

Computação Gráfica

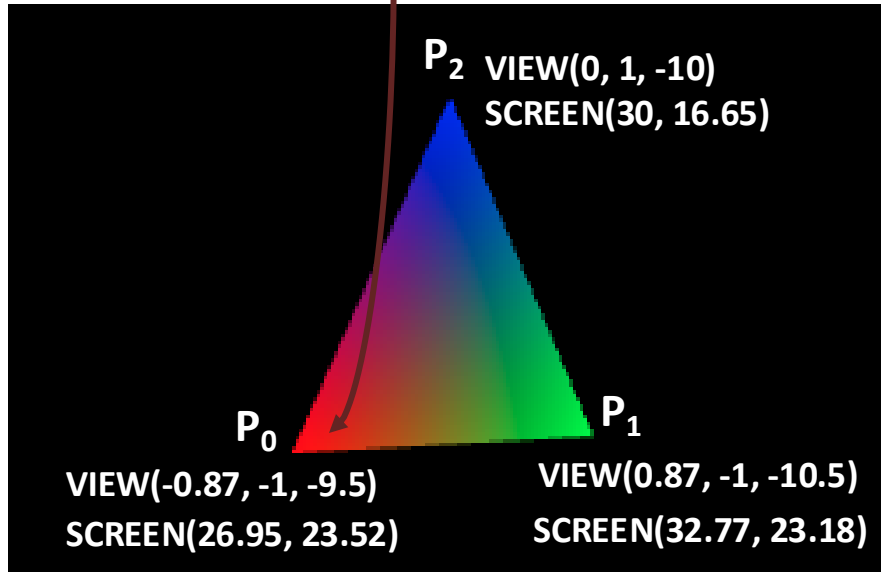
Aula 12: Revisão 3 Interpolação, MipMap, Visibilidade

Interpolação em Triângulos



Triângulo com vértices de cores diferentes

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52"> ← 30°
    <Shape>
      <IndexedFaceSet colorPerVertex='true' coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0' />
        <Color color='1 0 0 0 1 0 0 0 1' />
      </IndexedFaceSet>
    </Shape>
  </Transform>
</Scene>
```



Distâncias do ponto(28, 22) as arestas

$$L(x, y) = (x - x_0)(y_1 - y_0) - (y - y_0)(x_1 - x_0)$$

$$L_0 = (28.5 - 32.77)(16.65 - 23.18) - (22.5 - 23.18)(30 - 32.77)$$

$$L_0 = (-4.27)(-6.53) - (-0.68)(-2.77)$$

$$L_0 = 27.8831 - 1.8836$$

$$L_0 = 25.9995$$

$$L_1 = (28.5 - 30)(23.52 - 16.65) - (22.5 - 16.65)(26.95 - 30)$$

$$L_1 = (-1.5)(6.87) - (5.85)(-3.05)$$

$$L_1 = -10.305 + 17.8425$$

$$L_1 = 7.5375$$

$$L_2 = (28.5 - 26.95)(23.18 - 23.52) - (22.5 - 23.52)(32.77 - 26.95)$$

$$L_2 = (1.55)(-0.34) - (-1.02)(5.82)$$

$$L_2 = -0.527 + 5.9364$$

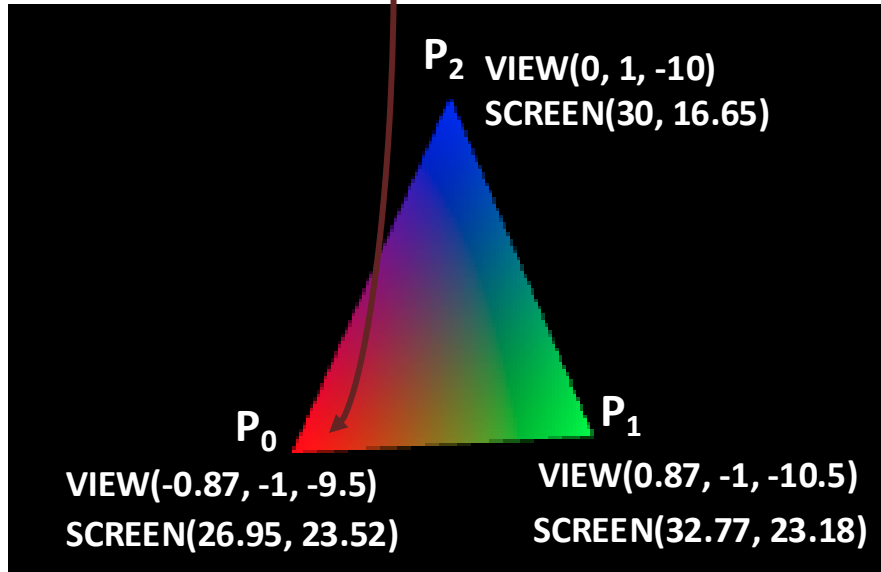
$$L_2 = 5.4094$$

Todos positivos: Dentro !!!

Triângulo com vértices de cores diferentes

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52"> ← 30°
    <Shape>
      <IndexedFaceSet colorPerVertex='true' coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0' />
        <Color color='1 0 0 0 1 0 0 0 1' />
      </IndexedFaceSet>
    </Shape>
  </Transform>
</Scene>
```

