Insper

Computação Gráfica

Aula 12: Revisão 2 Interpolaçãoo, MipMap, Visibilidade

Interpolação em Triângulos

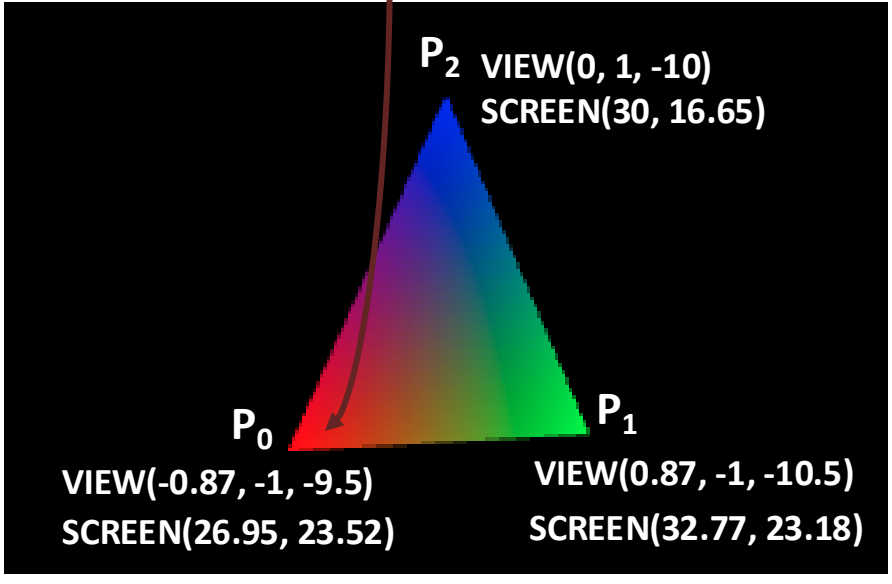
Triângulo com vértices de cores diferentes

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52">
    <Shape>
      <IndexedFaceSet colorPerVertex='true' coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0'/>
        <Color color='1 0 0 0 1 0 0 0 1'/>
      </IndexedFaceSet>
    </Shape>
  </Transform>
</Scene>
```

30°

É triângulo?
Qual a cor?

Pixel (28, 22)



Distâncias do ponto(28, 22) as arestas

L(x, y) = (x – x0)(y1 – y0) – (y – y0)(x1 – x0)

centro do pixel

L_0 = (28.5 - 32.77)(16.65 - 23.18) - (22.5 - 23.18)(30 - 32.77)
L_0 = (-4.27)(-6.53) - (-0.68)(-2.77)
L_0 = 27.8831 - 1.8836
L_0 = 25.9995

L_1 = (28.5 - 30)(23.52 - 16.65) - (22.5 – 16.65)(26.95 - 30)
L_1 = (-1.5)(6.87) - (5.85)(-3.05)
L_1 = -10.305 + 17.8425
L_1 = 7.5375

L_2 = (28.5 - 26.95)(23.18 - 23.52) - (22.5 - 23.52)(32.77 - 26.95)
L_2 = (1.55)(-0.34) - (-1.02)(5.82)
L_2 = -0.527 + 5.9364
L_2 = 5.4094

Todos positivos: Dentro !!!

Insper

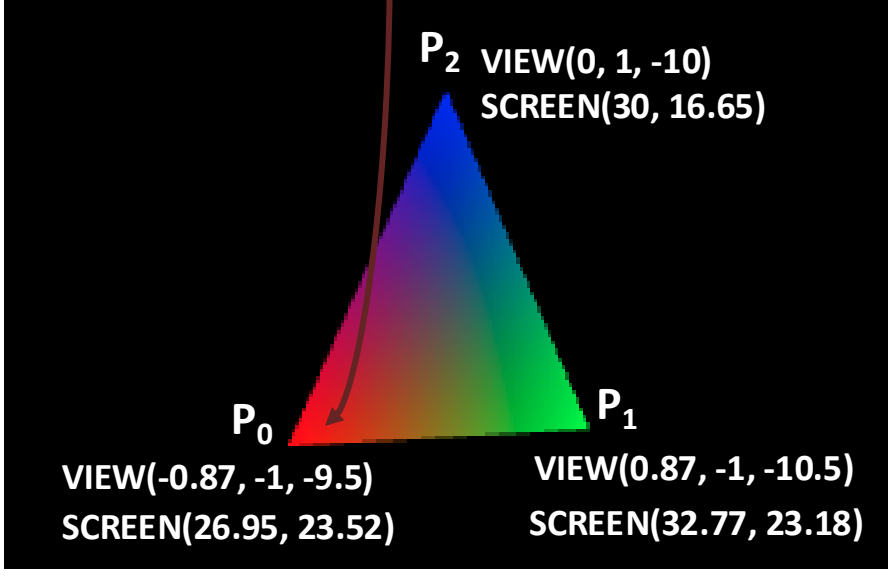
Triângulo com vértices de cores diferentes

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52">
    <Shape>
      <IndexedFaceSet colorPerVertex='true' coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0'/>
        <Color color='1 0 0 0 1 0 0 0 1'/>
      </IndexedFaceSet>
    </Shape>
  </Transform>
</Scene>
```

