: EM Izhikevich (2003). *Simple model of spiking neuron*, IEEE Transactions on Neural Networks **14**, pp. 1569–1572.

Book: EM Izhikevich (2007). Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting. The MIT Press, Cambridge, Chapters 7,8.

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