

Computação Gráfica

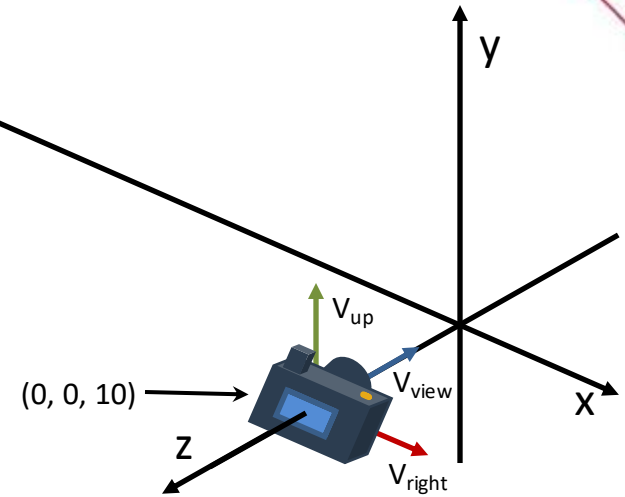
Revisão: Transformações e Projeções

Exemplo Completo:

- Transformações Geométricas
- Quatérnios
- Transformação Look-at
- Transformação Perspectiva
- Coordenadas normalizadas (NDC)
- Divisão Homogênea
- Transformação de tela
- Supersampling

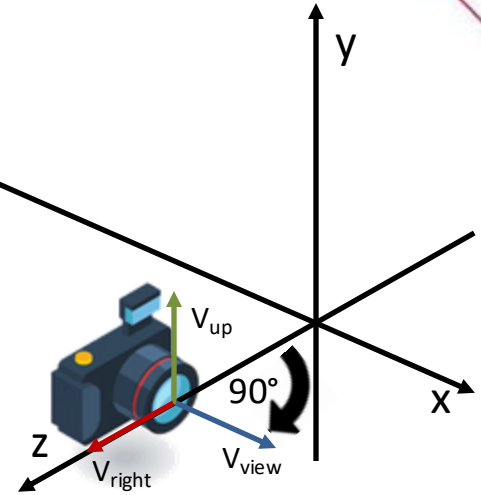
Definindo posição da câmera virtual

```
<Scene>  
<Viewpoint/>  
</Scene>
```



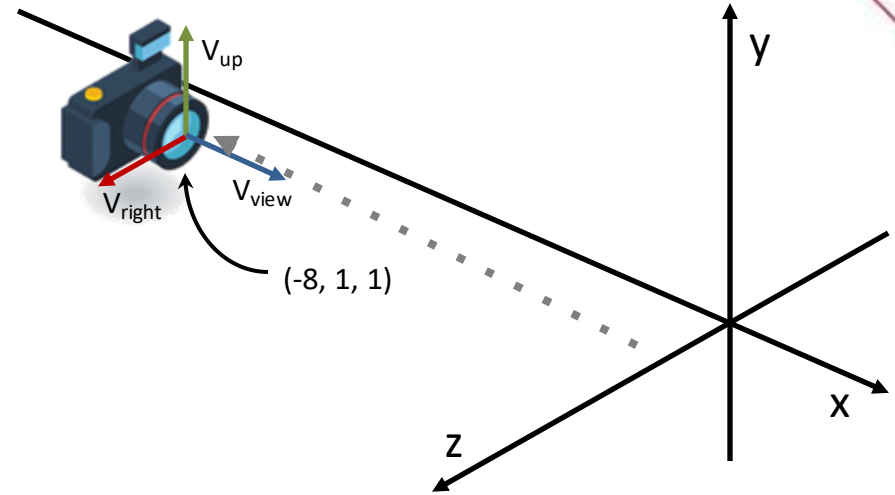
Definindo posição da câmera virtual

```
<Scene>  
<Viewpoint orientation="0 1 0 -1.57"/>  
</Scene>
```



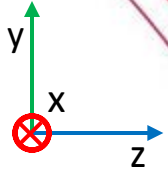
Definindo posição da câmera virtual

```
<Scene>  
<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>  
</Scene>
```



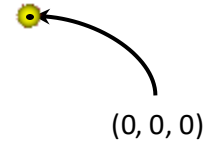
Criando uma esfera no centro do mundo

Visualização do ponto de vista da câmera mas no sistema de coordenadas do mundo.



```
<Scene>  
<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>  
<Transform>  
  <Shape>  
    <Sphere radius='0.1'/>  
    <Appearance>  
      <Material diffuseColor="1 1 0"/>  
    </Appearance>  
  </Shape>  
</Transform>  
</Scene>
```

amarelo



centro da esfera : $[0 \ 0 \ 0 \ 1]^T$

ou

centro da esfera : $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Criando um triângulo no centro do mundo

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius="0.1"/>
      <Appearance>
        <Material diffuseColor="1 1 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="1 0 0"/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```

vermelho

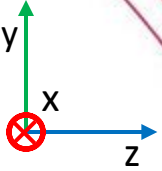
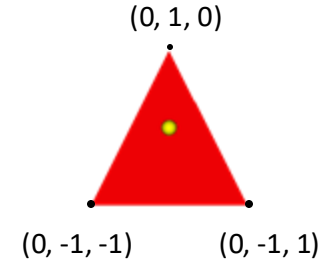


Diagram illustrating the vertices of a triangle in a 3D coordinate system, labeled P_0 , P_1 , and P_2 .

$$\text{vértices do triângulo} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

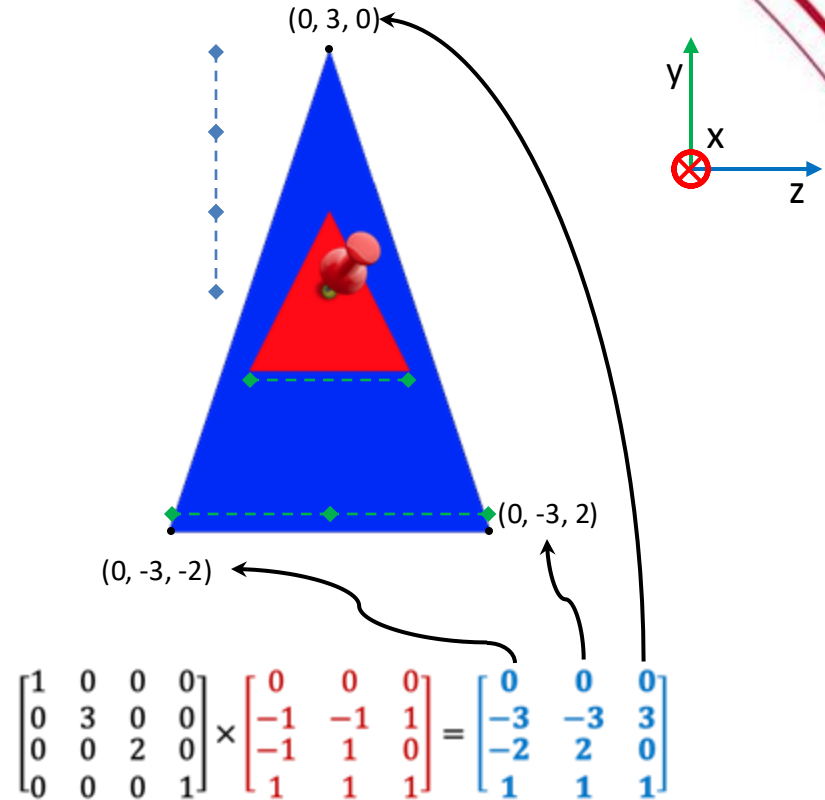
Novo triângulo ampliado

```

<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius="0.1"/>
      <Appearance>
        <Material diffuseColor="1 1 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="1 0 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2">
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="0 0 1"/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>

