Insper

# Computação Gráfica

Aula 21: SDF (Signed Distance Function)

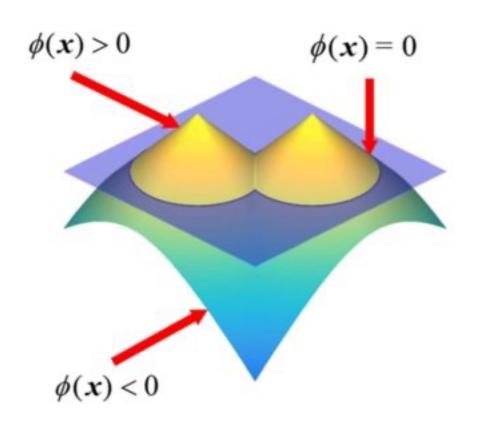
## Signed Distance Function (SDF)

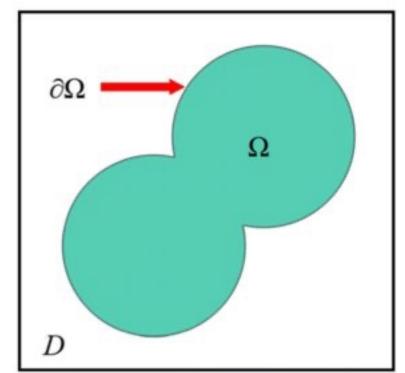
Em matemática, a Função de Distância com Sinal (Signed Distance Function) é a distância ortogonal de um determinado ponto x ao limite de um conjunto  $\Omega$  em um espaço métrico, com o sinal determinado por x estar ou não no interior de  $\Omega$ .

A função tem valores positivos nos pontos x dentro de  $\Omega$ , diminui em valor quando x se aproxima do limite de  $\Omega$  onde a função de distância com sinal é zero e assume valores negativos fora de  $\Omega$ . No entanto, a convenção alternativa às vezes também é adotada (isto é, negativo dentro de  $\Omega$  e positivo fora).

#### Signed Distance Function (SDF)

#### Explicar...

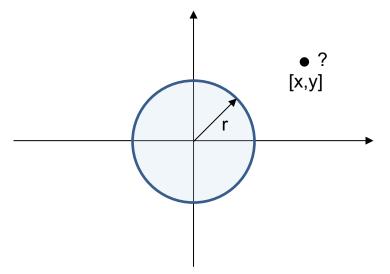






## Função para círculo 2D

Imagine que estou testando um ponto *uv*. Como saber se este ponto está dentro de um círculo de raio r?



```
float sdfCircle(vec2 uv, float r) {
   float d = length(uv) - r;
   return d;
}
```

#### Signed Distance Function (SDF)

#### Exemplo em GLSL para Shadertoy

```
vec3 sdfCircle(vec2 uv, float r) {
    float d = length(uv) - r;
    return d > 0. ? vec3(1.) : vec3(0., 0., 1.);
}

void mainImage( out vec4 fragColor, in vec2 fragCoord )
{
    vec2 uv = fragCoord/iResolution.xy;
    vec3 col = sdfCircle(uv, .2);
    fragColor = vec4(col,1.0);
}
```

O que acontece?



### Signed Distance Function (SDF)

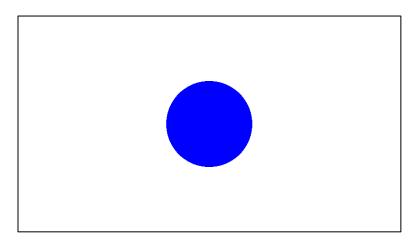
Como melhorar?

#### Centralizar:

```
uv = 0.5;
```

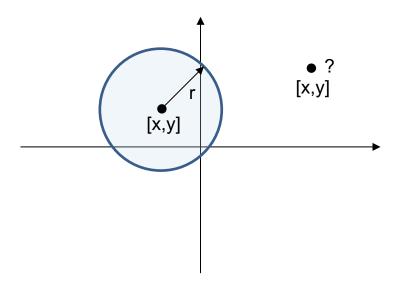
#### Acertar a razão de aspecto:

```
uv.x *= iResolution.x/iResolution.y;
```



## Círculo em outras posições

Como podemos fazer o círculo aparecer em outra posição?



