

DISCRETE STRUCTURES

Lab 9

Graph

1. Introduction

In this tutorial, we will practice graph presentation on computer and matrix techniques.

2. Graph's Presentations

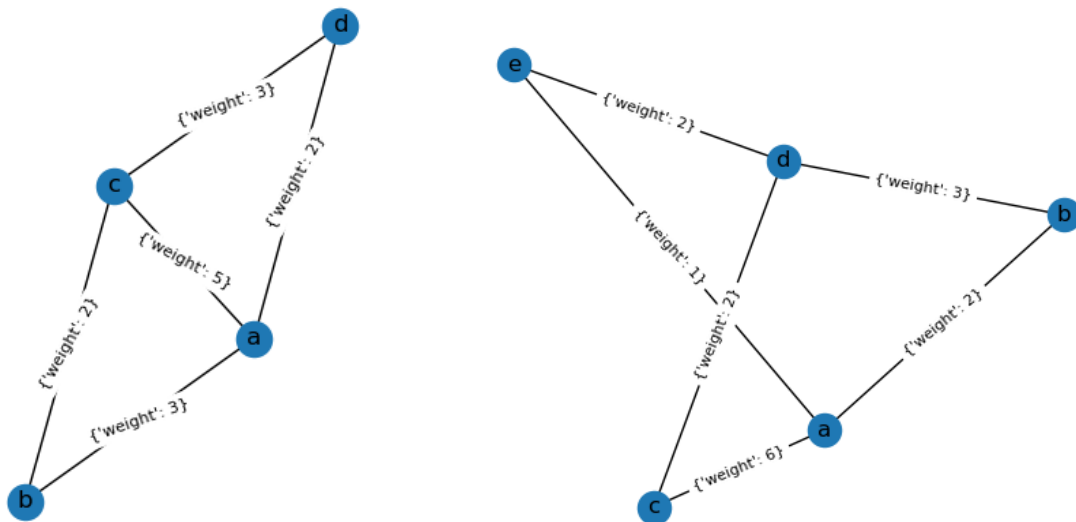
Read following lecture notes of **Discrete Structures** on site elit.tdtu.edu.vn

Week14_Graphs_and_Trees_1.pdf

2.1. Adjacency Matrix - Weighted Matrix

A graph $G(V,E)$ can be represented by its Adjacency Matrix - Weighted Matrix.

Adjacency Matrix is an $n \times n$ matrix A with $A(i,j) = 1$ if (i,j) in E and the Weighted Matrix is an $n \times n$ matrix A with $A(i,j) = \text{weight}(i,j)$ if (i,j) in E . In other word, Weighted matrix is a form of Adjacency Matrix where 1 is replaced with the weight of edge (i,j)



For these graphs, the Weighted Matrix are:

$$A_1 = \begin{bmatrix} 0 & 3 & 5 & 2 \\ 3 & 0 & 2 & 0 \\ 5 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 2 & 6 & 0 & 1 \\ 2 & 0 & 0 & 3 & 0 \\ 6 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 1 & 0 & 0 & 2 & 0 \end{bmatrix}$$

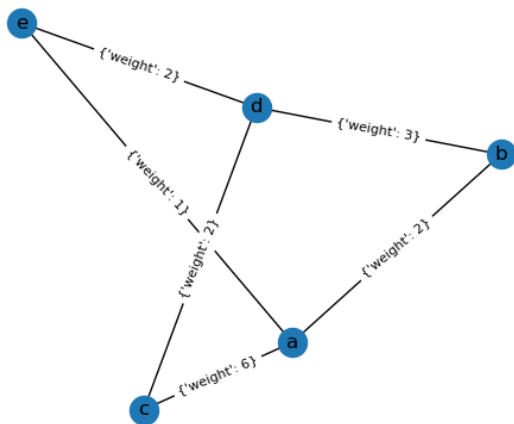
Since our graph are not directed graph, the Weighted Matrix can also be represented as only the upper or lower triangle since they are symmetric about the matrix diagonal:

$$A_1 = \begin{bmatrix} 0 & 3 & 5 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 2 & 6 & 0 & 1 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2.2. List of edges

Another common way to represent a matrix is by using list of edges and their weights (if available). For example, the 2nd graph from 2.1 can be represented as:



(A,B,2)
(A,C,6)
(A,E,1)
(B,D,3)
(C,D,2)
(D,E,2)

An edge on list often represented in the format of start point, end point then its weight.