```

azul



Novo triângulo rotacionado

```
<Scene>
<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
<Transform>
  <Shape>
    <Sphere radius="0.1"/>
    <Appearance>
      <Material diffuseColor="1 1 0"/>
    </Appearance>
  </Shape>
</Transform>
<Transform>
  <Shape>
    <TriangleSet>
      <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor="1 0 0"/>
    </Appearance>
  </Shape>
</Transform>
<Transform scale="1 3 2"
  rotation="1 0 0 -1.57">
  <Shape>
    <TriangleSet>
      <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor="0 0 1"/>
    </Appearance>
  </Shape>
</Transform>
</Scene>
```

$$\mathbf{R}_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & \sin \theta & 0 \\ 0 & \sin \theta & -\cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$q = \cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)u_x i + \sin\left(\frac{\theta}{2}\right)u_y j + \sin\left(\frac{\theta}{2}\right)u_z k$$

$$q = \cos(-0.79) + \sin(-0.79)1i + \sin(-0.79)0j + \sin(-0.79)0k$$

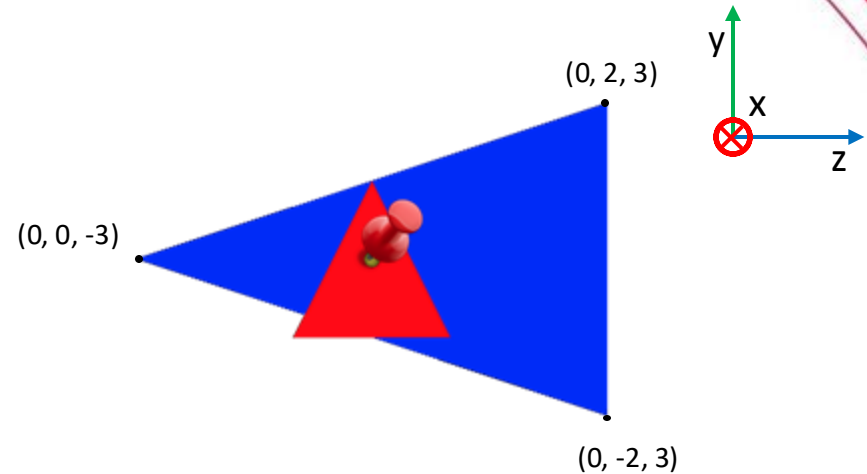
$$q = 0.71 - 0.71i$$

$$R = \begin{bmatrix} 1 - 2(q_j^2 + q_k^2) & 2(q_i q_j - q_k q_r) & 2(q_i q_k + q_j q_r) & 0 \\ 2(q_i q_j + q_k q_r) & 1 - 2(q_i^2 + q_k^2) & 2(q_j q_k - q_i q_r) & 0 \\ 2(q_i q_k - q_j q_r) & 2(q_j q_k + q_i q_r) & 1 - 2(q_i^2 + q_j^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Novo triângulo rotacionado

```
<Scene>
<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
<Transform>
  <Shape>
    <Sphere radius="0.1"/>
    <Appearance>
      <Material diffuseColor="1 1 0"/>
    </Appearance>
  </Shape>
</Transform>
<Transform>
  <Shape>
    <TriangleSet>
      <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor="1 0 0"/>
    </Appearance>
  </Shape>
</Transform>
<Transform scale="1 3 2"
  rotation="1 0 0 -1.57">
  <Shape>
    <TriangleSet>
      <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor="0 0 1"/>
    </Appearance>
  </Shape>
</Transform>
</Scene>
```



$$\begin{matrix} \text{rotação} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix} \times \begin{matrix} \text{escala} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ -2 & 2 & 0 \\ 3 & 3 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

Novo triângulo transladando

<Scene>

<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>

<Transform>

<Shape>

<Sphere radius="0.1"/>

<Appearance>

<Material diffuseColor="1 1 0"/>

</Appearance>

</Shape>

</Transform>

<Transform>

<Shape>

<TriangleSet>

<Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>

</TriangleSet>

<Appearance>

<Material diffuseColor="1 0 0"/>

</Appearance>

</Shape>

</Transform>

<Transform scale="1 3 2"

rotation="1 0 0 -1.57"

translation="2 1 1">

<Shape>

<TriangleSet>

<Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>

</TriangleSet>

<Appearance>

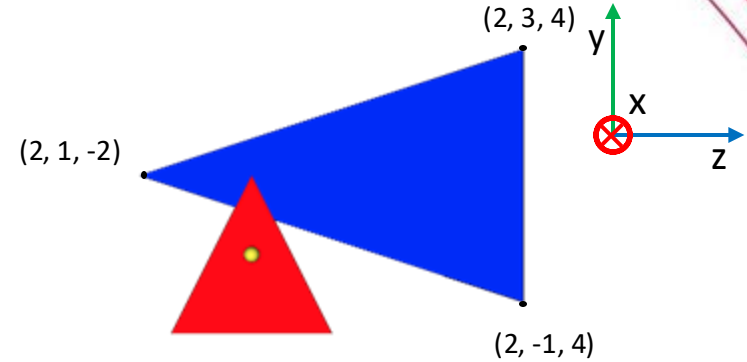
<Material diffuseColor="0 0 1"/>

</Appearance>

</Shape>

</Transform>

</Scene>



translação

rotação

escala

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ -1 & 3 & 1 \\ 4 & 4 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

