ENEL422 Assignment Cover Sheet

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Please complete your details above and include this cover sheet as the first page on your assignment submission. Please also remember to upload your source file (Matlab or otherwise), in a format that can be executed to confirm functionality, i.e. ensure all support files are included, and files are logically names and commented. A 'zip' file with subdirectories for each question and/or a 'README' file is highly recommended.

The allocation of marks will be:

Question	Max [%]	Mark [%]
Transmitter/20		
a	5	
b	5	
c	5	
d	5	
Receiver/30		
e	10	
f	10	
g	10	
Equaliser/30		
h	20	
i	10	
Lab / 20	Device under test (A, B or C)	A
j	20	
TOTAL	**100**	

A Modulation type feasibility

All of the modulation types (binary polar, 4-PAM, 8-PAM) would be feasible for real-world implementation. Since the bandwidth and power spectrum for each modulation type is entirely determined by the pulse shape, each of these modulation schemes could be used within the bit rate and spectrum using a the pulse shape outlined in Section C. Larger constallation sizes (8-PAM) can provide higher data rates over a given bandwith at the expense of a higher bit error rate.

Out of these, I would reccommend 8-PAM, because there is no mention of the channel noise level. 8-PAM can provide better data rates and/or lower bandwidth at the cost of noise susceptibility.

B Power spectral density of 4-PAM

The power spectral density of a pulse-shaped data sequence is given by the energy spectral density of the pulse shape multiplied by power spectral density of the impulse chain (data sequence). The power spectral density of the impulse chain can be found be calculating the Fourier transform of the autocorrelation of the impulse chain.

$$S_y(f) = |P(f)|^2 S_x(f) = |P(f)|^2 \mathcal{F}\{R_x(\tau)\} \tag{1}$$

$R_x(\tau)$ is a continuous autocorrelation, but due to the fact that the data is comprised of delta functions, $R_x(\tau)$ will only be non-zero when these delta functions line up, ie. when τ is an integer multiple of the sampling period T_s. Because of this, $R_x(\tau)$ can be represented as a sum rather than an integral.

R_n represents the value of the autocorrelation at an integer sample offset of n. N represents the window size in units of samples ($N = \frac{T}{T_s}$).

$$R_n = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k-n} \tag{2}$$

Since the continuous time correlation is only non-zero at multiples of sample period, it can be represented as an impulse chain (ie. sum of time-offset delta functions), where the height of each delta function is given by the discreet autocorrelation value at that offset.

$$R_x(\tau) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} R_n \delta(\tau - nT_s) \tag{3}$$

(i) Calculating R_n

From Equation (2), we can see that the discreet autocorrelation values are dependent on two adjacent values of $x[n]$. No specific data sequence is given in the question, so I will assume the input data is random. A table of all the possible combinations of adjacent signals can be produced. Since this is 4-PAM, the possible values for a single sample are $x[n] \in \{-3, -1, 1, 3\}$.

	-3	-1	1	3
-3	9	3	-3	-9
-1	3	1	-1	-3
1	-3	-1	1	3
3	-9	-3	3	9

Table 1: Symbol combinations

From Table 1, there are 16 possibilities for adjacent symbol multiplication. $\pm\{1,9\}$ occur twice each, while ± 3 occur four times each. Over a suffiently long dataset, the sum in Equation (2) will become an average of all the combinations.

$$\begin{gathered}\frac{1}{N}\sum_k a_k a_{k-n} = \frac{2}{16}(1) + \frac{2}{16}(-1) + \frac{2}{16}(9) + \frac{2}{16}(-9) + \frac{4}{16}(3) + \frac{4}{16}(-3) \\ \frac{1}{N}\sum_k a_k a_{k-n} = 0\end{gathered} \tag{4}$$

(ii) Calculating R_0

R_0 is the autocorrelation of the impulse chain at time of zero. The sum from Equation (2) becomes

$$\lim_{N\to\infty}\frac{1}{N}\sum_k a_k^2 \tag{5}$$

Like in Section (i), it can be assumed that all possibilities of $\{-3,-1,1,3\}$ are equally likelely, and the sum becomes the average.

$$R_0 = \frac{1}{4}(-3)^2 + \frac{1}{4}(-1)^2 + \frac{1}{4}(1)^2 + \frac{1}{4}(3)^2 = 5 \tag{6}$$

(iii) Calculating PSD

From Equation (3), since R_n is zero for all $n \neq 0$:

$$R_x(\tau) = R_0\delta(\tau) \tag{7}$$

$$R_x(f) = \mathcal{F}\{\frac{5}{T_s}\delta(\tau)\} = \frac{5}{T_s} = S_x(f) \tag{8}$$

$$\begin{gathered}S_y(f) = |P(f)|^2 S_x(f) = S_x(f)\int_{-\infty}^{\infty} rect(T_s)\,dt \\ S_y(f) = \frac{5}{T_s}T_s = \mathbf{5}\end{gathered} \tag{9}$$

A flat PSD of 5 has an infinite bandwith, and therefore does not meet the bandwidth requirement of 750KHz.