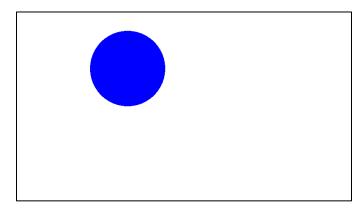
```
float sdfCircle(vec2 uv, float r, vec3 c) {
   float d = length(uv - c) - r;
   return d;
}
```

### Exemplo

#### Um exemplo com o seguinte ponto:

```
vec3 sdfCircle(vec2 uv, float r, vec2 c) {
    float d = length(uv - c) - r;
    return d > 0. ? vec3(1.) : vec3(0., 0., 1.);
}

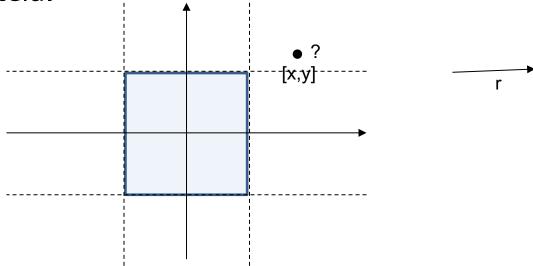
void mainImage( out vec4 fragColor, in vec2 fragCoord ) {
    vec2 uv = fragCoord/iResolution.xy;
    uv -= 0.5;
    uv.x *= iResolution.x/iResolution.y;
    vec3 col = sdfCircle(uv, .2, vec2(-0.3, 0.2));
    fragColor = vec4(col,1.0);
}
```



#### Atividade em Aula: Faça um quadrado

Usando os conceitos aprendidos de SDF. Desenhe um

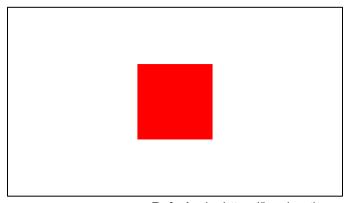
quadrado na tela.



```
bool sdfSquare(vec2 uv, float size, vec2 c) {
   float x = uv.x - c.x;
   float y = uv.y - c.y;
   float d = max(abs(x), abs(y)) - size;
   return d;
}
```

#### **Exemplo Completo**

```
vec3 sdfSquare(vec2 uv, float size, vec2 c) {
   float x = uv_x - c_x;
   float y = uv.y - c.y;
   float d = max(abs(x), abs(y)) - size;
   return d > 0. ? vec3(1.) : vec3(1., 0., 0.);
void mainImage( out vec4 fragColor, in vec2 fragCoord ) {
   vec2 uv = fragCoord/iResolution.xy;
   uv = 0.5;
   uv.x *= iResolution.x/iResolution.y;
   vec3 col = sdfSquare(uv, 0.2, vec2(0.0, 0.0));
   fragColor = vec4(col,1.0);
```



### Transformando objetos (Rotação)

Podemos aplicar a matriz de rotação em um objeto.

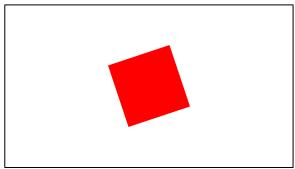
$$R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Cuidado que as matrizes no GLSL são column-first

```
vec2 rotate(vec2 uv, float th) {
    return mat2(cos(th), sin(th), -sin(th), cos(th)) * uv;
}
```

### Exemplo com rotação animada

```
vec2 rotate(vec2 uv, float th) {
    return mat2(cos(th), sin(th), -sin(th), cos(th)) * uv;
vec3 sdfSquare(vec2 uv, float size, vec2 c) {
    float x = uv.x - c.x;
    float y = uv.y - c.y;
   vec2 rotated = rotate(vec2(x,y), iTime);
   float d = max(abs(rotated.x), abs(rotated.y)) - size;
    return d > 0. ? vec3(1.) : vec3(1., 0., 0.);
void mainImage( out vec4 fragColor, in vec2 fragCoord ) {
   vec2 uv = fragCoord/iResolution.xy;
   uv = 0.5;
   uv.x *= iResolution.x/iResolution.y;
   vec3 col = sdfSquare(uv, 0.2, vec2(0.0, 0.0));
    fragColor = vec4(col,1.0);
```