Visualizando de outro ângulo

<Scene>

<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>

<Transform>

<Shape>

<Sphere radius="0.1"/>

<Appearance>

<Material diffuseColor='1 1 0'/>

</Appearance>

</Shape>

</Transform>

<Transform>

<Shape>

<TriangleSet>

<Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>

</TriangleSet>

<Appearance>

<Material diffuseColor='1 0 0'/>

</Appearance>

</Shape>

</Transform>

<Transform scale="1 3 2"

rotation="1 0 0 -1.57"

translation="2 1 1">

<Shape>

<TriangleSet>

<Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>

</TriangleSet>

<Appearance>

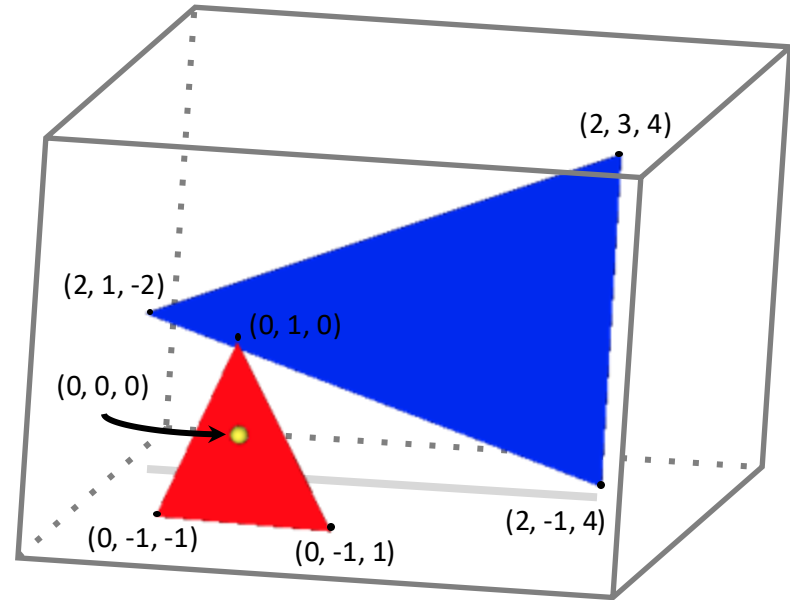
<Material diffuseColor='0 0 1'/>

</Appearance>

</Shape>

</Transform>

</Scene>



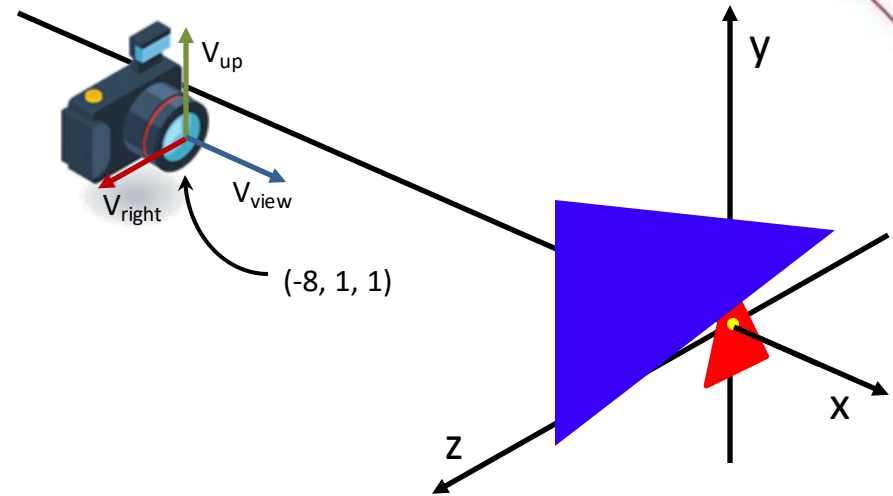
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 2 & 2 & 2 \\ -1 & 3 & 1 \\ 4 & 4 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

Transformação da Câmera

```

<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius="0.1"/>
      <Appearance>
        <Material diffuseColor="1 1 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="1 0 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="0 0 1"/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>

```



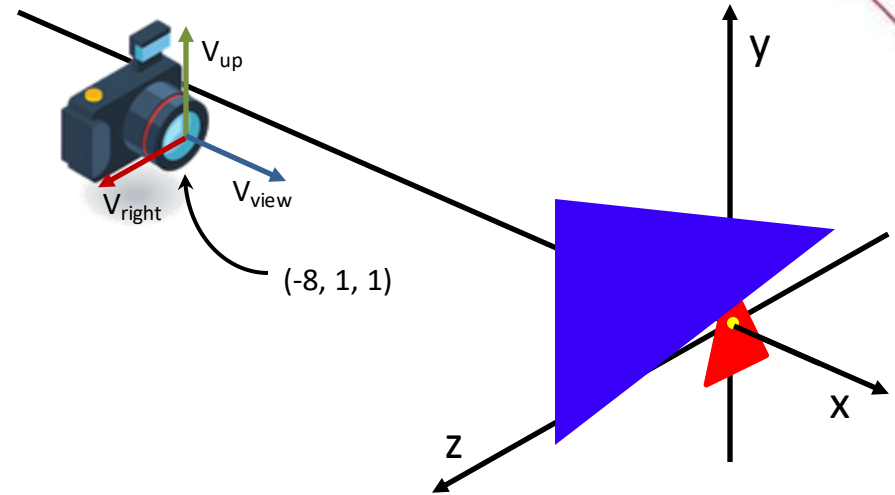
$$\begin{aligned} \mathbf{u} &= V_{right} \\ \mathbf{v} &= V_{up} \\ \mathbf{w} &= V_{view} \end{aligned}$$

$$\begin{bmatrix} u_x & v_x & -w_x & e_x \\ u_y & v_y & -w_y & e_y \\ u_z & v_z & -w_z & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$(T \cdot R)^{-1} = R^{-1} \cdot T^{-1}$$

Transformação da Câmera (Rotação)

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius="0.1"/>
      <Appearance>
        <Material diffuseColor="1 1 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="1 0 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="0 0 1"/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



Rotação:

Como temos uma câmera que estava na base da cena (ou seja identidade), podemos usar diretamente a rotação realizada para calcular a nova matriz.