30°

É triângulo?
Qual a cor?

Pixel (28, 22)



Área dos triângulos:

Area = $|x_0(y_1–y_2) + x_1(y_2–y_0) + x_2(y_0–y_1)| / 2$

A = |26.95(23.18-16.65)+32.77(16.65-23.52)+30(23.52-23.18)|/2
A = |26.95(6.53)+32.77(-6.87)+30(0.34)|/2
A = |175.9835-225.1299+10.2|/2

A = 19.4732

A_0 = |28.5(23.18-16.65)+32.77(16.65-22.5)+30(22.5-23.18)|/2
A_0 = |28.5(6.53)+32.77(-5.85)+30(-0.68)|/2
A_0 = |186.105-191.7045-20.4|/2

A_0 = 12.99975

A_1 = |28.5(16.65-23.52)+30(23.52-22.5)+26.95(22.5-16.65)|/2
A_1 = |28.5(-6.87)+30(1.02)+26.95(5.85)|/2
A_1 = |-195.795+30.6+157.6575|/2

A_1 = 3.76875

A_2 = |28.5(23.52-23.18)+26.95(23.18-22.5)+32.77(22.5-23.52)|/2
A_2 = |28.5(0.34)+26.95(0.68)+32.77(-1.02)|/2
A_2 = |9.69+18.326-33.4254|/2

A_2 = 2.7047

Insper

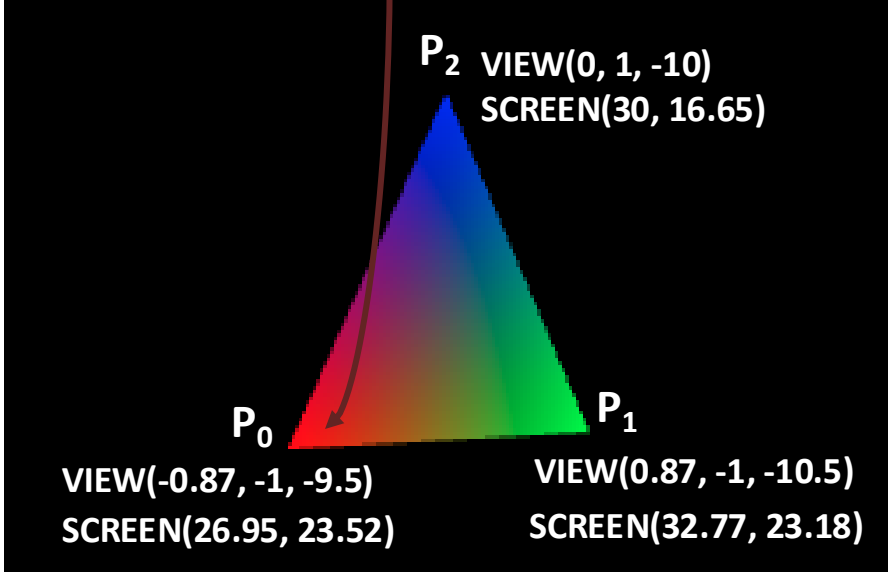
Triângulo com vértices de cores diferentes