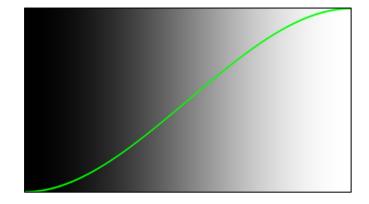
### Exemplo mix (LERP)

```
void mainImage( out vec4 fragColor, in vec2 fragCoord )
{
    vec2 uv = fragCoord/iResolution.xy;
    float interpolatedValue = mix(0., 1., uv.x);
    vec3 col = vec3(interpolatedValue);
    fragColor = vec4(col,1.0);
}
```



#### Exemplo smoothstep (Hermite)

```
#define diameter 0.01
float smooth step( float edge0, float edge1, float x ) {
    float p = clamp((x - edge0) / (edge1 - edge0), 0.0, 1.0);
   //float v = p * p * (3.0 - 2.0 * p); // smoothstep formula.
    float v = smoothstep( edge0, edge1, x ); // built-in
    return v;
float plot(vec2 st, float y)
    return smooth_step( y-diameter, y , st.y) -
    smooth step( y , y+diameter, st.y);
void mainImage( out vec4 fragColor, in vec2 fragCoord ){
    vec2 st = fragCoord.xy/iResolution.xy;
    float y = smooth step(0.0, 1.0, st.x);
   // grey gradient
   vec3 color = vec3(y);
    // draw smoothstep curve in green
    float percent = plot(st,y);
   color = (1.0-percent) *color + percent*vec3(0.0,1.0,0.0);
    fragColor = vec4(color,1.0);
```



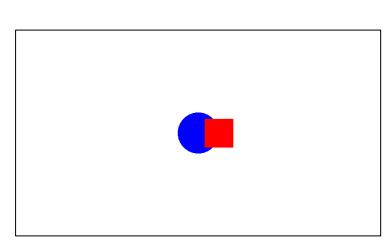
### Atividade: Faça um degrade para fundo de tela

Usando os conceitos aprendidos em aula, faça um degrade

```
void mainImage( out vec4 fragColor, in vec2 fragCoord ) {
   vec2 uv = fragCoord/iResolution.xy;
   vec3 gradientStartColor = vec3(0.6, 0.3, 0.0);
   vec3 gradientEndColor = vec3(0.2, 0.8, 1.);
   vec3 col = mix(gradientStartColor, gradientEndColor, uv.y);
   fragColor = vec4(col,1.0);
}
```

## Organizando código

```
float sdfCircle(vec2 uv, float r, vec2 c) {
  return length(uv - c) - r;
float sdfSquare(vec2 uv, float size, vec2 c) {
 float x = uv.x - c.x;
 float y = uv.y - c.y;
  return max(abs(x), abs(y)) - size;
vec3 drawScene(vec2 uv) {
 float circle = sdfCircle(uv, 0.1, vec2(0, 0));
  float square = sdfSquare(uv, 0.07, vec2(0.1, 0));
 vec3 col = vec3(1):
  col = mix(vec3(0, 0, 1), col, step(0., circle));
  col = mix(vec3(1, 0, 0), col, step(0., square));
  return col;
void mainImage( out vec4 fragColor, in vec2 fragCoord ) {
 vec2 uv = fragCoord/iResolution.xy;
 uv = 0.5;
 uv.x *= iResolution.x/iResolution.y;
 vec3 col = drawScene(uv);
  fragColor = vec4(col,1.0);
```



#### Combinando formas

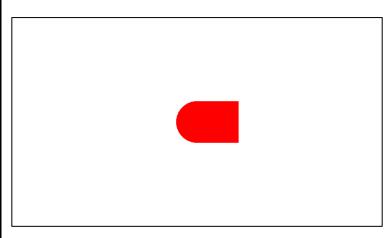
Um dos truques interessantes do SDF é poder combinar as formas de diversas formas.

Aqui veremos as principais possibilidades.