Pixel (28, 22)



Área dos triângulos:

$$Area = |x_0(y_1 - y_2) + x_1(y_2 - y_0) + x_2(y_0 - y_1)| / 2$$

$$A = |26.95(23.18 - 16.65) + 32.77(16.65 - 23.52) + 30(23.52 - 23.18)| / 2$$

$$A = |26.95(6.53) + 32.77(-6.87) + 30(0.34)| / 2$$

$$A = |175.9835 - 225.1299 + 10.2| / 2$$

$$A = 19.4732$$

$$A_0 = |28.5(23.18 - 16.65) + 32.77(16.65 - 22.5) + 30(22.5 - 23.18)| / 2$$

$$A_0 = |28.5(6.53) + 32.77(-5.85) + 30(-0.68)| / 2$$

$$A_0 = |186.105 - 191.7045 - 20.4| / 2$$

$$A_0 = 12.99975$$

$$A_1 = |28.5(16.65 - 23.52) + 30(23.52 - 22.5) + 26.95(22.5 - 16.65)| / 2$$

$$A_1 = |28.5(-6.87) + 30(1.02) + 26.95(5.85)| / 2$$

$$A_1 = |-195.795 + 30.6 + 157.6575| / 2$$

$$A_1 = 3.76875$$

$$A_2 = |28.5(23.52 - 23.18) + 26.95(23.18 - 22.5) + 32.77(22.5 - 23.52)| / 2$$

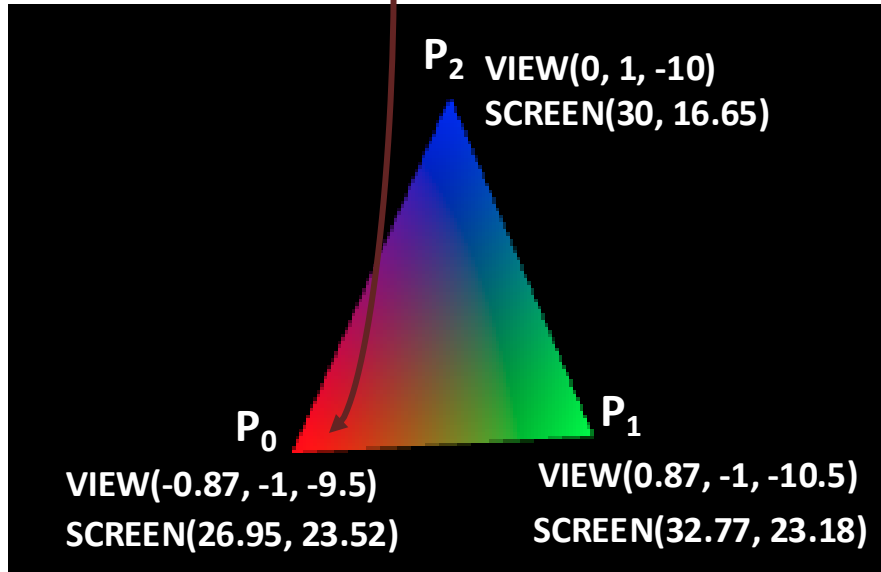
$$A_2 = |28.5(0.34) + 26.95(0.68) + 32.77(-1.02)| / 2$$

$$A_2 = |9.69 + 18.326 - 33.4254| / 2$$

$$A_2 = 2.7047$$

Triângulo com vértices de cores diferentes

```
<Scene>
<Viewpoint position="0 0 10"/>
<Transform rotation="0 1 0 0.52"> ← 30°
  <Shape>
    <IndexedFaceSet colorPerVertex='true' coordIndex='0 1 2 -1'>
      <Coordinate point='-1 -1 0 1 -1 0 0 1 0'>
        <Color color='1 0 0 0 1 0 0 0 1'>
          </IndexedFaceSet>
        </Shape>
      </Transform>
    </Scene>
```