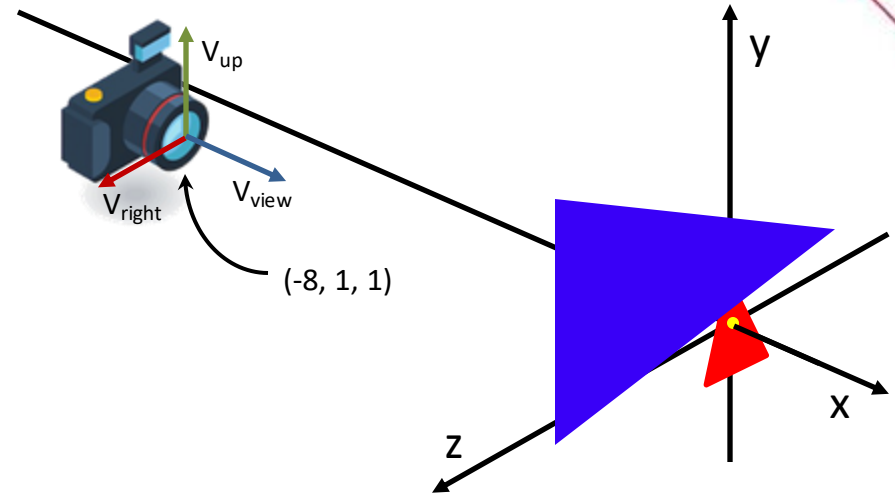
$$q = \cos(-0.79) + \sin(-0.79)0i + \sin(-0.79)1j + \sin(-0.79)0k$$

$$q = 0.71 - 0.71j$$

$$R = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformação da Câmera (Translação)

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius="0.1"/>
      <Appearance>
        <Material diffuseColor="1 1 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="1 0 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="0 0 1"/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



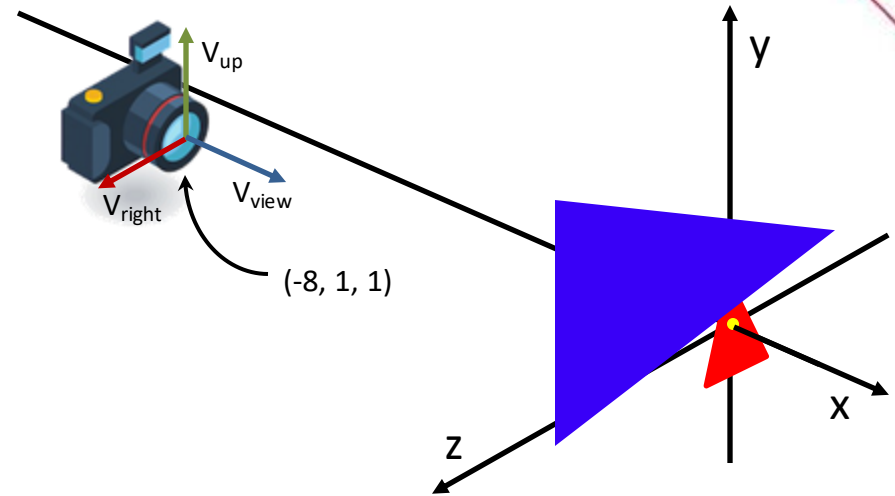
Translação:

$$T = \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformação da Câmera

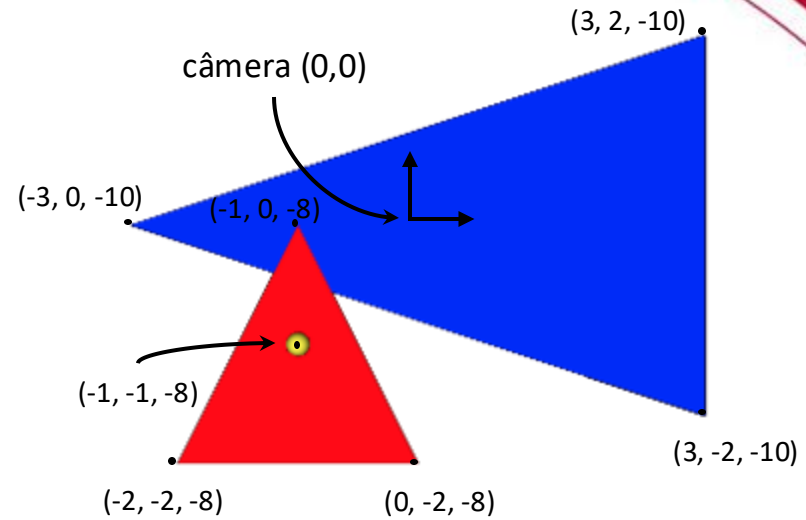
```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius="0.1"/>
      <Appearance>
        <Material diffuseColor="1 1 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="1 0 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="0 0 1"/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



$$\begin{array}{c} \text{Orientação}^{-1} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \times \begin{array}{c} \text{Translação}^{-1} \\ \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \text{Matriz de Visualização} \\ \text{(view)} \\ \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Aplicando Transformação

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius="0.1"/>
      <Appearance>
        <Material diffuseColor="1 1 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="1 0 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="0 0 1"/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



Matriz de Visualização

$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -8 \\ 1 \end{bmatrix}$$

Matriz de Visualização

$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -1 \\ -2 & -2 & 0 \\ -8 & -8 & -8 \\ 1 & 1 & 1 \end{bmatrix}$$

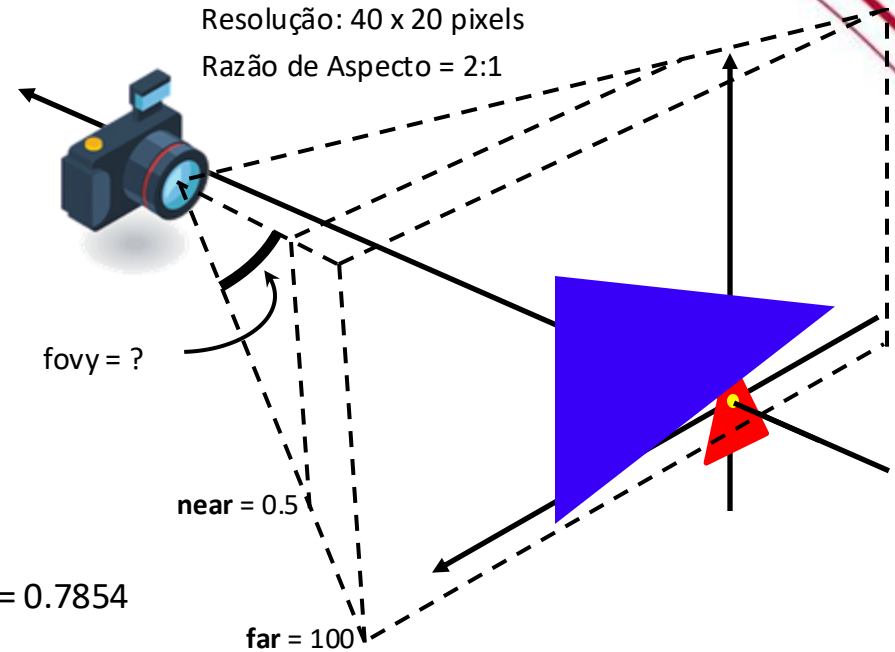
Matriz de Visualização

$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 2 & 2 \\ -1 & 3 & 1 \\ 4 & 4 & -2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -3 \\ -2 & 2 & 0 \\ -10 & -10 & -10 \\ 1 & 1 & 1 \end{bmatrix}$$

Matriz Perspectiva