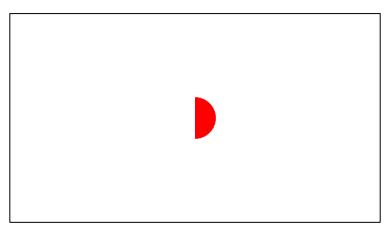
#### União

```
float sdfCircle(vec2 uv, float r, vec2 c) {
  return length(uv - c) - r;
float sdfSquare(vec2 uv, float size, vec2 c) {
 float x = uv.x - c.x;
 float y = uv.y - c.y;
  return max(abs(x), abs(y)) - size;
vec3 drawScene(vec2 uv) {
 float circle = sdfCircle(uv, 0.1, vec2(0, 0));
 float square = sdfSquare(uv, 0.1, vec2(0.1, 0));
 vec3 col = vec3(1);
 float res = min(circle, square);
  col = mix(vec3(1, 0, 0), col, step(0., res));
  return col;
void mainImage( out vec4 fragColor, in vec2 fragCoord ) {
 vec2 uv = fragCoord/iResolution.xy;
 uv = 0.5;
 uv.x *= iResolution.x/iResolution.y;
 vec3 col = drawScene(uv);
 fragColor = vec4(col,1.0);
```



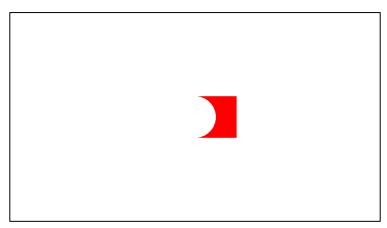
## Intersecção

```
float sdfCircle(vec2 uv, float r, vec2 c) {
  return length(uv - c) - r;
float sdfSquare(vec2 uv, float size, vec2 c) {
 float x = uv.x - c.x;
 float y = uv_y - c_y;
  return max(abs(x), abs(y)) - size;
vec3 drawScene(vec2 uv) {
 float circle = sdfCircle(uv, 0.1, vec2(0, 0));
 float square = sdfSquare(uv, 0.1, vec2(0.1, 0));
 vec3 col = vec3(1);
 float res = max(circle, square);
  col = mix(vec3(1, 0, 0), col, step(0., res));
  return col;
void mainImage( out vec4 fragColor, in vec2 fragCoord ) {
 vec2 uv = fragCoord/iResolution.xy;
 uv = 0.5;
 uv.x *= iResolution.x/iResolution.y;
 vec3 col = drawScene(uv);
 fragColor = vec4(col,1.0);
```



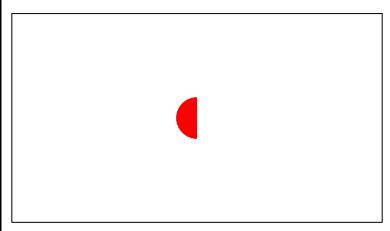
### Subtrair o círculo do quadrado

```
float sdfCircle(vec2 uv, float r, vec2 c) {
  return length(uv - c) - r;
float sdfSquare(vec2 uv, float size, vec2 c) {
  float x = uv \cdot x - c \cdot x;
 float y = uv_1y - c_1y_1
  return max(abs(x), abs(y)) - size;
vec3 drawScene(vec2 uv) {
  float circle = sdfCircle(uv, 0.1, vec2(0, 0));
  float square = sdfSquare(uv, 0.1, vec2(0.1, 0));
  vec3 col = vec3(1);
  float res = max(-circle, square);
  col = mix(vec3(1, 0, 0), col, step(0., res));
  return col;
void mainImage( out vec4 fragColor, in vec2 fragCoord ) {
  vec2 uv = fragCoord/iResolution.xy;
  uv = 0.5;
  uv.x *= iResolution.x/iResolution.y;
  vec3 col = drawScene(uv);
  fragColor = vec4(col,1.0);
```