C Design of pulse shape

A raised cosine filter is used for pulse shaping. Choose $B_T = 750KHz$ and $\alpha = 1$ so that the filter produces maximum power transfer while staying within the bandwidth limit.

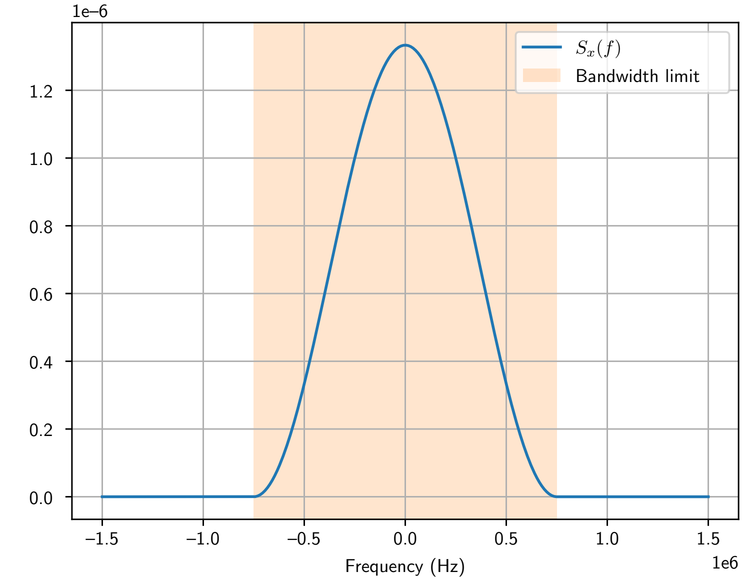

Figure 1: Power spectral density of raised cosine filter

The PSD calculated in Section B (using a rect pulse shape) has a flat frequency response across all frequencies. This is not possible to transmit as it requires infinite energy. It also violates the 750KHz bandwidth limit. The raised cosine pulse shape constrains the PSD to lie within the bandwith limits.