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52">
    <Shape>
      <IndexedFaceSet colorPerVertex='true' coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0'/>
        <Color color='1 0 0 0 1 0 0 0 1'/>
      </IndexedFaceSet>
    </Shape>
  </Transform>
</Scene>
```

30°

É triângulo?
Qual a cor?

Pixel (28, 22)



Área dos triângulos:

A = 19.4732

A_0 = 12.99975

A_1 = 3.76875

A_2 = 2.7047

Pesos

α = 12.99975 / 19.4732 ~= 0.668

β = 3.76875 / 19. 4732 ~= 0.194

γ = 2.7047 / 19. 4732 ~= 0.139

ou

γ = 1 – α - β ~= 0.138

Cor do pixel:

$R = \alpha R_0 + \beta R_1 + \gamma R_2 = 0.668$

$G = \alpha G_0 + \beta G_1 + \gamma G_2 = 0.194$

$B = \alpha B_0 + \beta B_1 + \gamma B_2 = 0.139$

C = (0.668, 0.194, 0.139)

Percebeu que só usou coordenadas 2D da tela?

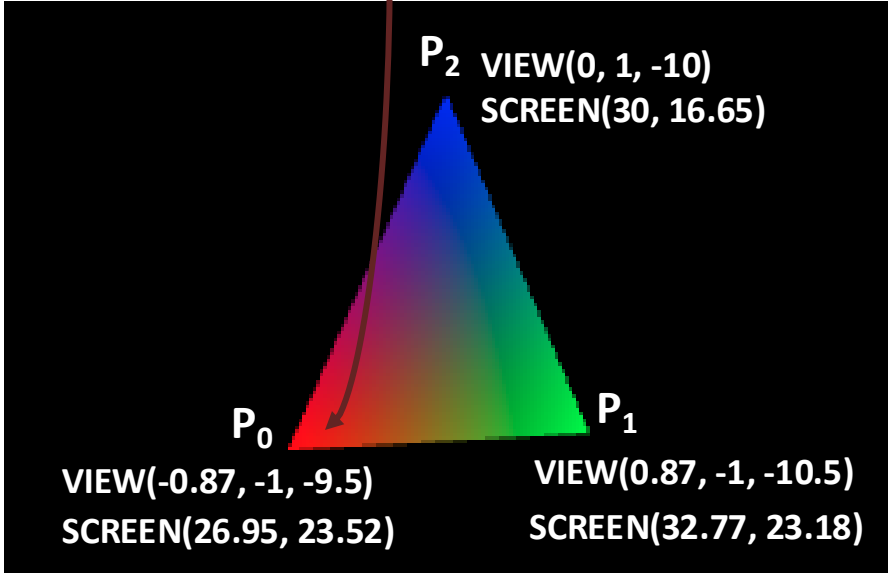
Cores do Triângulo com Correção Perspectiva

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52">
    <Shape>
      <IndexedFaceSet colorPerVertex='true' coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0'/>
        <Color color='1 0 0 0 1 0 0 0 1'/>
      </IndexedFaceSet>
    </Shape>
  </Transform>
</Scene>
```

30°

É triângulo?
Qual a cor?

Pixel (28, 22)



