Assignment Cover Sheet

Name	Liam Pribis
Student Number	81326643
Student Email	jlp90@uclive.ac.nz
Date of submission	24/05/2021

Please complete your details above and include this cover sheet as the first page on your assignment submission. Please also remember to upload your source file (Matlab or otherwise), in a format that can be executed to confirm functionality, i.e. ensure all support files are included, and files are logically names and commented. A 'zip' file with subdirectories for each question and/or a 'README' file is highly recommended.

The allocation of marks will be:

Question	Max [%]	Mark [%]
Transmitter/20		
a	5	
b	5	
c	5	
d	5	
Receiver/30		
e	10	
f	10	
g	10	
Equaliser/30		
h	20	
i	10	
Lab / 20	Device under test (A, B or C)	$\boxed{A}$
j	20	
TOTAL	**100**	

A Modulation type feasibility

B Power spectral density of 4-PAM

The power spectral density of a pulse-shaped data sequence is given by the energy spectral density of the pulse shape multiplied by power spectral density of the impulse chain (data sequence). The power spectral density of the impulse chain can be found be calculating the Fourier transform of the autocorrelation of the impulse chain.

$$S_y(f) = |P(f)|^2 S_x(f) = |P(f)|^2 \mathcal{F}\{R_x(\tau)\} \tag{1}$$

$R_x(\tau)$ is a continuous autocorrelation, but due to the fact that the data is comprised of delta functions, $R_x(\tau)$ will only be non-zero when these delta functions line up, ie. when τ is an integer multiple of the sampling period T_s. Because of this, $R_x(\tau)$ can be represented as a sum rather than an integral.

R_n represents the value of the autocorrelation at an integer sample offset of n. N represents the window size in units of samples ($N = \frac{T}{T_s}$).

$$R_n = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k-n} \tag{2}$$

Since the continuous time correlation is only non-zero at multiples of sample period, it can be represented as an impulse chain (ie. sum of time-offset delta functions), where the height of each delta function is given by the discreet autocorrelation value at that offset.

$$R_x(\tau) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} R_n \delta(\tau - nT_s) \tag{3}$$

(i) Calculating R_n

From Equation (2), we can see that the discreet autocorrelation values are dependent on two adjacent values of $x[n]$. No specific data sequence is given in the question, so I will assume the input data is random. A table of all the possible combinations of adjacent signals can be produced. Since this is 4-PAM, the possible values for a single sample are $x[n] \in \{-3, -1, 1, 3\}$.

	-3	-1	1	3
-3	9	3	-3	-9
-1	3	1	-1	-3
1	-3	-1	1	3
3	-9	-3	3	9

Table 1: Symbol combinations

From Table 1, there are 16 possibilities for adjacent symbol multiplication. $\pm\{1, 9\}$ occur twice each, while ± 3 occur four times each. Over a suffiently long dataset, the sum in

Equation (2) will become an average of all the combinations.

$$\begin{gathered}\frac{1}{N}\sum_k a_k a_{k-n} = \frac{2}{16}(1) + \frac{2}{16}(-1) + \frac{2}{16}(9) + \frac{2}{16}(-9) + \frac{4}{16}(3) + \frac{4}{16}(-3) \\ \frac{1}{N}\sum_k a_k a_{k-n} = 0\end{gathered} \tag{4}$$

(ii) Calculating R_0

R_0 is the autocorrelation of the impulse chain at time of zero. The sum from Equation (2) becomes

$$\lim_{N\to\infty} \frac{1}{N}\sum_k a_k^2 \tag{5}$$

Like in Section (i), it can be assumed that all possibilities of $\{-3, -1, 1, 3\}$ are equally likelely, and the sum becomes the average.

$$R_0 = \frac{1}{4}(-3)^2 + \frac{1}{4}(-1)^2 + \frac{1}{4}(1)^2 + \frac{1}{4}(3)^2 = 5 \tag{6}$$

(iii) Calculating PSD

From Equation (3), since R_n is zero for all $n \neq 0$:

$$R_x(\tau) = R_0\delta(\tau) \tag{7}$$

$$R_x(f) = \mathcal{F}\{\frac{5}{T_s}\delta(\tau)\} = \frac{5}{T_s} = S_x(f) \tag{8}$$

$$\begin{gathered}S_y(f) = |P(f)|^2 S_x(f) = S_x(f)\int_{-\infty}^{\infty} rect(T_s)\, dt \\ S_y(f) = \frac{5}{T_s}T_s = \mathbf{5}\end{gathered} \tag{9}$$

C Design of pulse shape

A raised cosine filter is used for pulse shaping. Choose $B_T = 750KHz$ and $\alpha = 1$ so that the filter produces maximum power transfer while staying within the bandwidth limit.