Área dos triângulos:

$$A = 19.4732$$

$$A_0 = 12.99975$$

$$A_1 = 3.76875$$

$$A_2 = 2.7047$$

Pesos

$$\alpha = 12.99975 / 19.4732 \approx 0.668$$

$$\beta = 3.76875 / 19.4732 \approx 0.194$$

$$\gamma = 2.7047 / 19.4732 \approx 0.139$$

ou

$$\gamma = 1 - \alpha - \beta \approx 0.138$$

Cor do pixel:

$$R = \alpha R_0 + \beta R_1 + \gamma R_2 = 0.668$$

$$G = \alpha G_0 + \beta G_1 + \gamma G_2 = 0.194$$

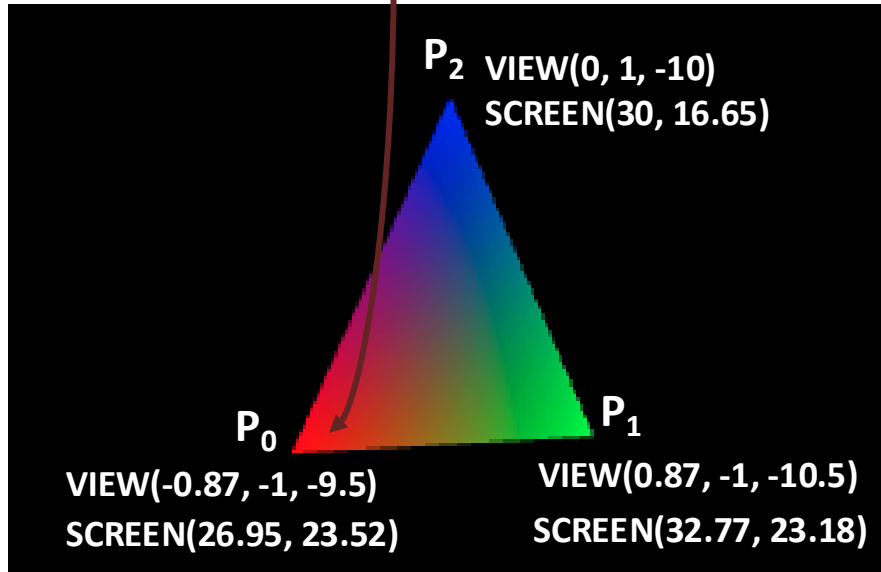
$$B = \alpha B_0 + \beta B_1 + \gamma B_2 = 0.139$$