Pesos

$\alpha = 0.668$ $\quad Z_0 = |-9.5| = 9.5$

$\beta = 0.194$ $\quad Z_1 = |-10.5| = 10.5$

$\gamma = 0.139$ $\quad Z_2 = |-10.0| = 10$

$$Z = \frac{1}{\alpha\frac{1}{Z_0} + \beta\frac{1}{Z_1} + \gamma\frac{1}{Z_2}}$$

$$Z = \frac{1}{0.668\frac{1}{9.5} + 0.194\frac{1}{10.5} + 0.139\frac{1}{10}}$$

$$Z = \frac{1}{0.0703 + 0.0185 + 0.0139}$$

$$Z = 9.74$$

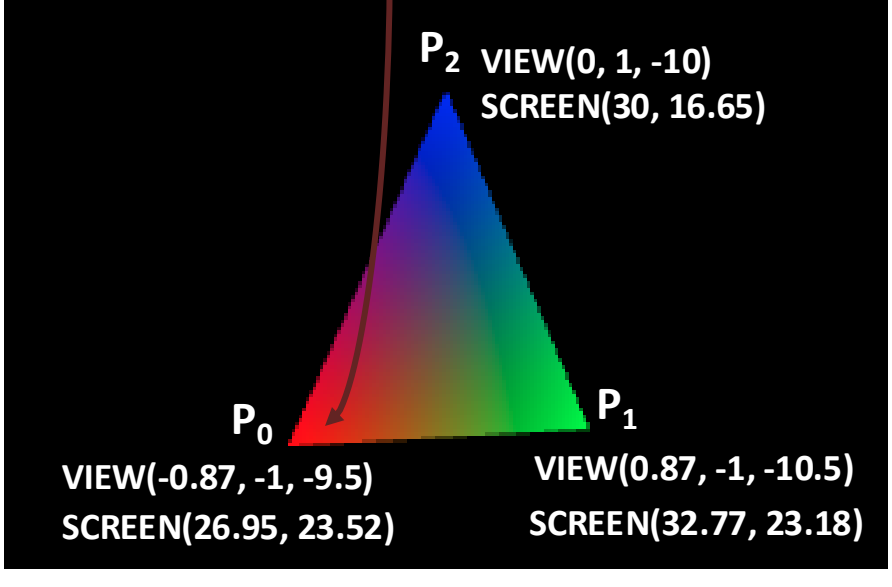
Cores do Triângulo com Correção Perspectiva