3. Draw graph on python

To draw graph on python we can use the following method:

```
import matplotlib.pyplot as plt
import networkx as nx
import numpy as np
```

```
A1=np.array([[0,3,5,2],
             [0,0,2,0],
             [0,0,0,3],
             [0,0,0,0]])
G1 = nx.from_numpy_matrix(A1)
pos=nx.spring_layout(G1)
nx.draw_networkx(G1,pos=pos,with_labels=True,labels={a:b for
a,b in enumerate('abcd')})
edge_labels = nx.draw_networkx_edge_labels(G1,font_size=6,
pos=pos,label_pos=0.5)
plt.axis('equal')
plt.show()
```

The above code shows how to draw the graph of A_1 Weighted Matrix from 2.1

4. Exercise

1. Matrix practice:

- Write function `mPlus(A,B)` to calculate the sum of two matrix A, B knowing that $A + B = C$ where $C_{i,j} = A_{i,j} + B_{i,j}$
- Write function `mMinus(A,B)` to calculate the difference of two matrix A, B knowing that $A - B = C$ where $C_{i,j} = A_{i,j} - B_{i,j}$
- Write function `mMultiply(A,B)` to calculate the difference of two matrix A, B knowing that $A * B = C$ where $C_{i,j} = \sum_{k=1}^n A_{i,k} * B_{k,j}$ (remember that python count from 0 not 1)
- Write function `mTranspose(A)` to calculate the transpose matrix of A where $A_{i,j}^T = A_{j,i}$
- Test your functions with A is the blue matrix and B is the yellow matrix

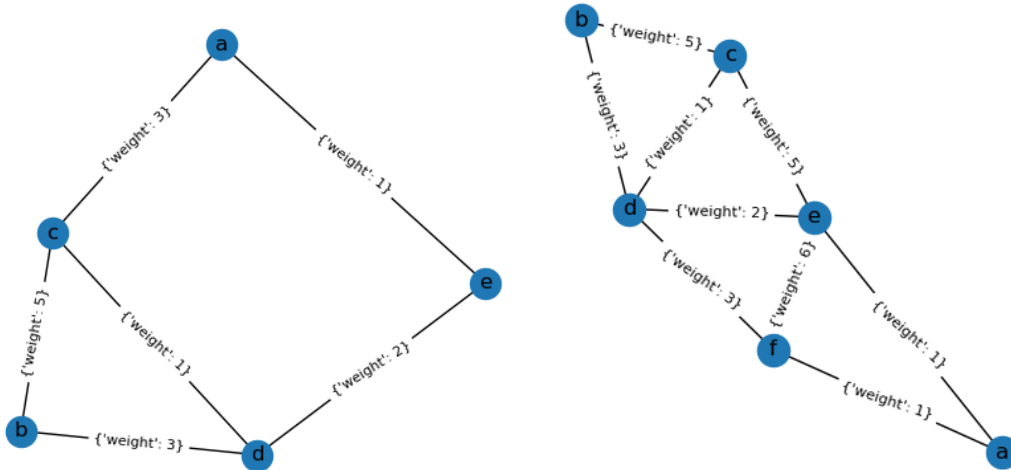
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 10 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 7 & 10 \end{bmatrix}$$

f. Test your multiply and transpose function with the following A and B matrices

$$\begin{matrix} \text{Matrix A} & & \text{Matrix B} & & \text{Product} \\ \left[\begin{array}{cccc} 1 & 4 & 6 & 10 \\ 2 & 7 & 5 & 3 \end{array} \right] & \cdot & \left[\begin{array}{ccc} 1 & 4 & 6 \\ 2 & 7 & 5 \\ 9 & 0 & 11 \\ 3 & 1 & 0 \end{array} \right] & = & \left[\begin{array}{ccc} 93 & 42 & 92 \\ 70 & 60 & 102 \end{array} \right]
 \end{matrix}$$

2. Calculate the Weighted Matrices of the following Graphs and graph them on your computer (the shape of graph might be difference)



3. Calculate the Weighted Matrices of the following List of Edges and graph them on your computer (the shape of graph might be difference)

a)	b)
(A,C,5)	(A,C,2)
(A,D,3)	(A,D,3)
(B,C,3)	(A,E,3)
(B,D,2)	(B,C,3)
(C,D,1)	(B,D,2)
(C,E,3)	(C,D,2)

(D,E,1)	(C,E,8)
(D,F,3)	(C,F,6)
(E,F,4)	(D,F,5)
	(E,F,3)

4. Write a function **toLoE(A)** convert a weighted matrix A to its list of edges.

*Note: In this Lab, we only use undirected graph, and the Weighted matrix is in upper triangle form.

5. We know that:

Monkeys, Apes, Gorillas are Primates.

Mice, Squirrels, Beavers are Rodents.

Crocodiles, Komodo dragons, Lizards are Reptiles.

Coconut trees, Grasses, Oaks are Plants

Mushrooms, Molds, Yeasts are Fungi.

And:

Primates and Rodents are Mammals

Mammals, Rodents, Reptiles are Animals

Animals, Plant, Mushrooms, Molds are Multicellular organisms

Yeasts are Unicellular organisms.

Draw a graph to represent the relation of the above terms.