$$C = (0.668, 0.194, 0.139)$$



Cores do Triângulo com Correção Perspectiva

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52"> ← 30°
    <Shape>
      <IndexedFaceSet colorPerVertex='true' coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0'>
          <Color color='1 0 0 0 1 0 0 0 1'>
            </IndexedFaceSet>
          </Shape>
        </Transform>
      </Scene>
```



Pesos

$$\alpha = 0.668$$

$$\beta = 0.194$$

$$\gamma = 0.139$$

$$Z_0 = |-9.5| = 9.5$$

$$Z_1 = |-10.5| = 10.5$$

$$Z_2 = |-10.0| = 10$$

$$Z = \frac{1}{\alpha \frac{1}{Z_0} + \beta \frac{1}{Z_1} + \gamma \frac{1}{Z_2}}$$

$$Z = \frac{1}{0.668 \frac{1}{9.5} + 0.194 \frac{1}{10.5} + 0.139 \frac{1}{10}}$$

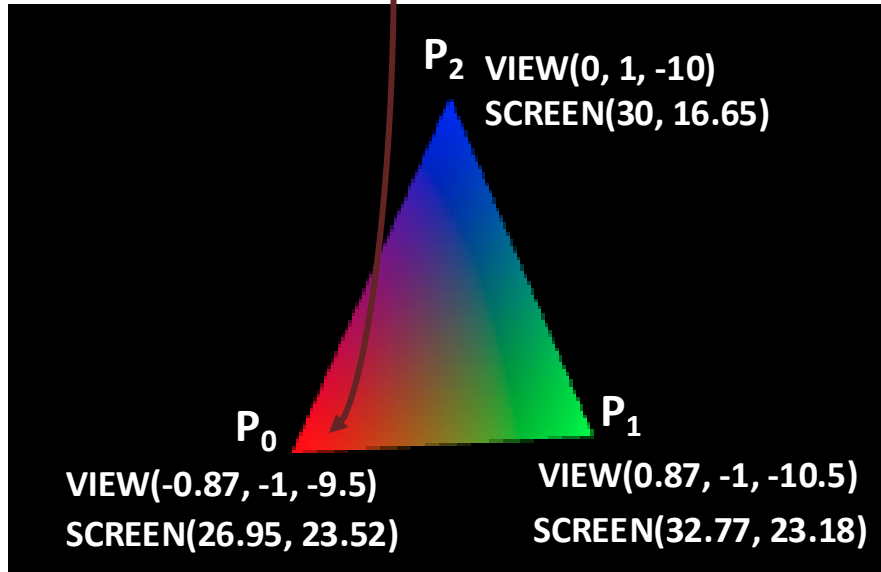
$$Z = \frac{1}{0.0703 + 0.0185 + 0.0139}$$

$$Z = 9.74$$

Cores do Triângulo com Correção Perspectiva

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52"> ← 30°
    <Shape>
      <IndexedFaceSet colorPerVertex='true' coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0' />
        <Color color='1 0 0 0 1 0 0 0 1' />
      </IndexedFaceSet>
    </Shape>
  </Transform>
</Scene>
```

Pixel (28, 22)



Pesos
 $\alpha = 0.668$
 $\beta = 0.194$
 $\gamma = 0.139$

$Z_0 = |-9.5| = 9.5$
 $Z_1 = |-10.5| = 10.5$
 $Z_2 = |-10.0| = 10$

$Z = 9.74$

$$C = Z \cdot \left(\alpha \frac{C_0}{Z_0} + \beta \frac{C_1}{Z_1} + \gamma \frac{C_2}{Z_2} \right)$$

$$C_R = 9.74 \left(0.668 \frac{1}{9.5} + 0.194 \frac{0}{10.5} + 0.139 \frac{0}{10} \right) = 0.685$$

$$C_G = 9.74 \left(0.668 \frac{0}{9.5} + 0.194 \frac{1}{10.5} + 0.139 \frac{0}{10} \right) = 0.180$$

$$C_B = 9.74 \left(0.668 \frac{0}{9.5} + 0.194 \frac{0}{10.5} + 0.139 \frac{1}{10} \right) = 0.135$$