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52">
    <Shape>
      <IndexedFaceSet colorPerVertex='true' coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0'/>
        <Color color='1 0 0 0 1 0 0 0 1'/>
      </IndexedFaceSet>
    </Shape>
  </Transform>
</Scene>
```

30°

É triângulo?
Qual a cor?

Pixel (28, 22)



Pesos
α = 0.668
β = 0.194
γ = 0.139

$Z_0 = |-9.5| = 9.5$
$Z_1 = |-10.5| = 10.5$
$Z_2 = |-10.0| = 10$

$Z = 9.74$

$$C = Z \cdot \left(\alpha \frac{C_0}{Z_0} + \beta \frac{C_1}{Z_1} + \gamma \frac{C_2}{Z_2} \right)$$

$$C_R = 9.74 \left(0.668 \frac{1}{9.5} + 0.194 \frac{0}{10.5} + 0.139 \frac{0}{10} \right) = 0.685$$

$$C_G = 9.74 \left(0.668 \frac{0}{9.5} + 0.194 \frac{1}{10.5} + 0.139 \frac{0}{10} \right) = 0.180$$

$$C_B = 9.74 \left(0.668 \frac{0}{9.5} + 0.194 \frac{0}{10.5} + 0.139 \frac{1}{10} \right) = 0.135$$

C = (0.685, 0.180, 0.135)

Antes C = (0.668, 0.194, 0.139)

Insper

Aplicando Texturas

Triângulo com Texturas

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52">
    <Shape>
      <IndexedFaceSet coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0'/>
          <TextureCoordinate point='0.0 0.0 1.0 0.0 0.5 1.0'/>
      </IndexedFaceSet>
      <Appearance>
        <ImageTexture url=' "chess.png" '/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```

Pesos

$\alpha = 0.668$

$\beta = 0.194$

$\gamma = 0.139$

$U = \alpha U_0 + \beta U_1 + \gamma U_2$

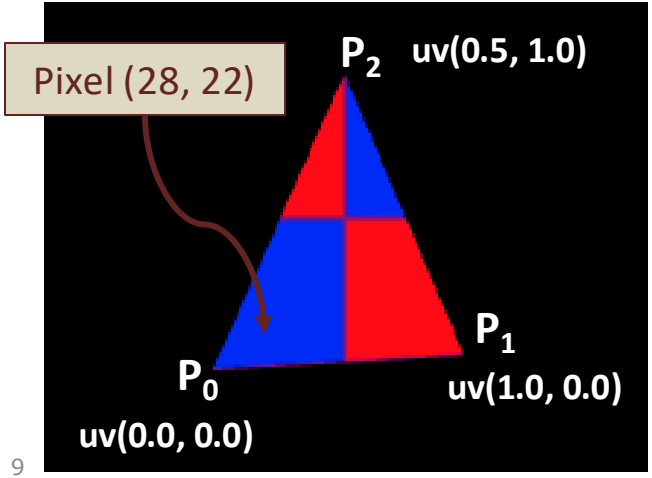
$U = 0.668*0.0 + 0.194*1.0 + 0.139*0.5$

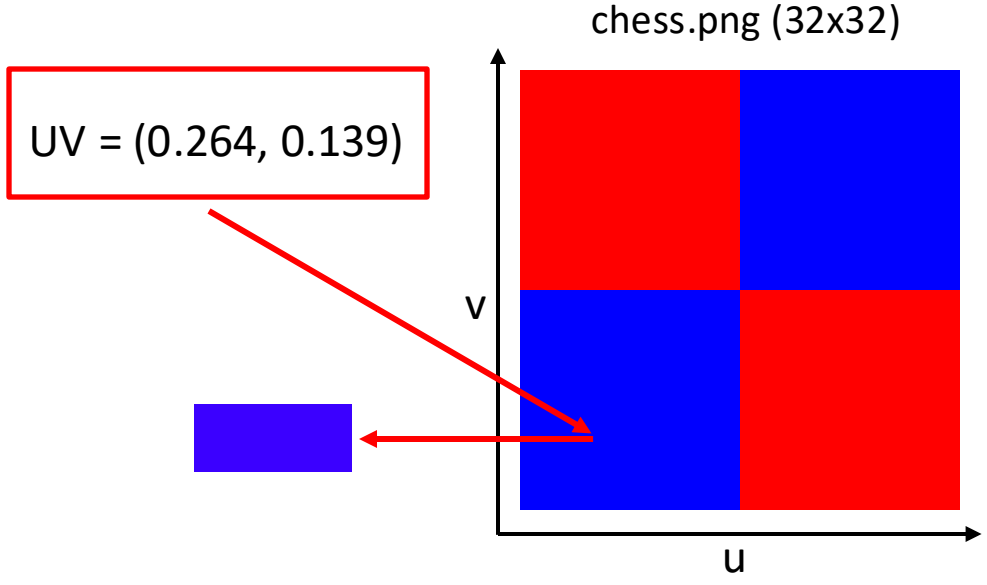
$U = 0.2635$

$V = \alpha V_0 + \beta V_1 + \gamma V_2$

$V = 0.668*0.0 + 0.194*0.0 + 0.139*1.0$

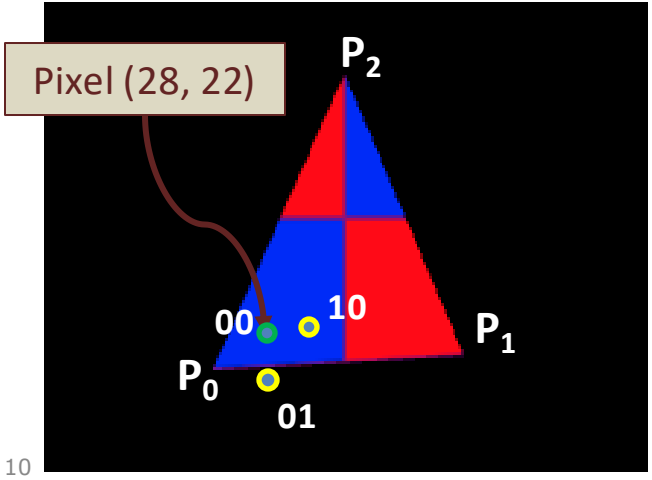
$V = 0.139$





Triângulo com Texturas MipMap

```
<Scene>
  <Viewpoint position="0 0 10"/>
  <Transform rotation="0 1 0 0.52">
    <Shape>
      <IndexedFaceSet coordIndex='0 1 2 -1'>
        <Coordinate point='-1 -1 0 1 -1 0 0 1 0'/>
          <TextureCoordinate point='0.0 0.0 1.0 0.0 0.5 1.0'/>
      </IndexedFaceSet>
      <Appearance>
        <ImageTexture url=' "chess.png" '/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



$pixel_{00}(28, 22) \Rightarrow UV(0.264, 0.139)$

Coordenadas (u,v) Vizinhas:

$pixel_{10}(29, 22) \Rightarrow UV(0.414, 0.129)$

$pixel_{01}(28, 23) \Rightarrow UV(0.248, -0.010)$

$$\frac{\partial u}{\partial x} = \frac{u_{10} - u_{00}}{29 - 28} \qquad \frac{\partial v}{\partial x} = \frac{v_{10} - v_{00}}{29 - 28}$$

$$\frac{\partial u}{\partial y} = \frac{u_{01} - u_{00}}{23 - 22} \qquad \frac{\partial v}{\partial y} = \frac{v_{01} - v_{00}}{23 - 22}$$

$$\frac{\partial u}{\partial x} = \frac{u_{10} - u_{00}}{1} \qquad \frac{\partial v}{\partial x} = \frac{v_{10} - v_{00}}{1}$$

$$\frac{\partial u}{\partial y} = \frac{u_{01} - u_{00}}{1} \qquad \frac{\partial v}{\partial y} = \frac{v_{01} - v_{00}}{1}$$

Triângulo com Texturas MipMap

U = 0.264
V = 0.139

Coordenadas (u,v)
$pixel_{10}(29, 22)$ => UV(0.414, 0.129)
$pixel_{01}(28, 23)$ => UV(0.248, -0.010)

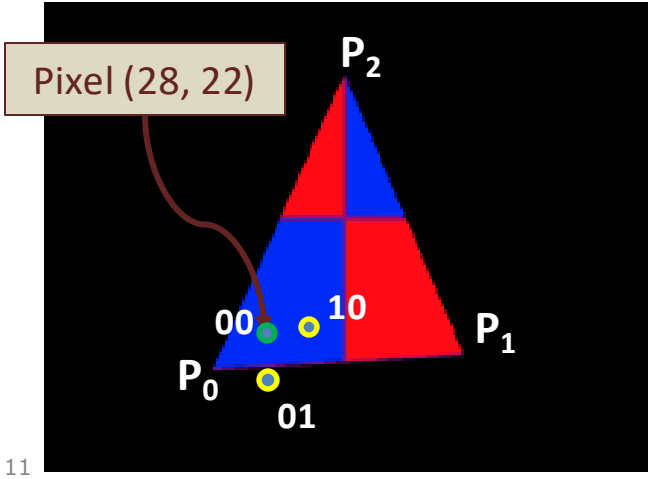
x 32

$$\frac{\partial u}{\partial x} = \frac{u_{10} - u_{00}}{1} = 32\left(\frac{0.414-0.264}{1}\right) = 4.8$$

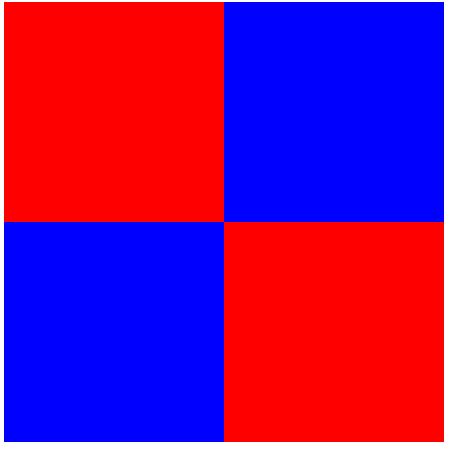
$$\frac{\partial v}{\partial x} = \frac{v_{10} - v_{00}}{1} = 32\left(\frac{0.129-0.139}{1}\right) = -0.32$$

$$\frac{\partial u}{\partial y} = \frac{u_{01} - u_{00}}{1} = 32\left(\frac{0.248-0.264}{1}\right) = -0.56$$

$$\frac{\partial v}{\partial y} = \frac{v_{01} - v_{00}}{1} = 32\left(\frac{-0.010-0.139}{1}\right) = -4.8$$