D Baseband transmission simulation

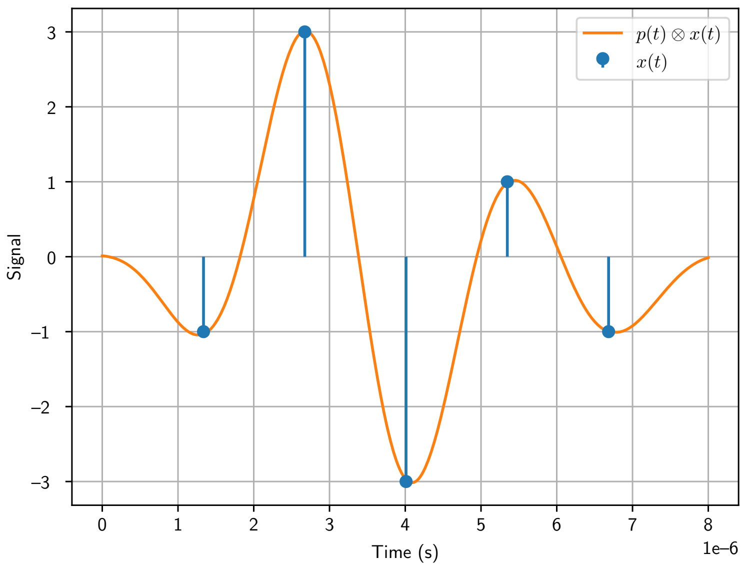

Figure 2: Baseband delta chain and pulse shaped output

E Ideal channel eye diagram

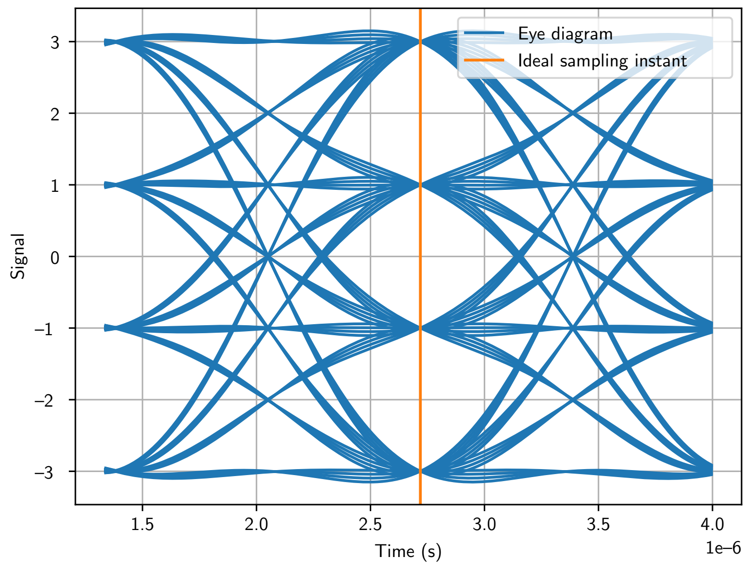

Figure 3: Eye diagram and sampling instant for a perfect channel

F AWGN effect

An addition of additive white gaussian noise in the channel will cause the eye to 'close' more than in the perfect channel case.

To simulate this, the noise power needs to be calculated. The average energy per symbol can be calculated as follows, where E_p is the energy of a pulse.

$$E_p = \int_{-\infty}^{\infty} |p(t)|^2 \, dt \tag{10}$$

Given a value for $\frac{E_b}{N_0}$, the value for N_0 can be calculated as follows:

$$N_0 = \left(\frac{E_b}{N_0} \right)^{-1} E_b \tag{11}$$

For the filter selected in the baseband receiver simulation, the energy is normalized such that $E_b = 1$. To generate noise to add to the channel, N_0 was used as the variance of a randomly sampled gaussian variable. The standard deviation is given by $\sqrt{N_0}$. AWGN always has a mean $\mu = 0$.

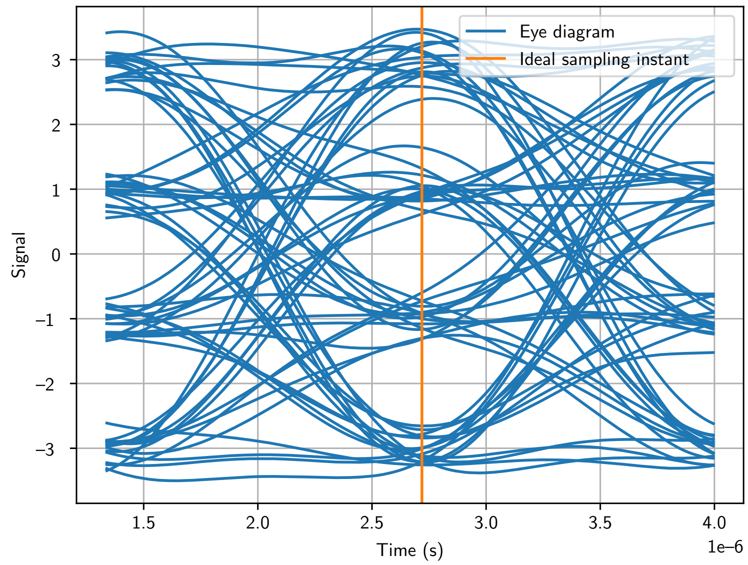

Figure 4: Eye diagram with AWGN, $\frac{E_b}{N_0} = 10dB$

G 4-PAM Detector

To produce a 4-PAM thresholding detector, thresholds were chosen at $\{-2, 0, 2\}$. These thresholds are in the midpoint between symbol signal levels. This was chosen because AWGN causes symbol errors at the same rate irregardless of which symbol is being transmitted. The detector samples at the ideal sampling instant.

The detector was run with AWGN in the channel for a variety of $\frac{E_b}{N_0}$ values. Theoretical error rate was calculated using