$C = (0.685, 0.180, 0.135)$

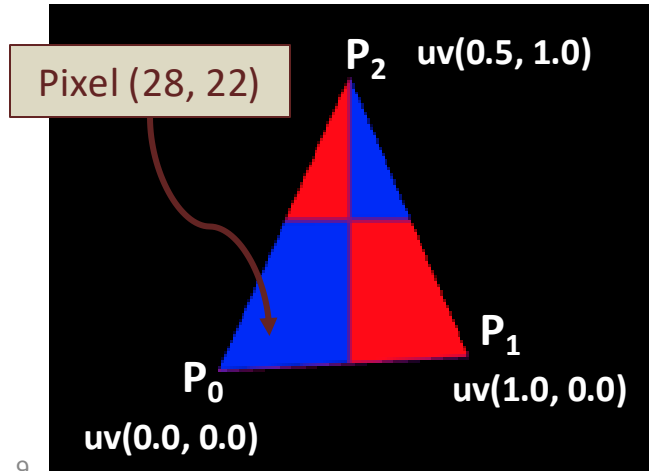


Aplicando Texturas



Triângulo com Texturas

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52">
    <Shape>
      <IndexedFaceSet coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0'>
          <TextureCoordinate point='0.0 0.0 1.0 0.0 0.5 1.0'>
            </IndexedFaceSet>
          <Appearance>
            <ImageTexture url=' "chess.png" '/>
          </Appearance>
        </Shape>
      </Transform>
    </Scene>
```



Pesos

$$\alpha = 0.668$$

$$\beta = 0.194$$

$$\gamma = 0.139$$

$$U = \alpha U_0 + \beta U_1 + \gamma U_2$$

$$U = 0.668 * 0.0 + 0.194 * 1.0 + 0.139 * 0.5$$

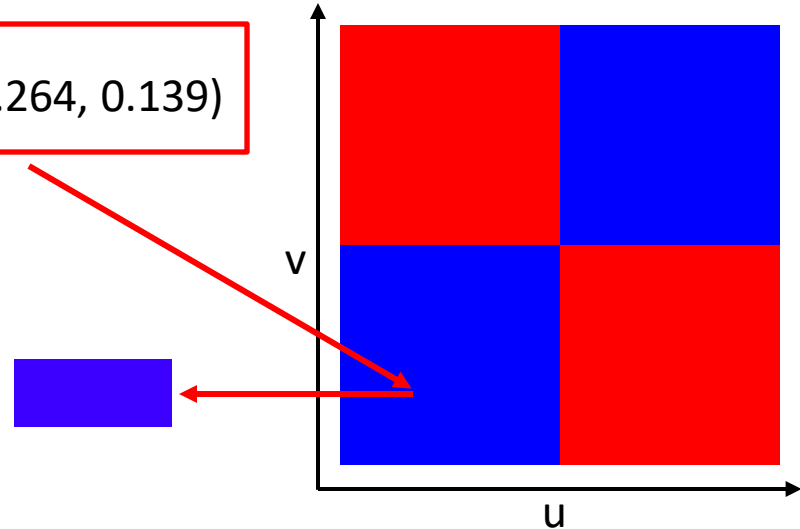
$$U = 0.2635$$

$$V = \alpha V_0 + \beta V_1 + \gamma V_2$$

$$V = 0.668 * 0.0 + 0.194 * 0.0 + 0.139 * 1.0$$

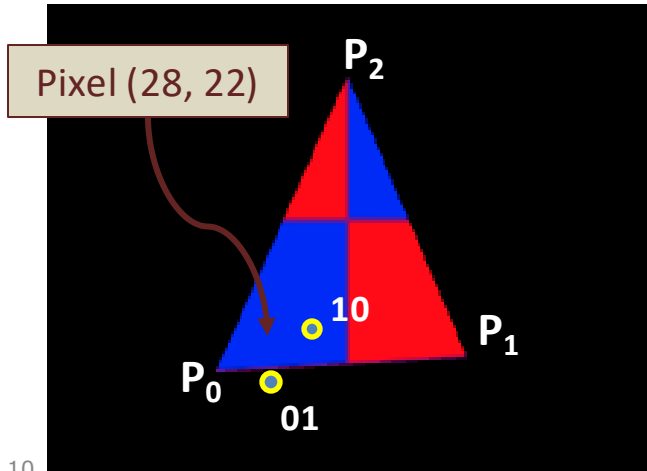
$$V = 0.139$$

$$UV = (0.264, 0.139)$$



Triângulo com Texturas MipMap

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52">
    <Shape>
      <IndexedFaceSet coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0'>
          <TextureCoordinate point='0.0 0.0 1.0 0.0 0.5 1.0'>
            </IndexedFaceSet>
          <Appearance>
            <ImageTexture url=' "chess.png" '/>
          </Appearance>
        </Shape>
      </Transform>
    </Scene>
```



UV = (0.264, 0.139)

Coordenadas (u,v)

pixel₁₀(29, 22) => UV(0.414, 0.129)

pixel₀₁(28, 23) => UV(0.248, -0.010)

$$\frac{\partial u}{\partial x} = \frac{u_{10} - u_{00}}{1} \quad \frac{\partial v}{\partial x} = \frac{v_{10} - v_{00}}{1}$$

$$\frac{\partial u}{\partial y} = \frac{u_{01} - u_{00}}{1} \quad \frac{\partial v}{\partial y} = \frac{v_{01} - v_{00}}{1}$$

Triângulo com Texturas MipMap

Coordenadas (u,v)
U = 0.264
V = 0.139
pixel₁₀(29, 22) => UV(0.414, 0.129)
pixel₀₁(28, 23) => UV(0.248, -0.010)