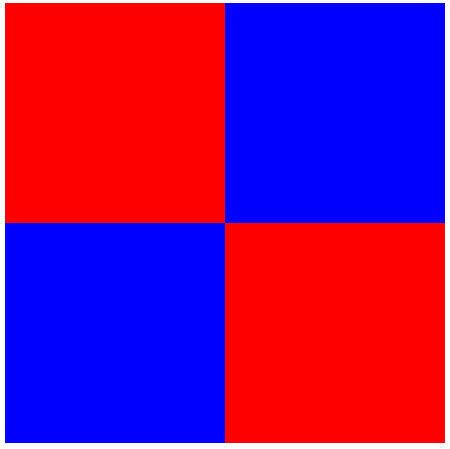
chess.png (32x32)



Insper

Triângulo com Texturas MipMap

chess.png (32x32)



0

Coordenadas (u,v)

U = 0.264

V = 0.139

$pixel_{10}(29, 22) \Rightarrow UV(0.414, 0.129)$

$pixel_{01}(28, 23) \Rightarrow UV(0.248, -0.010)$

$$\frac{du}{dx} = 4.8 \qquad \frac{dv}{dx} = -0.32$$

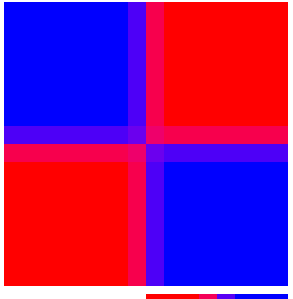
$$\frac{du}{dy} = -0.56 \qquad \frac{dv}{dy} = -4.8$$

$$D = \log_2 L$$

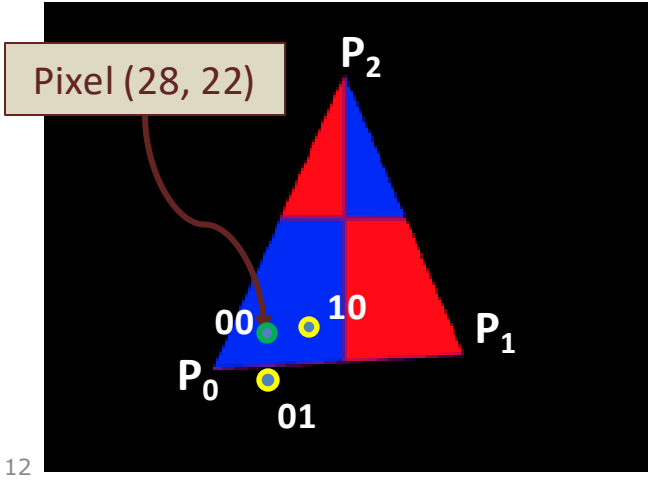
$$L = \max\left(\sqrt{\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2}, \sqrt{\left(\frac{du}{dy}\right)^2 + \left(\frac{dv}{dy}\right)^2}\right)$$

$$L = \max\left(\sqrt{(4.8)^2 + (-0.32)^2}, \sqrt{(-0.56)^2 + (-4.8)^2}\right)$$

$$L = \max(4.81, 4.83)$$



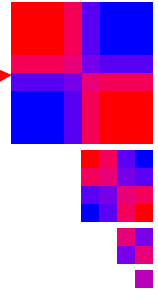
1 (16x16)



L = 4.83

D = 2.27

D = 2



2 (8x8)

3 (4x4)