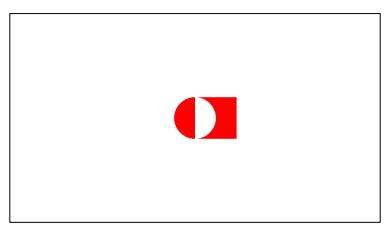
#### Subtrair o quadrado do círculo

```
float sdfCircle(vec2 uv, float r, vec2 c) {
  return length(uv - c) - r;
float sdfSquare(vec2 uv, float size, vec2 c) {
 float x = uv.x - c.x;
 float y = uv_1y - c_1y_1
  return max(abs(x), abs(y)) - size;
vec3 drawScene(vec2 uv) {
 float circle = sdfCircle(uv, 0.1, vec2(0, 0));
 float square = sdfSquare(uv, 0.1, vec2(0.1, 0));
 vec3 col = vec3(1):
 float res = max(circle, -square);
  col = mix(vec3(1, 0, 0), col, step(0., res));
  return col;
void mainImage( out vec4 fragColor, in vec2 fragCoord ) {
 vec2 uv = fragCoord/iResolution.xy;
 uv = 0.5;
 uv.x *= iResolution.x/iResolution.y;
 vec3 col = drawScene(uv);
 fragColor = vec4(col,1.0);
```



#### Ou exclusivo (XOR)

```
float sdfCircle(vec2 uv, float r, vec2 c) {
  return length(uv - c) - r;
float sdfSquare(vec2 uv, float size, vec2 c) {
  float x = uv \cdot x - c \cdot x;
 float y = uv_1y - c_1y_1
  return max(abs(x), abs(y)) - size;
vec3 drawScene(vec2 uv) {
  float circle = sdfCircle(uv, 0.1, vec2(0, 0));
  float square = sdfSquare(uv, 0.1, vec2(0.1, 0));
  vec3 col = vec3(1);
  float res = max(min(circle, square), -max(circle, square));
  col = mix(vec3(1, 0, 0), col, step(0., res));
  return col;
void mainImage( out vec4 fragColor, in vec2 fragCoord ) {
  vec2 uv = fragCoord/iResolution.xy;
 uv = 0.5:
  uv.x *= iResolution.x/iResolution.y;
  vec3 col = drawScene(uv);
  fragColor = vec4(col,1.0);
```



#### Resumindo

```
res = min(d1, d2); // união
res = max(d1, d2); // intersecção
res = max(-d1, d2); // subtração - d1 menos d2
res = max(d1, -d2); // subtração - d2 menos d1
res = \max(\min(d1, d2), -\max(d1, d2)); // xor
```



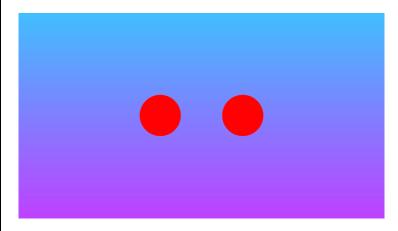
#### Posicionamento 2D

Inspirado originalmente no trabalho de Inigo Quilez. A seguir serão apresentadas algumas estratégias de posicionar e exibir padrões de imagens.



### opSymX

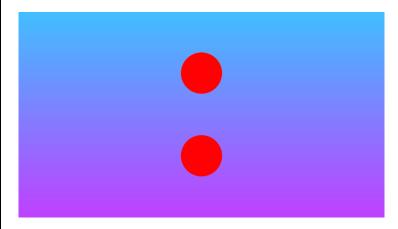
```
float opSymX(vec2 p, float r)
  p.x = abs(p.x);
  return sdCircle(p, r, vec2(0.2, 0));
vec3 drawScene(vec2 uv) {
  vec3 col = getBackgroundColor(uv);
 float res; // result
  res = opSymX(uv, 0.1);
  res = step(0., res);
  col = mix(vec3(1,0,0), col, res);
  return col;
```





## opSymY

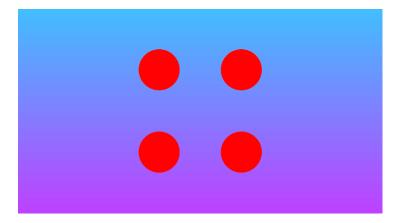
```
float opSymY(vec2 p, float r)
  p.y = abs(p.y);
  return sdCircle(p, r, vec2(0, 0.2));
vec3 drawScene(vec2 uv) {
  vec3 col = getBackgroundColor(uv);
 float res; // result
  res = opSymY(uv, 0.1);
  res = step(0., res);
  col = mix(vec3(1,0,0), col, res);
  return col;
```