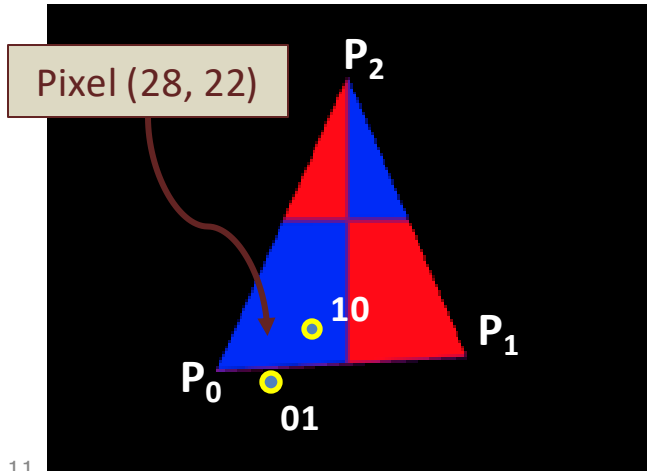
x 32

$$\frac{\partial u}{\partial x} = \frac{u_{10} - u_{00}}{1} = 32 \left(\frac{0.414 - 0.264}{1} \right) = 4.8$$

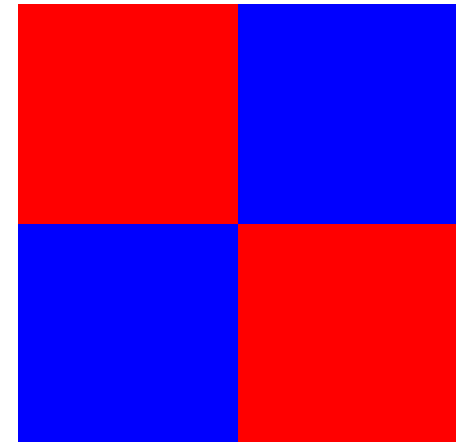
$$\frac{\partial v}{\partial x} = \frac{v_{10} - v_{00}}{1} = 32 \left(\frac{0.129 - 0.139}{1} \right) = -0.32$$

$$\frac{\partial u}{\partial y} = \frac{u_{01} - u_{00}}{1} = 32 \left(\frac{0.248 - 0.264}{1} \right) = -0.56$$

$$\frac{\partial v}{\partial y} = \frac{v_{01} - v_{00}}{1} = 32 \left(\frac{-0.010 - 0.139}{1} \right) = -4.8$$



chess.png (32x32)



Triângulo com Texturas MipMap

$$U = 0.264$$

$$V = 0.139$$

Coordenadas (u,v)

$$\text{pixel}_{10}(29, 22) \Rightarrow UV(0.414, 0.129)$$

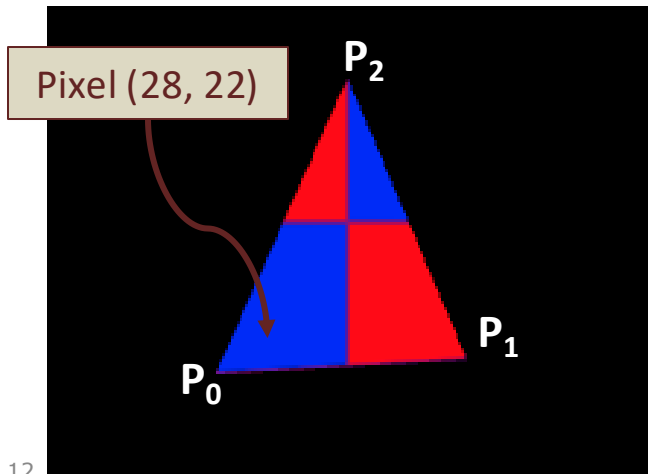
$$\text{pixel}_{01}(28, 23) \Rightarrow UV(0.248, -0.010)$$

$$\frac{du}{dx} = 4.8$$

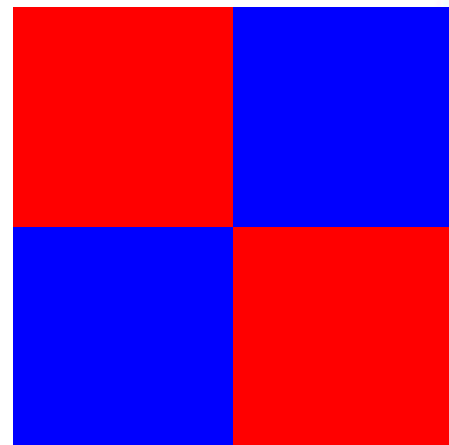
$$\frac{dv}{dx} = -0.32$$

$$\frac{du}{dy} = -0.56$$

$$\frac{dv}{dy} = -4.8$$



chess.png (32x32)