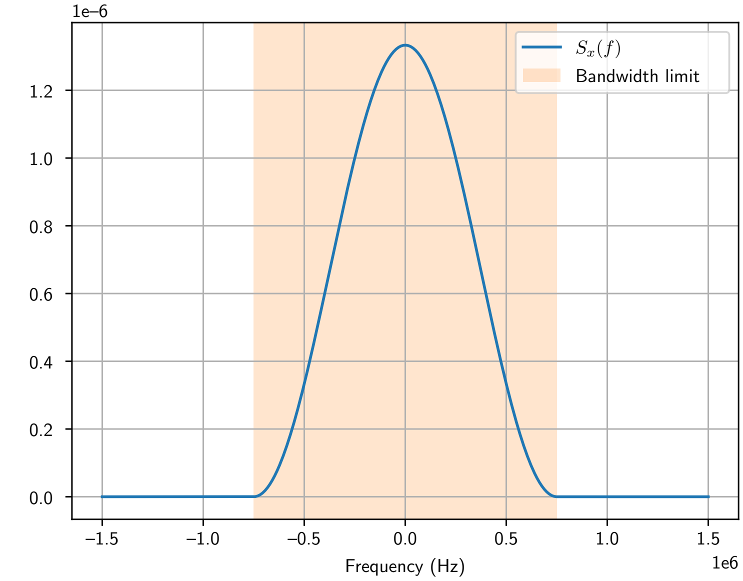

Figure 1: Power spectral density of raised cosine filter

The PSD calculated in Section B (using a rect pulse shape) has a flat frequency response across all frequencies. This is not possible to transmit as it requires infinite energy. It also violates the 750KHz bandwidth limit. The raised cosine pulse shape constrains the PSD to lie within the bandwith limits.