$$R_e = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) \tag{12}$$

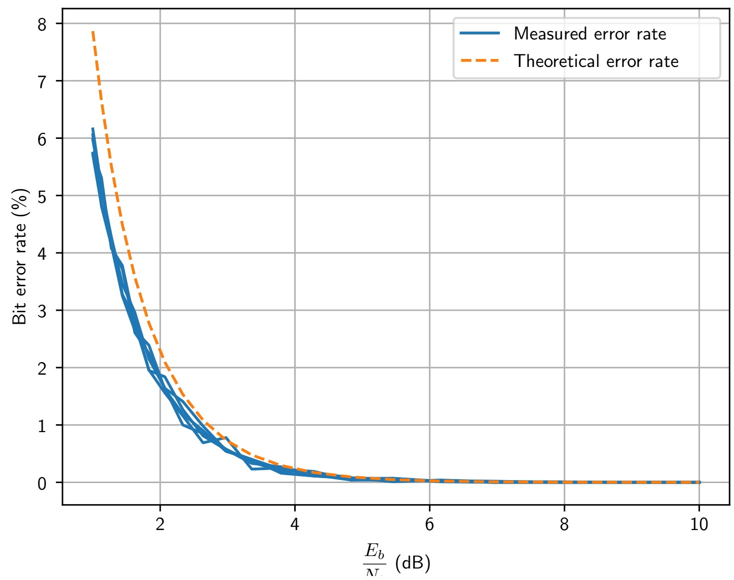

Figure 5: Bit error rate for simulated channel

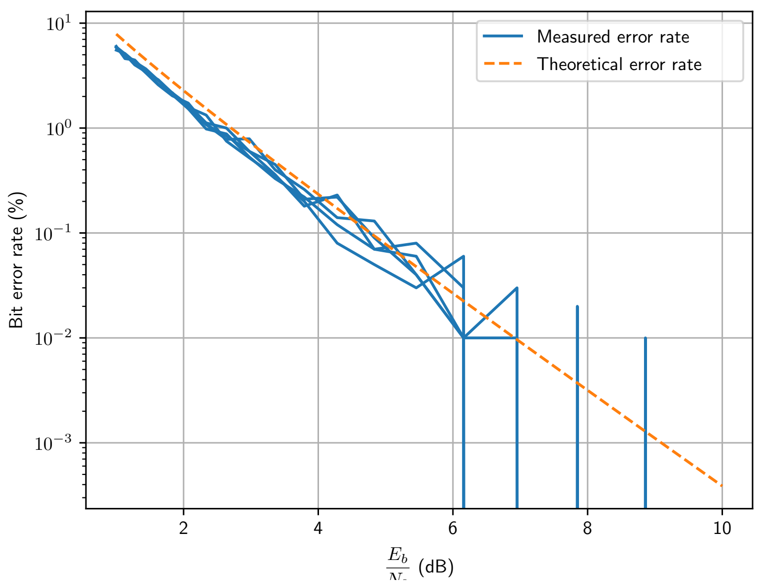

Figure 6: Bit error rate for simulated channel, log Y axis

As shown in Figure 5, the bit error rate is nearly zero for higher values of $\frac{E_b}{N_0}$. This is very apparent in Figure 6, where the error rate is zero and therefor undefined/$-\infty$ on a log scale. The SNR of these data points is high enough that not a single bit error occurs in the 10000 simulated bits. In order to predict the BER for these values of $\frac{E_b}{N_0}$, a much longer bit stream would need to be simulated to trigger enough bit errors to get a good statistical sample.

H ISI and Equalization

$$y[i] = \sum_{k=-\infty}^{\infty} a_k h[i-k] + w[i] \tag{13}$$

The sampled receiver output $y[i]$ is the sum over all transmitted symbols a_k multiplied by the effective channel filter at the offset of each symbol. This models ISI because the recieved sample may be influenced by other transmitted samples (a_k) if the impulse response of the effective channel filter at that symbol ($h[i-k]$) is nonzero.

$$R_y[m] \triangleq E\left[y[i+m]y^*[i]\right] \tag{14}$$

For a wide sense stationary process, the autocorrelation of a signal is the expected value of the random variable times it's conjugate. This is due to autocorrelation effectively being a time delayed sum, which is a realization of an average/mean, giving the expected value definition.

$$R_y[m] = E\left[\left(\sum_k a_k h[i+m-k] + w[i+m]\right)\left(\sum_j a_j^* h^*[i-j] + w^*[i]\right)\right] \tag{15}$$

Plugging Equation (13) into Equation (14).

$$E[a_k a_j^*] = \begin{cases} E_a & j = k \\ 0 & j \neq k \end{cases} \tag{16}$$

This case is discussed in Section B. For any combination of different symbols, there are an equal number of positive and negative results, due to the symbol levels being spread around zero. Because of this, the mean (expected value) of two disjoint symbols is zero. If a symbol is multiplied with the conjugate of itself (equivalent to $|a_k|^2$), its expected value will be the average of the squares of the symbol levels, represented here as E_a.