```
<Scene>
<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
<Transform>
  <Shape>
    <Sphere radius="0.1"/>
    <Appearance>
      <Material diffuseColor="1 1 0"/>
    </Appearance>
  </Shape>
</Transform>
<Transform>
  <Shape>
    <TriangleSet>
      <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor="1 0 0"/>
    </Appearance>
  </Shape>
</Transform>
<Transform scale="1 3 2"
  rotation="1 0 0 -1.57"
  translation="2 1 1">
  <Shape>
    <TriangleSet>
      <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor="0 0 1"/>
    </Appearance>
  </Shape>
</Transform>
</Scene>
```

$$\text{fieldOfView} = \pi/4 = 0.7854$$



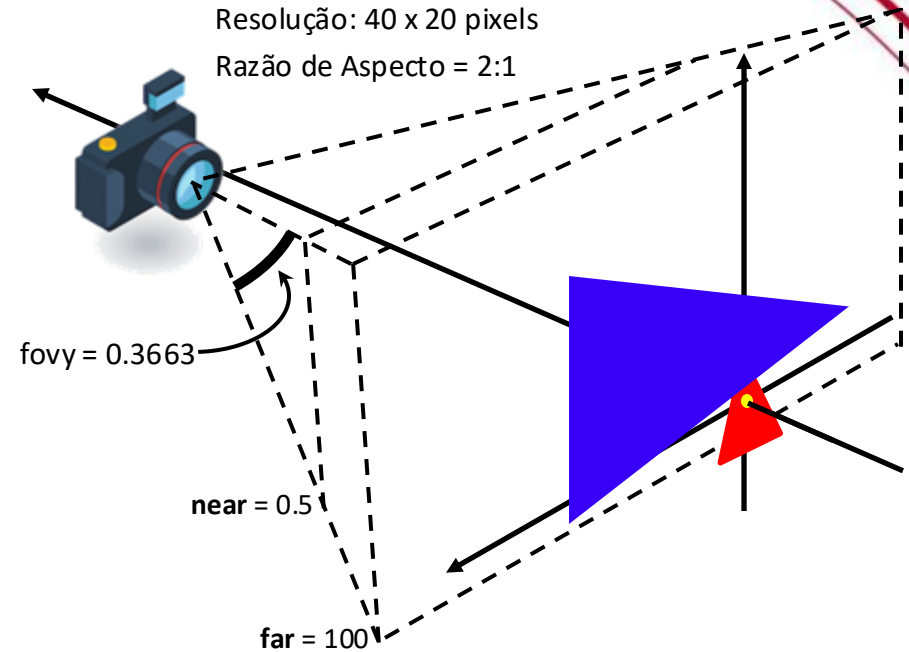
$$\text{FOV}_y = 2 \cdot \arctan \left(\tan \left(\frac{\text{FOV}_d}{2} \right) \cdot \frac{\text{Altura}}{\sqrt{\text{Altura}^2 + \text{Largura}^2}} \right)$$

$$\text{FOV}_y = 2 \cdot \arctan \left(\tan \left(\frac{0.7854}{2} \right) \cdot \frac{20}{\sqrt{20^2 + 40^2}} \right)$$

$$\text{FOV}_y = 0.3663$$

Matriz Perspectiva

```
<Scene>
<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
<Transform>
  <Shape>
    <Sphere radius="0.1"/>
    <Appearance>
      <Material diffuseColor="1 1 0"/>
    </Appearance>
  </Shape>
</Transform>
<Transform>
  <Shape>
    <TriangleSet>
      <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor="1 0 0"/>
    </Appearance>
  </Shape>
</Transform>
<Transform scale="1 3 2"
  rotation="1 0 0 -1.57"
  translation="2 1 1">
  <Shape>
    <TriangleSet>
      <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor="0 0 1"/>
    </Appearance>
  </Shape>
</Transform>
</Scene>
```



$$P = \begin{bmatrix} \frac{near}{right} & 0 & 0 & 0 \\ 0 & \frac{near}{top} & 0 & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & \frac{-2far*near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

top = near * tan (fovy) = 0.5 * 0.3663 = 0.1832

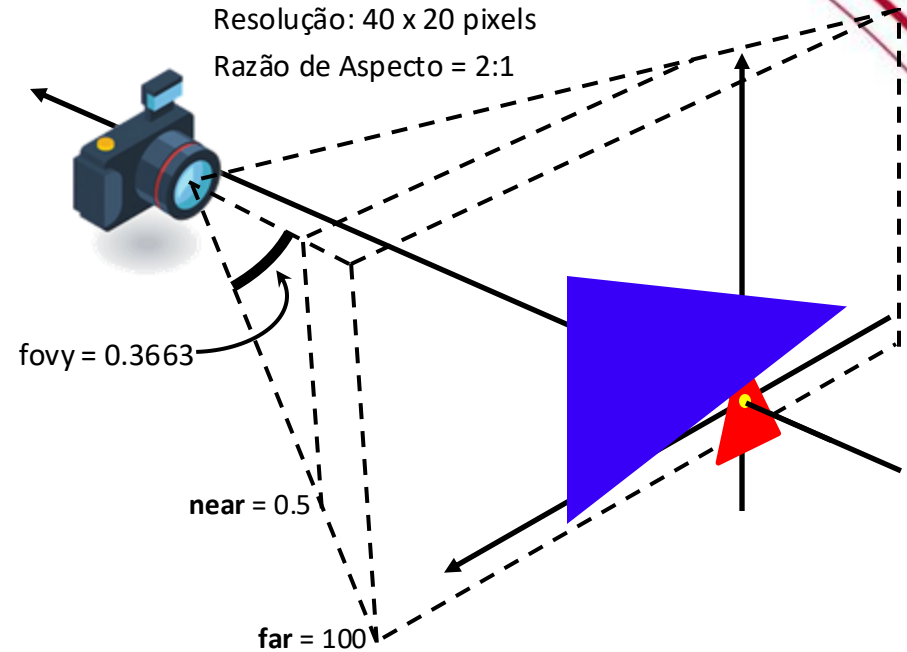
bottom = -top = -top = -0.1832

right = top * aspect = top * aspect = 0.3663

left = -right = -right = -0.3663

Matriz Perspectiva