0

$$D = \log_2 L$$

$$L = \max \left(\sqrt{\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2}, \sqrt{\left(\frac{du}{dy}\right)^2 + \left(\frac{dv}{dy}\right)^2} \right)$$

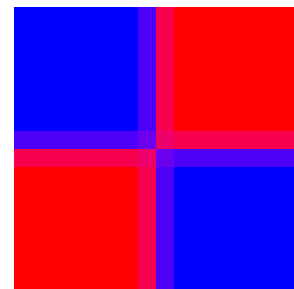
$$L = \max \left(\sqrt{(4.8)^2 + (-0.32)^2}, \sqrt{(-0.56)^2 + (-4.8)^2} \right)$$

$$L = \max(4.81, 4.83)$$

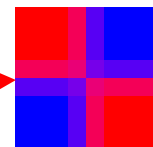
$$L = 4.83$$

$$D = 2.27$$

$$D = 2$$



1 (16x16)



2 (8x8)



3 (4x4)

Insper

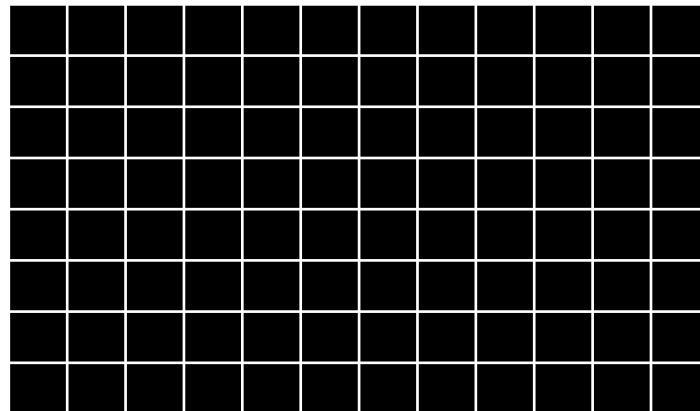
Z-Buffer



Desenhando com Z-Buffer

```
<Scene>
  <Transform translation="1 0 0">
    <Shape>
      <TriangleSet>
        <Coordinate point='-2 -2 0 2 -2 0 0 2 0'>
        </TriangleSet>
      <Appearance>
        <Material emissiveColor='1 0 0'>
        </Appearance>
      </Shape>
    </Transform>
    <Transform translation="-1 0 0">
      <Shape>
        <TriangleSet>
          <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'>
          </TriangleSet>
        <Appearance>
          <Material emissiveColor='0 0 1'>
          </Appearance>
        </Shape>
      </Transform>
    </Scene>
```

Cores



Profundidade

1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1

Desenhando com Z-Buffer

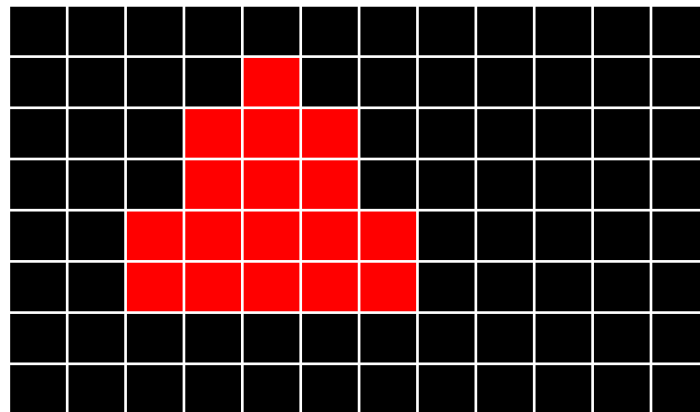
```

<Scene>
  <Transform translation="1 0 0">
    <Shape>
      <TriangleSet>
        <Coordinate point='-2 -2 0 2 -2 0 0 2 0'>
        </TriangleSet>
      <Appearance>
        <Material emissiveColor='1 0 0'>
        </Material>
      </Appearance>
    </Shape>
  </Transform>
  <Transform translation="-1 0 0">
    <Shape>
      <TriangleSet>
        <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'>
        </TriangleSet>
      <Appearance>
        <Material emissiveColor='0 0 1'>
        </Material>
      </Appearance>
    </Shape>
  </Transform>
</Scene>