$$E\left[a_k w^*[i]\right] = 0 \tag{17}$$

The expected value of AWGN $w^*[i]$ is 0. Since the signals a_k and $w^*[i]$ are statistically independent, $E\left[a_k w^*[i]\right]$ can be rewritten as $E[a_k]E[w^*[i]] = E[a_k] \cdot 0 = 0$

$$E\left[w[l]w^*[i]\right] = \begin{cases} \sigma_w^2 = \frac{N_0}{2} & l = i \\ 0 & l \neq i \end{cases} \tag{18}$$

Two samples of AWGN are independent. The expected value of any AWGN sample $E[w[l]] = 0$. If two different samples are multiplied, the expected value will also be zero ($E[xy] = E[x]E[y]$ for independent variables), hence $E[w[l]w^*[i]] = 0$ for $l \neq i$. For a single sample ($l = i$), the expected value is the energy of the noise $\frac{N_0}{2}$, which is also the variance of the noise's gaussian distribution σ_w^2.

$$R_y[m] = E\left[\left(\sum_k a_k h[i+m-k] + \sum_k w[i+m]\right)\left(\sum_j a_j^* h^*[i-j] + \sum_j w^*[i]\right)\right] \quad (19)$$

Distribute sum operators from Equation (15).

$$\begin{aligned} R_y[m] = & \\ & E\left[\sum_k a_k h[i+m-k] \sum_j a_j^* h^*[i-j]\right] + \\ & E\left[\sum_k a_k h[i+m-k] \sum_j w^*[i]\right] + \\ & E\left[\sum_k w[i+m] \sum_j a_j^* h^*[i-j]\right] + \\ & E\left[\sum_k w[i+m] \sum_j w^*[i]\right] \end{aligned} \quad (20)$$

Multilply the two binomials and distribute the expected value operation.

$$R_y[m] = E\left[\sum_k a_k h[i+m-k] \sum_j a_j^* h^*[i-j]\right] + E\left[\sum_k w[i+m] \sum_j w^*[i]\right] \quad (21)$$

Using the Equation (17), we are able to eliminate the second and third terms, since they include an expected value for the AWGN, which is zero.

$$\therefore R_y[m] = \sum_k E_a h[i+m-k] h^*[i-k] + \frac{N_0}{2}\delta[m] \quad (22)$$

Using Equation (16), the expected value of the product $a_k a_j^*$ is equal to E_a. Using Equation (18), the expected value of the product $w[i+m]w^*[i]$ is zero except for when $j = k$, ie. at the sample m, when it is $\frac{N_0}{2}$. This can be represented by a delta function at time m. The filter coefficients $h[x]$ and $h^*[x]$ are left intact.

$$R_y[m] = E_a \sum_j h[m+j] h^*[j] + \frac{N_0}{2}\delta[m] \quad (23)$$

Pull E_a out of the sum, and set $j = i - k$ as the sum iterator.