```
<Scene>
<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
<Transform>
  <Shape>
    <Sphere radius="0.1"/>
    <Appearance>
      <Material diffuseColor="1 1 0"/>
    </Appearance>
  </Shape>
</Transform>
<Transform>
  <Shape>
    <TriangleSet>
      <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor="1 0 0"/>
    </Appearance>
  </Shape>
</Transform>
<Transform scale="1 3 2"
  rotation="1 0 0 -1.57"
  translation="2 1 1">
  <Shape>
    <TriangleSet>
      <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor="0 0 1"/>
    </Appearance>
  </Shape>
</Transform>
</Scene>
```



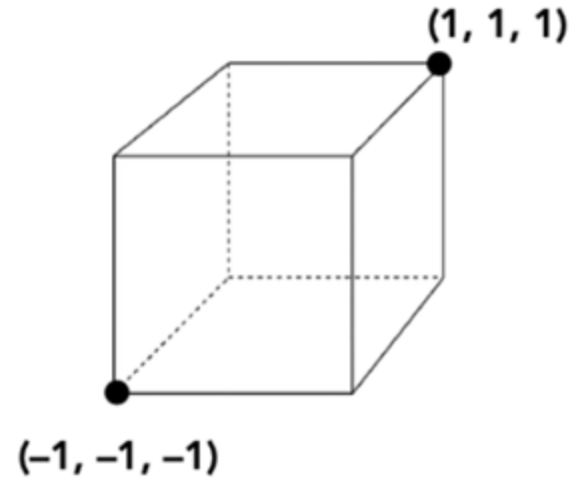
$$P = \begin{bmatrix} \frac{\text{near}}{\text{right}} & 0 & 0 & 0 \\ 0 & \frac{\text{near}}{\text{top}} & 0 & 0 \\ 0 & 0 & -\frac{\text{far}+\text{near}}{\text{far}-\text{near}} & -\frac{2\text{far}\cdot\text{near}}{\text{far}-\text{near}} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

top = 0.1832
bottom = -0.1832
right = 0.3663
left = -0.3663

$$P = \begin{bmatrix} \frac{0.5}{0.3663} & 0 & 0 & 0 \\ 0 & \frac{0.5}{0.1832} & 0 & 0 \\ 0 & 0 & -\frac{100+0.5}{100-0.5} & -\frac{2 \cdot 100 \cdot 0.5}{100-0.5} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1.3649 & 0 & 0 & 0 \\ 0 & 2.7298 & 0 & 0 \\ 0 & 0 & -1.01 & -1.005 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Aplicando Matriz de Projeção Perspectiva

Projeção em NDC
(Normalized Device Coordinate)



Projeção Perspectiva

$$\begin{bmatrix} 1.3649 & 0 & 0 & 0 \\ 0 & 2.7298 & 0 & 0 \\ 0 & 0 & -1.01 & -1.005 \\ 0 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 \\ -1 \\ -8 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.3649 \\ -2.7298 \\ 7.075 \\ 8 \end{bmatrix}$$

Projeção Perspectiva

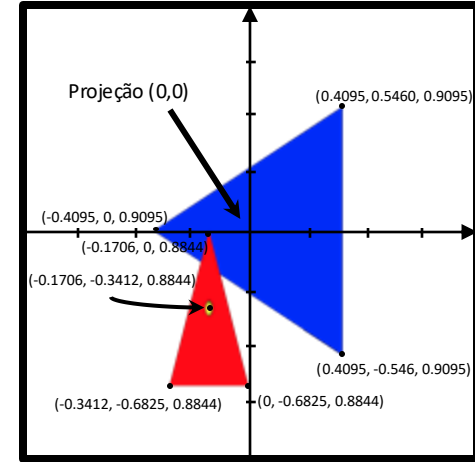
$$\begin{bmatrix} 1.3649 & 0 & 0 & 0 \\ 0 & 2.7298 & 0 & 0 \\ 0 & 0 & -1.01 & -1.005 \\ 0 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} -2 & 0 & -1 \\ -2 & -2 & 0 \\ -8 & -8 & -8 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2.7298 & 0 & -1.3649 \\ -5.4596 & -5.4596 & 0 \\ 7.075 & 7.075 & 7.075 \\ 8 & 8 & 8 \end{bmatrix}$$

Projeção Perspectiva

$$\begin{bmatrix} 1.3649 & 0 & 0 & 0 \\ 0 & 2.7298 & 0 & 0 \\ 0 & 0 & -1.01 & -1.005 \\ 0 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 3 & -3 \\ -2 & 2 & 0 \\ -10 & -10 & -10 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -4.0947 & 4.0947 & -4.0947 \\ -5.4596 & 5.4596 & 0 \\ 9.095 & 9.095 & 9.095 \\ 10 & 10 & 10 \end{bmatrix}$$

Aplicando Divisão Homogênea

Projeção em NDC
(Normalized Device Coordinate)



$$\begin{bmatrix} -1.3649 \\ -2.7298 \\ 7.075 \\ 8 \end{bmatrix} \rightarrow \begin{bmatrix} -0.1706 \\ -0.3412 \\ 0.8844 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2.7298 & 0 & -1.3649 \\ -5.4596 & -5.4596 & 0 \\ 7.075 & 7.075 & 7.075 \\ 8 & 8 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} -0.3412 & 0 & -0.1706 \\ -0.6825 & -0.6825 & 0 \\ 0.8844 & 0.8844 & 0.8844 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -4.0947 & 4.0947 & -4.0947 \\ -5.4596 & 5.4596 & 0 \\ 9.095 & 9.095 & 9.095 \\ 10 & 10 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} -0.4095 & 0.4095 & -0.4095 \\ -0.5460 & 0.5460 & 0 \\ 0.9095 & 0.9095 & 0.9095 \\ 1 & 1 & 1 \end{bmatrix}$$