D Baseband transmission simulation

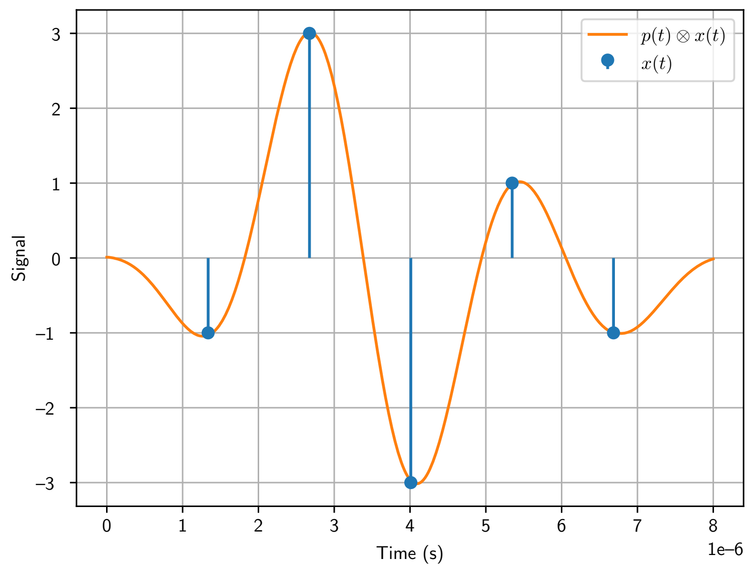

Figure 2: Baseband delta chain and pulse shaped output

E Ideal channel eye diagram

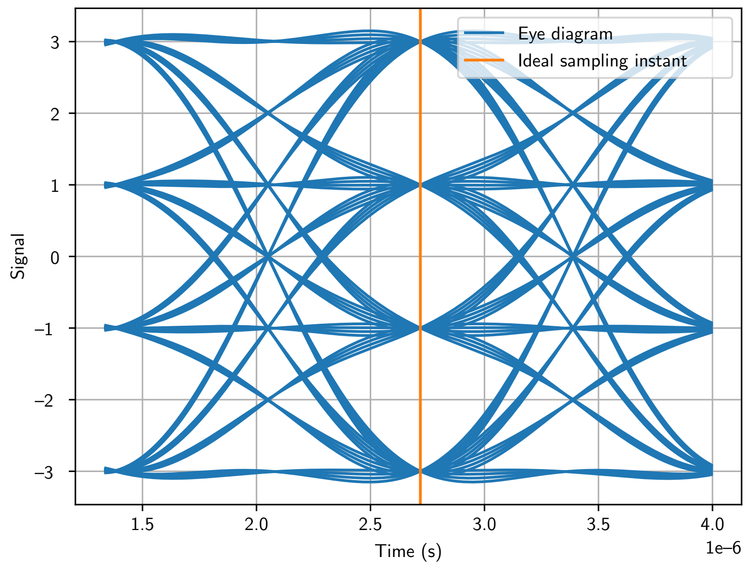

Figure 3: Eye diagram and sampling instant for a perfect channel

F AWGN effect

An addition of additive white gaussian noise in the channel will cause the eye to 'close' more than in the perfect channel case.

To simulate this, the noise power needs to be calculated. The average energy per symbol can be calculated as follows, where E_p is the energy of a pulse.

$$
\begin{aligned}
E_s &= \frac{E_p(4^2-1)}{3} \\
E_p &= \int_{-\infty}^{\infty} |p(t)|^2 dt
\end{aligned}
\tag{10}
$$

The signal energy of the root-raised-cosine pulse shaping filter is 1, meaning the energy per symbol is 5. We can derive the noise power spectral density, N_0 by relating the symbol energy to bit energy:

$$
\begin{aligned}
\frac{E_s}{N_0} = 2\frac{E_b}{N_0} = 20^{\left(\frac{(E_b/N_0)_{dB}}{10}\right)} = 20^1 = 20 \\
N_0 = \frac{E_s}{20} = 0.25
\end{aligned}
\tag{11}
$$

In the simulation, oversampling is also compensated for by dividing the noise power by the oversampling factor.

To generate the noise, samples were taken from a Gaussian distribution with a variance of

$$
\sigma^2 = \frac{N_0}{2\alpha} \int_{-\infty}^{\infty} |p(t)|^2 dt \tag{12}
$$

where α is the oversampling factor.

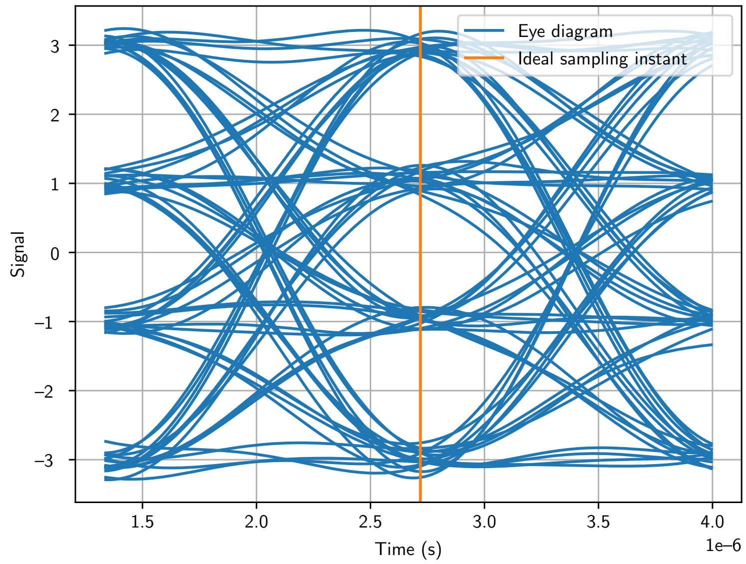

Figure 4: Eye diagram with $\frac{E_b}{N_0}$ of $10dB$

G 4-PAM Detector