I MMSE equaliser simulation

(i) MATLAB plots

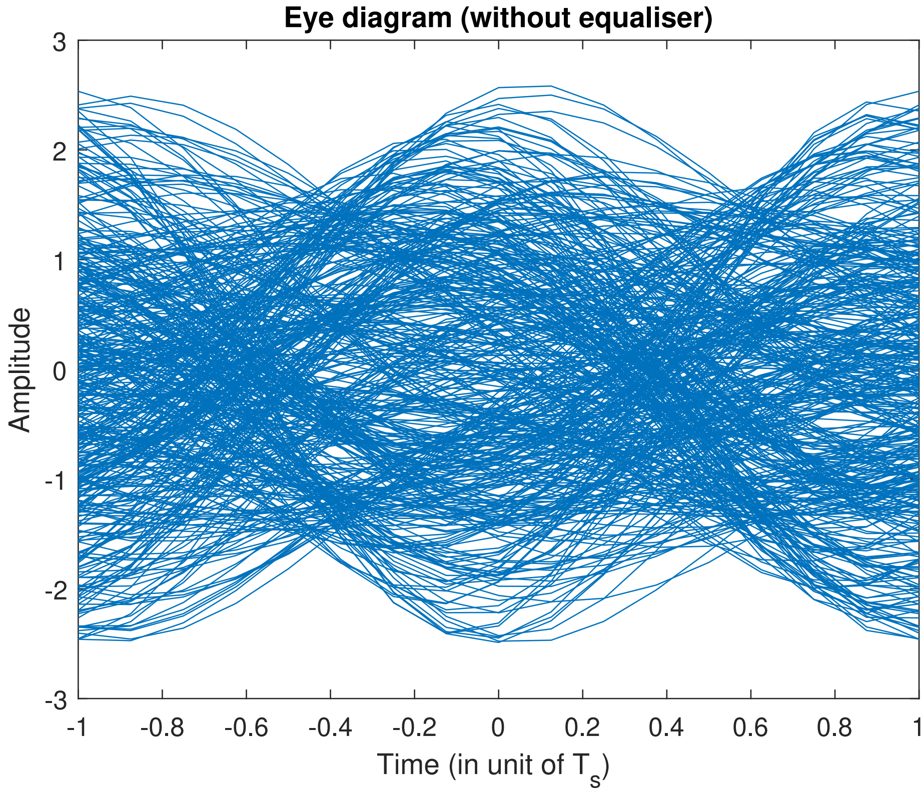

Figure 7: Eye diagram without equaliser

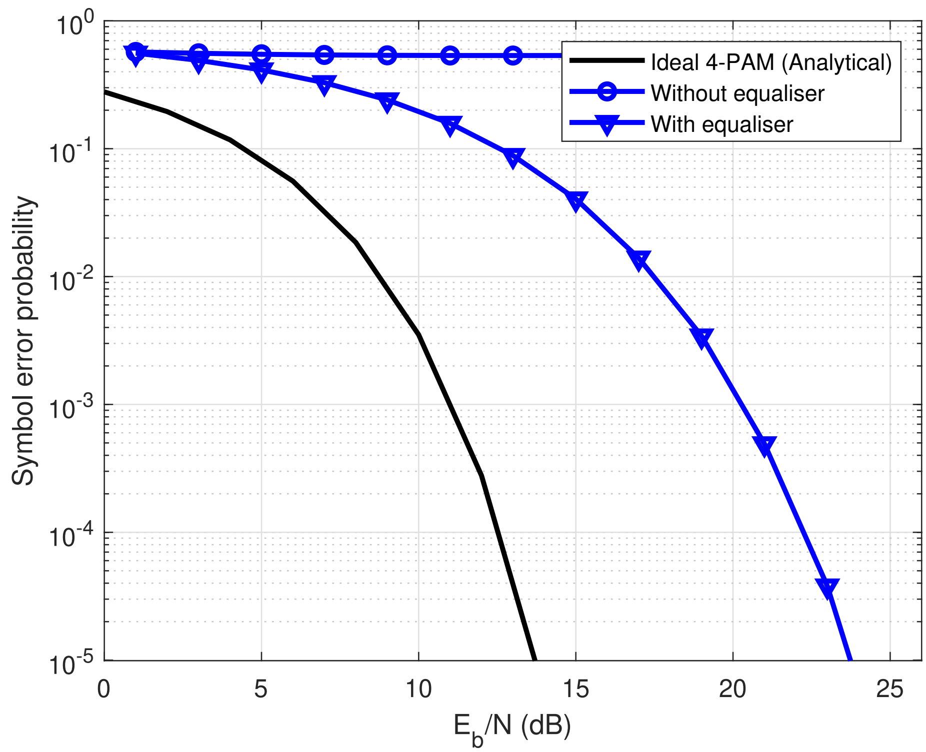

Figure 8: Symbol error probability with and without equaliser

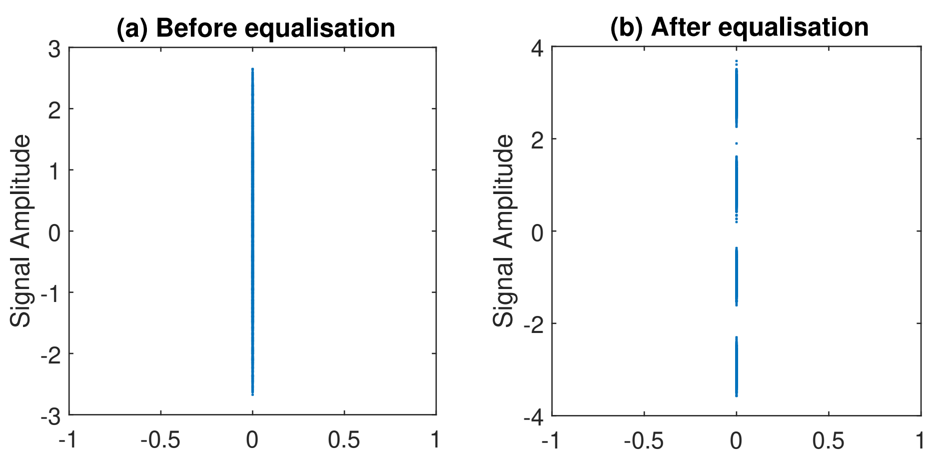

Figure 9: Signal amplitude levels (constellation) with and without equaliser

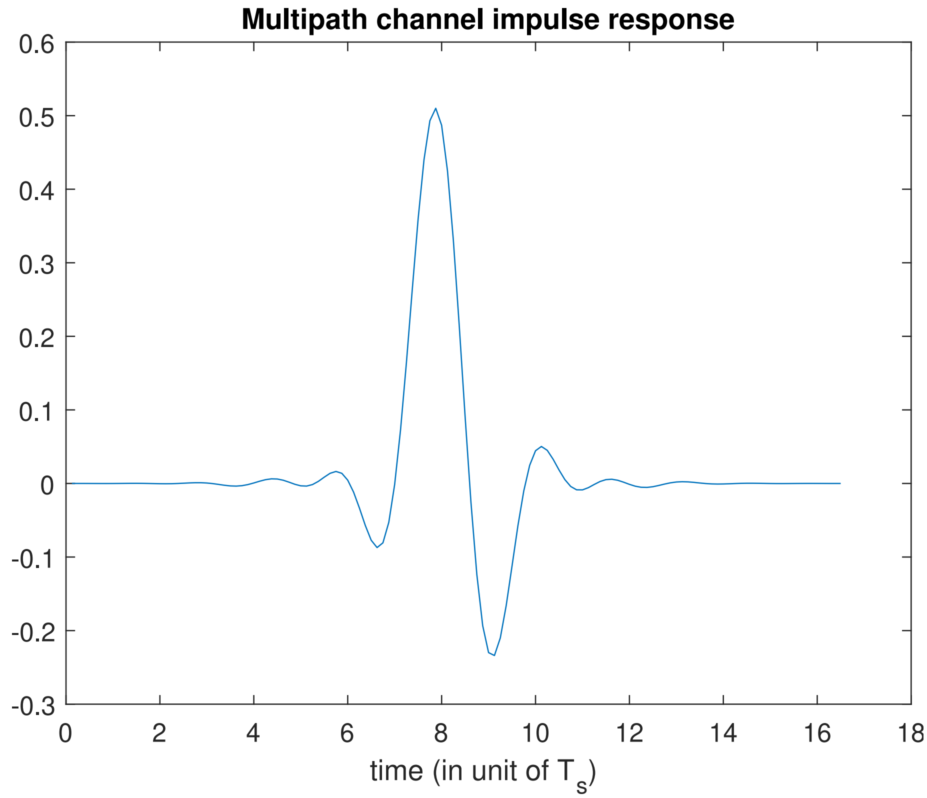

Figure 10: Impulse response of multipath channel

(ii) Analysis

Equalization can greatly improve the performance of a communication system. Prior to use of an equalizer, the ‘eye’ is completely closed as shown in Figure 7. There is no margin over noise. After equalization, a larger margin over noise is produced, meaning the eye is ‘open’ and signals can be decoded with an error rate less than 100%. This difference can be shown in the constellations in Figure 9, with gaps showing margin over noise only appearing after equalization.

Due to there being zero margin over noise prior to equalization, the receiver is effectively sampling nothing but ISI noise. Because of this, the error rate is a flat 60% for all SNRs, illustrated by the ‘without equalizer’ plot in Figure 8.