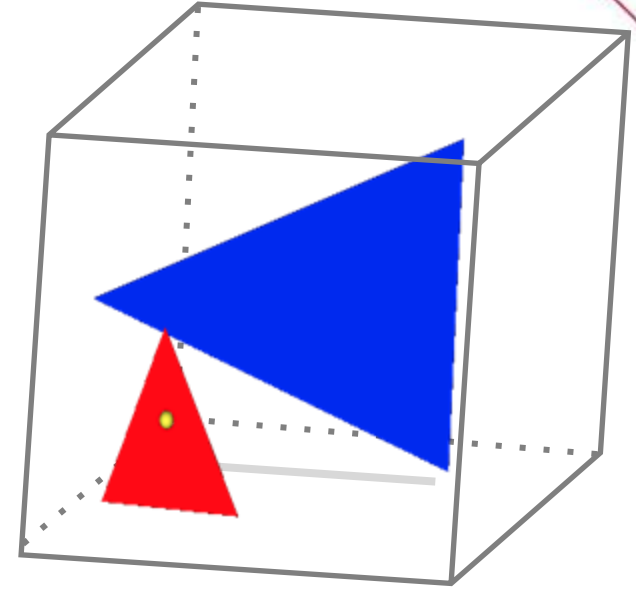
Aplicando Divisão Homogênea

Projeção em NDC
(Normalized Device Coordinate)

$$\begin{bmatrix} -1.3649 \\ -2.7298 \\ 7.075 \\ 8 \end{bmatrix} \rightarrow \begin{bmatrix} -0.1706 \\ -0.3412 \\ 0.8844 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2.7298 & 0 & -1.3649 \\ -5.4596 & -5.4596 & 0 \\ 7.075 & 7.075 & 7.075 \\ 8 & 8 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} -0.3412 & 0 & -0.1706 \\ -0.6825 & -0.6825 & 0 \\ 0.8844 & 0.8844 & 0.8844 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -4.0947 & 4.0947 & -4.0947 \\ -5.4596 & 5.4596 & 0 \\ 9.095 & 9.095 & 9.095 \\ 10 & 10 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} -0.4095 & 0.4095 & -0.4095 \\ -0.5460 & 0.5460 & 0 \\ 0.9095 & 0.9095 & 0.9095 \\ 1 & 1 & 1 \end{bmatrix}$$



Mapeando coordenadas para tela (screen)

$$\begin{array}{ccc}
 \text{Escala para (W/2, H/2)} & \text{Translate por (1,1)} & \text{Espelha em Y} \\
 \begin{bmatrix} \frac{W}{2} & 0 & 0 & 1 \\ 0 & \frac{H}{2} & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \cdot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{W}{2} & 0 & 0 & \frac{W}{2} \\ 0 & -\frac{H}{2} & 0 & \frac{H}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{array}$$

- Resolução = 40 x 20

$$\begin{array}{ccc}
 S_{40,20} & T_{1,1} & E_y \\
 \text{Coordenadas da Tela} = \begin{bmatrix} \frac{40}{2} & 0 & 0 & 0 \\ 0 & \frac{20}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 0 & 0 & 20 \\ 0 & -10 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{array}$$

Obs: Não há necessidade da matriz ser 4x4.

Coordenadas de Tela

<Scene>

<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>

<Transform>

<Shape>

<Sphere radius="0.1"/>

<Appearance>

<Material diffuseColor="1 1 0"/>

</Appearance>

</Shape>

</Transform>

<Transform>

<Shape>

<TriangleSet>

<Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>

</TriangleSet>

<Appearance>

<Material diffuseColor="1 0 0"/>

</Appearance>

</Shape>

</Transform>

<Transform scale="1 3 2"

rotation="1 0 0 -1.57"

translation="2 1 1">

<Shape>

<TriangleSet>

<Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>

</TriangleSet>

<Appearance>

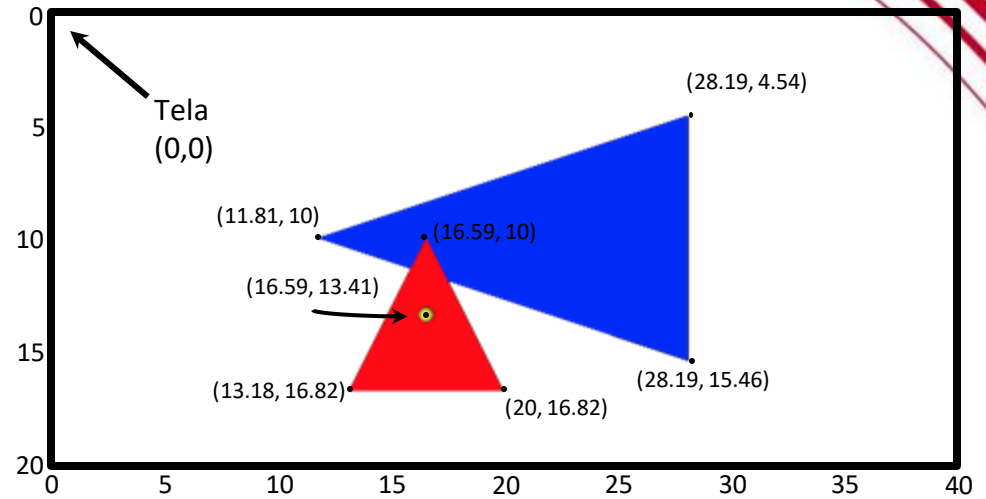
<Material diffuseColor="0 0 1"/>

</Appearance>

</Shape>

</Transform>

</Scene>



Tela

$$\begin{bmatrix} 20 & 0 & 0 & 20 \\ 0 & -10 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -0.1706 \\ -0.3412 \\ 0.8844 \\ 1 \end{bmatrix} = \begin{bmatrix} 16.59 \\ 13.41 \\ 0.88 \\ 1 \end{bmatrix}$$

Tela

$$\begin{bmatrix} 20 & 0 & 0 & 20 \\ 0 & -10 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -0.3412 & 0 & -0.1706 \\ -0.6825 & -0.6825 & 0 \\ 0.8844 & 0.8844 & 0.8844 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 13.18 & 20 & 16.59 \\ 16.82 & 16.82 & 10 \\ 0.88 & 0.88 & 0.88 \\ 1 & 1 & 1 \end{bmatrix}$$

Tela

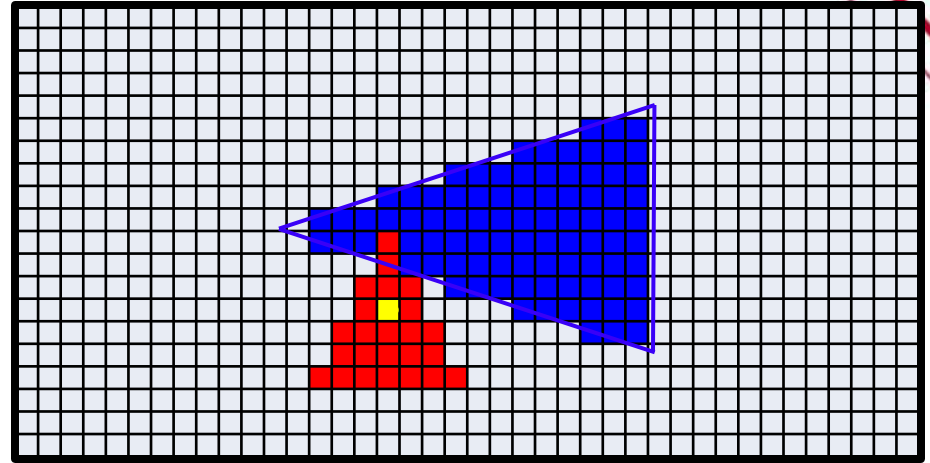
$$\begin{bmatrix} 20 & 0 & 0 & 20 \\ 0 & -10 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -0.4095 & 0.4095 & -0.4095 \\ -0.5460 & 0.5460 & 0 \\ 0.9095 & 0.9095 & 0.9095 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 28.19 & 28.19 & 11.81 \\ 15.46 & 4.54 & 10 \\ 0.91 & 0.91 & 0.91 \\ 1 & 1 & 1 \end{bmatrix}$$

Renderização