## opSymXY

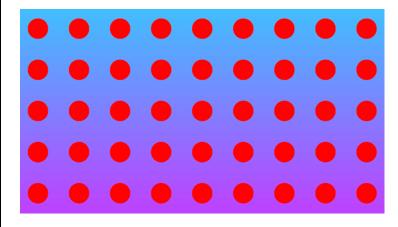
```
float opSymXY(vec2 p, float r)
  p = abs(p);
  return sdCircle(p, r, vec2(0.2));
vec3 drawScene(vec2 uv) {
  vec3 col = getBackgroundColor(uv);
 float res; // result
  res = opSymXY(uv, 0.1);
  res = step(0., res);
  col = mix(vec3(1,0,0), col, res);
  return col;
```





#### opRep

```
float opRep(vec2 p, float r, vec2 c)
  vec2 q = mod(p+0.5*c,c)-0.5*c;
  return sdCircle(q, r, vec2(0));
vec3 drawScene(vec2 uv) {
  vec3 col = getBackgroundColor(uv);
 float res; // result
  res = opRep(uv, 0.05, vec2(0.2, 0.2));
  res = step(0., res);
  col = mix(vec3(1,0,0), col, res);
  return col;
```

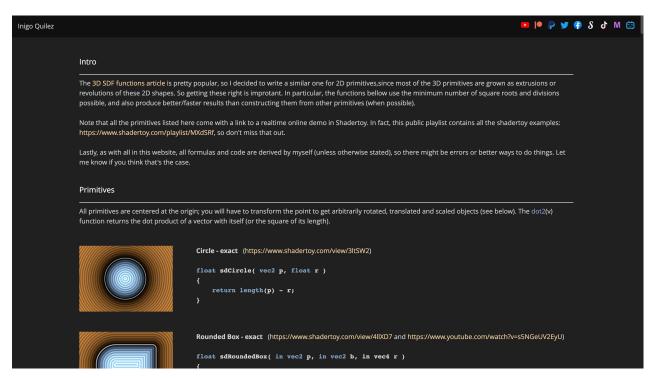




#### Funções SDF prontas

Muitas funcionalidades para SDF já existem. Um bom repositório é o site do Inigo Quilz:

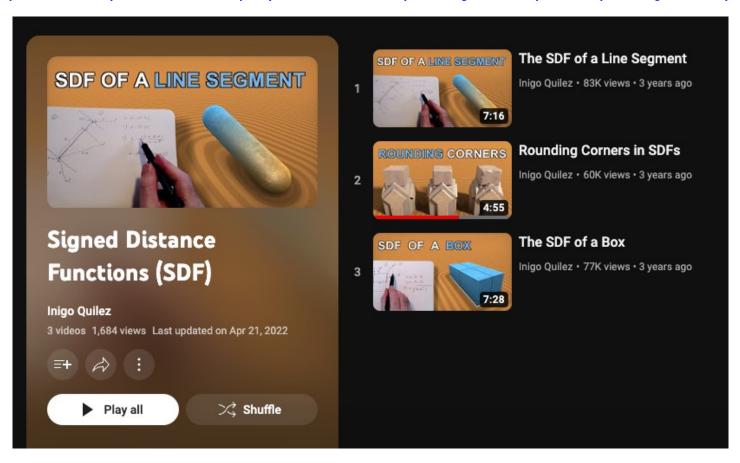
https://iquilezles.org/articles/distfunctions2d/





#### Vídeos sobre SDFs

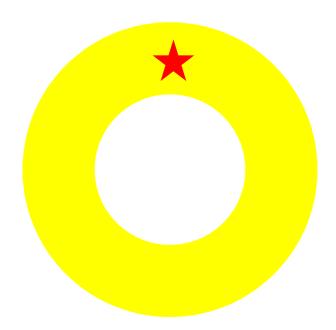
https://www.youtube.com/playlist?list=PL0EpikNmjs2AUFqRi3vmpkrO3j-zWuoyq





#### Projeto 2.1

Crie uma animação 2D no Fragment Shader de uma estrela (qualquer tipo) sobre um anel. A volta toda deve demorar 5 segundos. A velocidade na parte superior deve ser zero.



#### Referências

#### Baseado:

https://www.shadertoy.com/

#### Usando:

https://inspirnathan.com/posts/49-shadertoy-tutorial-part-3

#### Documentações:

https://iquilezles.org/

https://thebookofshaders.com/

# Insper

# Computação Gráfica

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