J Lab Exercise

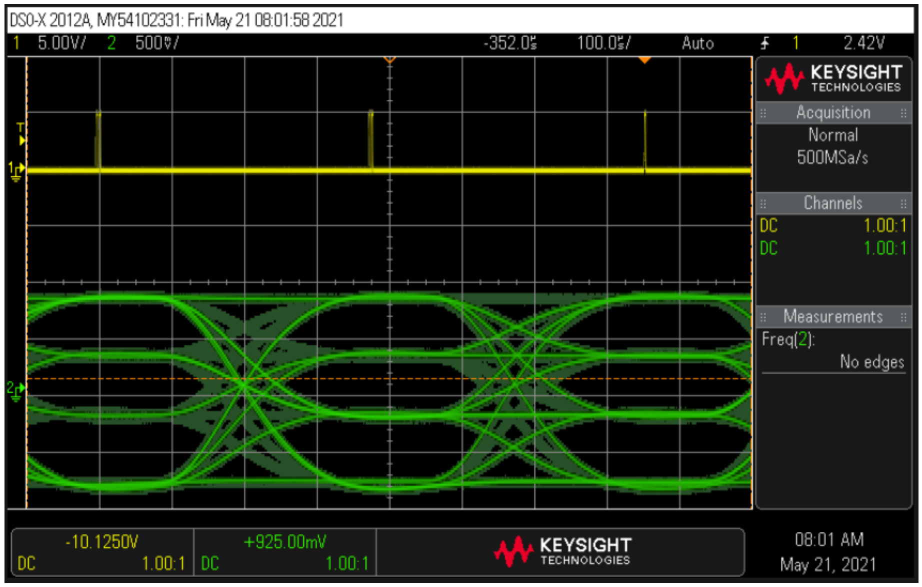

Figure 11: Oscilloscope output for signal generator **A**

(i) Steps required to generate image

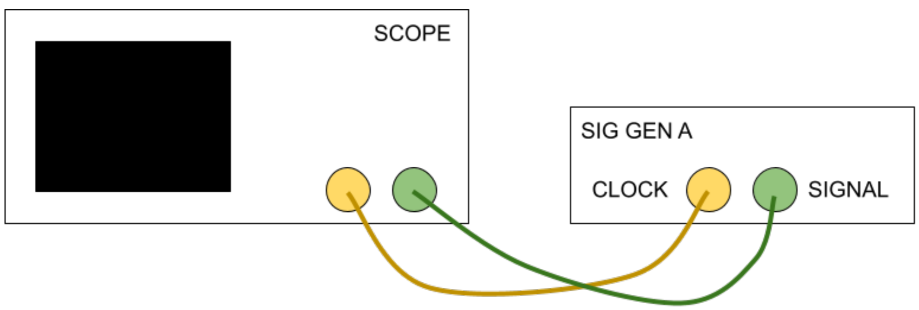

Figure 12: Experiment setup

These steps can be used to generate an eye diagram on a generic digital storage oscilloscope:

1. Connect the signal and clock outputs of the signal generator to two input channels of the oscilloscope.

2. Set the trigger of the oscilloscope to use the clock channel.

3. Adjust the trigger level to approximately half of the maximum amplitude of the clock signal, about 2.5V in this case. Any trigger level such that the refresh is only triggered on the edge of a pulse will work (ie. not above the max amplitude but not below the noise threshold of a clock '0' output).

4. Adjust the horizontal scaling factor of the oscilloscope such that about three clock pulses are visible in the oscilloscope output window.

5. Adjust the vertical scaling of both the signal and clock waveforms so main features are visible and large enough to measure using the volts per division and grid.

6. Turn the persistence of the oscilloscope to maximum (∞)

7. If the image needs to be adjusted (scaled or horizontally offset), the persistence must be cleared.

(ii) Modulation type

As shown in Figure 11, the signal output shows four main levels at the eye open instant. These are levels of voltage (signal *amplitude*). Because of this, the modulation type is 4-PAM.

(iii) Signal characteristics

Symbol rate The symbol rate is the frequency at which symbols are transmitted, which can be found by the reciprocal of the time between clock pulses. Measured at 2.66KHz.

Error free sampling interval The error free sampling interval is the horizontal distance between the left-most and right-most corners of the eye. In this interval, the signal has settled in to one of the threshold regions for symbol decoding and will be correctly decoded. Measured at 260us.

Margin over noise Noise margin is the voltage difference between a threshold (the horizontal axis of the eye) and the maximum value of a noisy signal at the widest opening point of the eye. If the signal were to be distorted by more than this margin, it would cause a bit error even with if the sampling instant was chosen optimally. Measured at 200mV.

Level crossing timing jitter Level crossing jitter is the variation in when signals cross the threshold level. It shows the clock stability of the transmitter or phase distortion in the transmission medium. Measured at 120us.

(iv) Sensitivity of signal to timing error

The error free sampling interval (260us) is a large proportion (approximately 70%) of the entire symbol period (376us). Because of this, errors in timing will be tolerated well, provided the timing error is not cumulative. If the timing error is purely a phase error, a wide range of errors will be tolerated. If the timing error is a sampling frequency error (ie. desynchronization of local oscillators), the symbols will become corrupted as the RX sampling oscillator moves out of the 260us window.

An eye diagram with a large error free sampling period may signify the signal has not been band limited inside the bandwidth of the original TX pulse shape. Since the measured signal exhibits this characteristic, it has not been band limited.