```

<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius="0.1"/>
      <Appearance>
        <Material diffuseColor="1 1 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <Triangle Set>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor="1 0 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="0 0 1"/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>

```



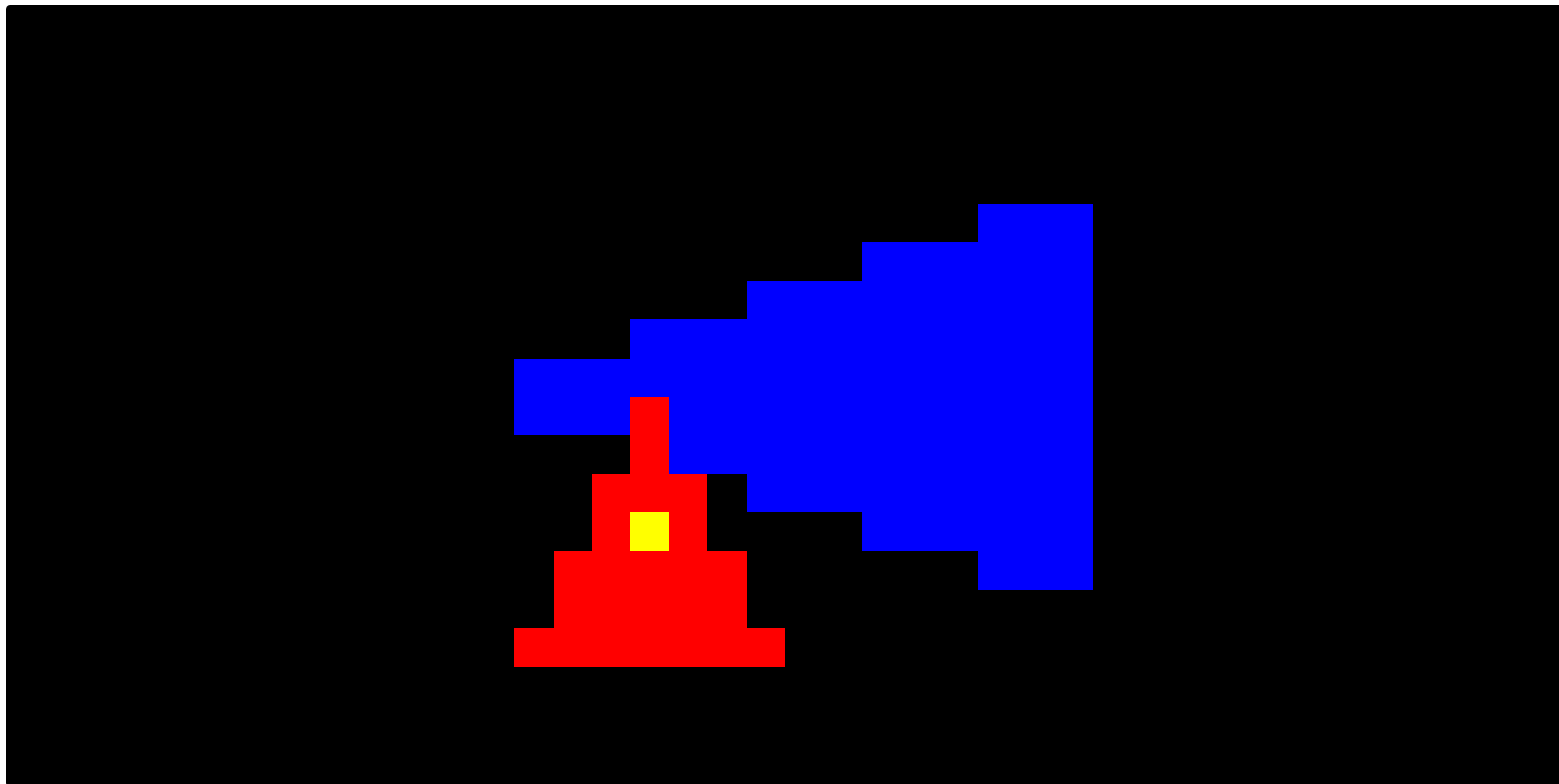
Rasterizar os Triângulos

Esfera : $\begin{bmatrix} 16.59 \\ 13.41 \end{bmatrix}$

Triângulo Vermelho : $\begin{bmatrix} 13.18 & 20 & 16.59 \\ 16.82 & 16.82 & 10 \end{bmatrix}$

Triângulo Azul : $\begin{bmatrix} 28.19 & 28.19 & 11.81 \\ 15.46 & 4.54 & 10 \end{bmatrix}$

Resultado Final



Resolução Final: 40 x 20 pixels

Revisão Numpy

- **numpy.array**: cria matrizes tipo numpy
- **numpy.matmul**: multiplica matrizes numpy
 - A partir do Python 3.5 você pode usar o arroba (@)
- ***** : multiplica todos os valores da matriz por um escalar
 - **numpy.multiply** se quiser fazer uma chamada de função
- **/** : divide todos os valores da matriz por um escalar
- **numpy.empty**: matriz iniciada com valores não iniciados
- **numpy.zeros**: matriz com todos os valores sendo zero
- **numpy.ones**: matriz com todos os valores sendo um
- **np.uint8, np.uint16, np.float32**: tipos de dados para numpy

CUIDADO

- **numpy.dot**: realiza o cálculo do produto escalar
(mas também algumas multiplicações de matrizes)
(mas é confuso e não funciona em todos os casos)

Resultados Esperados

No final dos arquivos X3D existe um comentários com os valores esperados das matrizes.

```
<!--
camera
[[-9.99999996e-01 0.00000000e+00 9.26535897e-05 4.63267948e-04]
 [ 0.00000000e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00]
 [-9.26535897e-05 0.00000000e+00 -9.99999996e-01 -4.99999998e+00]
 [ 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00]]

persp
[[ 1.37417368 0. 0. 0. ]
 [ 0. 2.06126053 0. 0. ]
 [ 0. 0. -1.00002 -0.0200002 ]
 [ 0. 0. -1. 0. ]]

screen
[[ 150. 0. 0. 150.]
 [ 0. -100. 0. 100.]
 [ 0. 0. 1. 0.]
 [ 0. 0. 0. 1.]]

view
[[-2.06112154e+02 0.00000000e+00 1.50019098e+02 7.50095488e+02]
 [ 9.26535897e-03 -2.06126053e+02 9.99999996e+01 4.99999998e+02]
 [ 9.26554428e-05 0.00000000e+00 1.00002000e+00 4.98009978e+00]
 [ 9.26535897e-05 0.00000000e+00 9.99999996e-01 4.99999998e+00]]
model (transform)
[[ 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]
 [ 0.00000000e+00 -9.99999996e-01 9.26535897e-05 0.00000000e+00]
 [ 0.00000000e+00 -9.26535897e-05 -9.99999996e-01 5.00000000e+00]
 [ 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00]]
-->
```

Computação Gráfica

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