```



Cores



Profundidade

1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0.8	1	1	1	1	1	1	1
1	1	1	0.8	0.8	0.8	1	1	1	1	1	1
1	1	1	0.8	0.8	0.8	1	1	1	1	1	1
1	1	0.8	0.8	0.8	0.8	0.8	1	1	1	1	1
1	1	0.8	0.8	0.8	0.8	0.8	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1

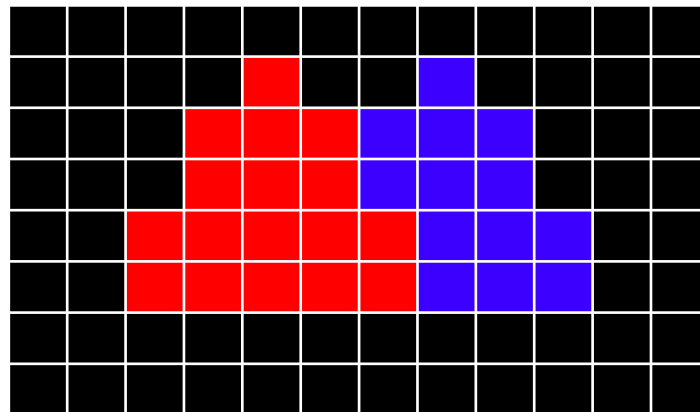
Desenhando com Z-Buffer

```

<Scene>
  <Transform translation="1 0 0">
    <Shape>
      <TriangleSet>
        <Coordinate point='-2 -2 0 2 -2 0 0 2 0'>
        </TriangleSet>
      <Appearance>
        <Material emissiveColor='1 0 0'>
        </Material>
      </Appearance>
    </Shape>
  </Transform>
  <Transform translation="-1 0 0">
    <Shape>
      <TriangleSet>
        <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'>
        </TriangleSet>
      <Appearance>
        <Material emissiveColor='0 0 1'>
        </Material>
      </Appearance>
    </Shape>
  </Transform>
</Scene>

```

Cores



Profundidade

1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0.8	1	1	0.9	1	1	1	1
1	1	1	0.8	0.8	0.8	0.9	0.9	0.9	1	1	1
1	1	1	0.8	0.8	0.8	0.9	0.9	0.9	1	1	1
1	1	0.8	0.8	0.8	0.8	0.8	0.9	0.9	0.9	1	1
1	1	0.8	0.8	0.8	0.8	0.8	0.9	0.9	0.9	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1

Transparência



Transparência

```
<Scene>
  <Transform translation="-1 0 0">
    <Shape>
      <TriangleSet>
        <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
      </TriangleSet>
      <Appearance>
        <Material emissiveColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform translation="1 0 0">
    <Shape>
      <TriangleSet>
        <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
      </TriangleSet>
      <Appearance>
        <Material emissiveColor='1 1 0'
          transparency='0.2'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```

Cores

(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

$\text{cor_anterior} = \text{framebuffer}[x,y] * \text{transparência}$

$\text{cor_nova} = \text{rbg} * (1 - \text{transparência})$

$\text{framebuffer}[x,y] = \text{cor_anterior} + \text{cor_nova}$

Transparência

```
<Scene>
  <Transform translation="-1 0 0">
    <Shape>
      <TriangleSet>
        <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
      </TriangleSet>
      <Appearance>
        <Material emissiveColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform translation="1 0 0">
    <Shape>
      <TriangleSet>
        <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
      </TriangleSet>
      <Appearance>
        <Material emissiveColor='0 1 0'
          transparency='0.4'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```

Cores

(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,6,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,6,0)	(0,0,6,0)	(0,0,6,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,6,0)	(0,0,6,0)	(0,0,6,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,6,0)	(0,0,6,0)	(0,0,6,0)	(0,0,6,0,4)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,6,0)	(0,0,6,0)	(0,0,6,0)	(0,0,6,0,4)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

$\text{cor_anterior} = \text{framebuffer}[x,y] * \text{transparência}$

$\text{cor_nova} = \text{rbg} * (1 - \text{transparência})$

$\text{framebuffer}[x,y] = \text{cor_anterior} + \text{cor_nova}$

Computação Gráfica

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