Insper

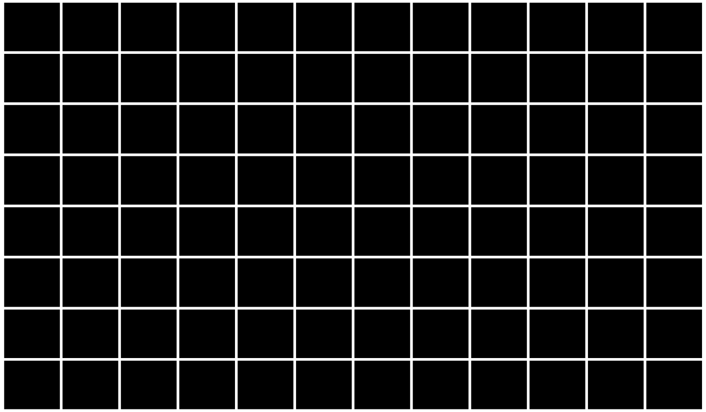
Z-Buffer

Desenhando com Z-Buffer

```
<Scene>
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='1 0 0'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

Por praticidade estamos definindo o Z-buffer variando de 0 a 1.
Sendo o 1 para valores distantes e 0 para próximos da câmera.

Cores



Profundidade

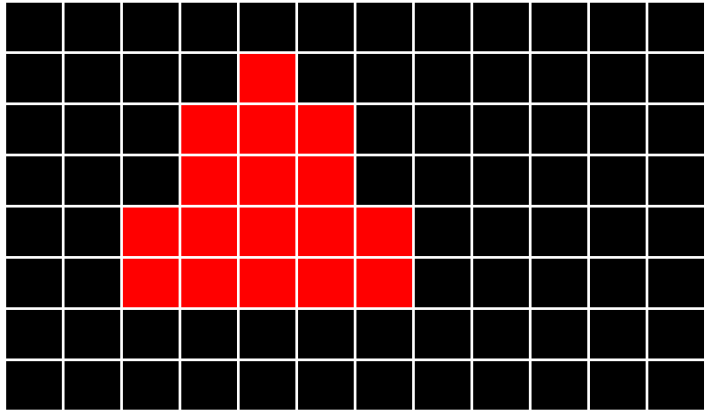
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1

Desenhando com Z-Buffer

```
<Scene>
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='1 0 0'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

Por praticidade estamos definindo o Z-buffer variando de 0 a 1.
Sendo o 1 para valores distantes e 0 para próximos da câmera.

Cores



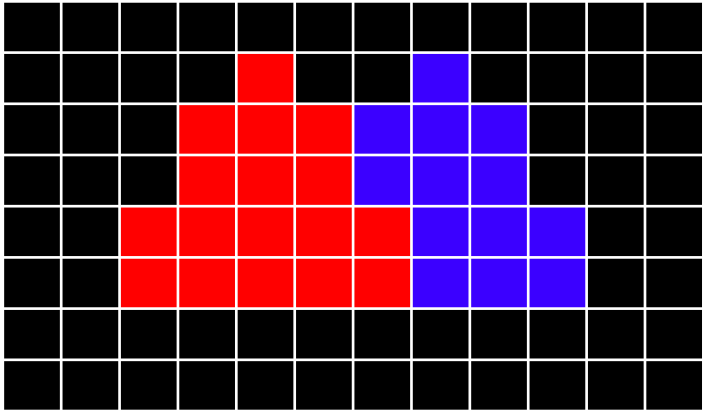
Profundidade

1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0.8	1	1	1	1	1	1	1
1	1	1	0.8	0.8	0.8	1	1	1	1	1	1
1	1	1	0.8	0.8	0.8	1	1	1	1	1	1
1	1	0.8	0.8	0.8	0.8	0.8	1	1	1	1	1
1	1	0.8	0.8	0.8	0.8	0.8	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1

Desenhando com Z-Buffer

```
<Scene>
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='1 0 0'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

Cores



Profundidade

1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0.8	1	1	0.9	1	1	1	1
1	1	1	0.8	0.8	0.8	0.9	0.9	0.9	1	1	1
1	1	1	0.8	0.8	0.8	0.9	0.9	0.9	1	1	1
1	1	0.8	0.8	0.8	0.8	0.8	0.9	0.9	0.9	1	1
1	1	0.8	0.8	0.8	0.8	0.8	0.9	0.9	0.9	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1

Por praticidade estamos definindo o Z-buffer variando de 0 a 1.
Sendo o 1 para valores distantes e 0 para próximos da câmera.

Transparência

Transparência

```
<Scene>
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor ='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='1 1 0'
              transparency='0.2'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

Cores (Vermelho, Verde, Azul)

(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

```
cor_anterior = framebuffer[x,y] * transparência
cor_nova = rbg * (1 – transparência)
framebuffer[x,y] = cor_anterior + cor_nova
```

Transparência

```
<Scene>
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor ='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='0 1 0'
              transparency='0.4'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

Cores

(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0.6,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0.4)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0.4)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

```
cor_anterior = framebuffer[x,y] * transparência
cor_nova = rbg * (1 – transparência)
framebuffer[x,y] = cor_anterior + cor_nova
```

Insper

Computação Gráfica

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