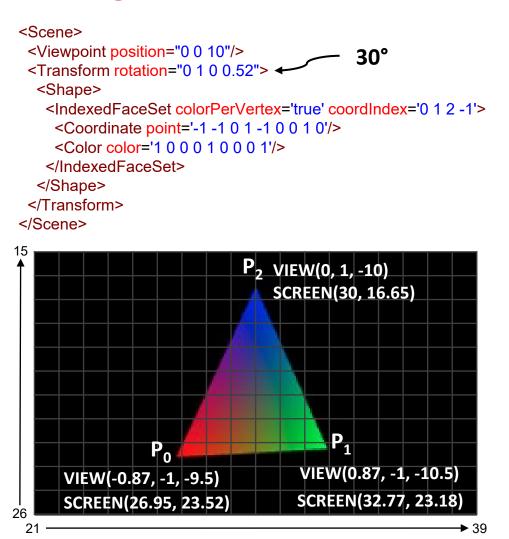
Insper

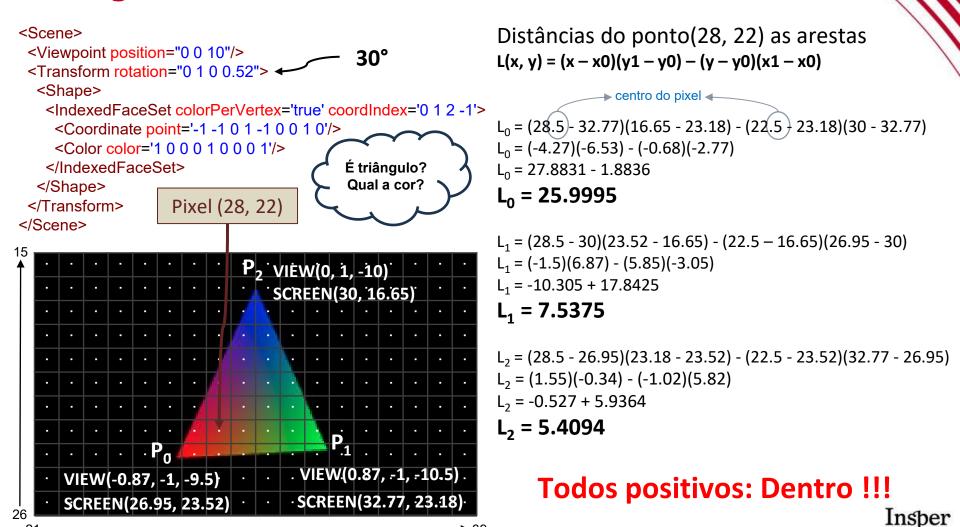
Computação Gráfica

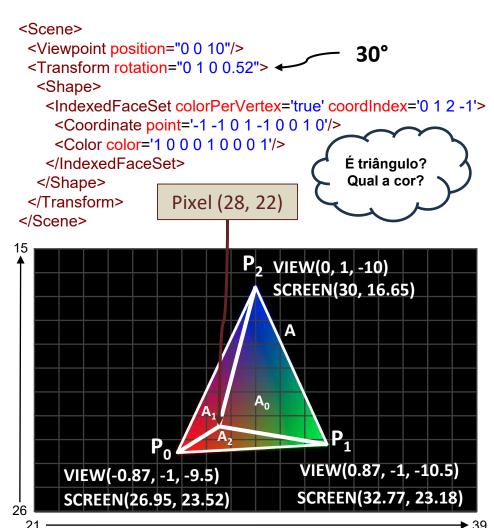
Aula 12: Revisão 2 Interpolaçãoo, MipMap, Visibilidade

Interpolação em Triângulos









Área dos triângulos:

Area = $|x_0(y_1-y_2) + x_1(y_2-y_0) + x_2(y_0-y_1)| / 2$

A = |26.95(23.18-16.65)+32.77(16.65-23.52)+30(23.52-23.18)|/2

A = |26.95(6.53)+32.77(-6.87)+30(0.34)|/2

A = |175.9835-225.1299+10.2|/2

A = 19.4732

 $A_0 = |28.5(23.18-16.65)+32.77(16.65-22.5)+30(22.5-23.18)|/2$

 $A_0 = |28.5(6.53)+32.77(-5.85)+30(-0.68)|/2$

 $A_0 = |186.105-191.7045-20.4|/2$

 $A_0 = 12.99975$

 $A_1 = |28.5(16.65-23.52)+30(23.52-22.5)+26.95(22.5-16.65)|/2$

 $A_1 = |28.5(-6.87)+30(1.02)+26.95(5.85)|/2$

 $A_1 = |-195.795+30.6+157.6575|/2$

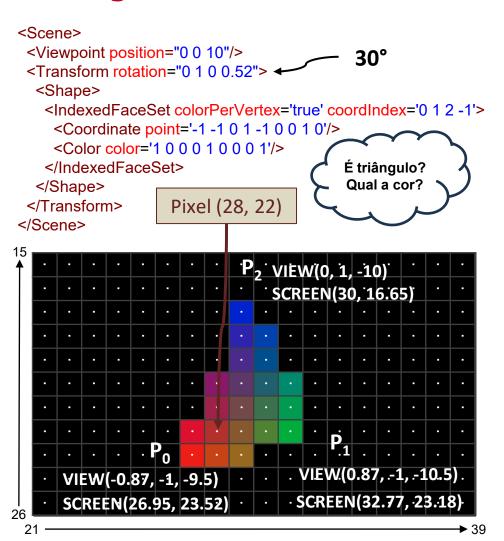
 $A_1 = 3.76875$

 $A_2 = |28.5(23.52-23.18)+26.95(23.18-22.5)+32.77(22.5-23.52)|/2$

 $A_2 = |28.5(0.34) + 26.95(0.68) + 32.77(-1.02)|/2$

 $A_2 = |9.69+18.326-33.4254|/2$

 $A_2 = 2.7047$



Área dos triângulos:

$$A = 19.4732$$

$$A_0 = 12.99975$$

$$A_1 = 3.76875$$

$$A_2 = 2.7047$$

Pesos

$$\alpha$$
 = 12.99975 / 19.4732 ~= 0.668

$$\beta = 3.76875 / 19.4732 \approx 0.194$$

$$\gamma = 2.7047 / 19.4732 \approx 0.139$$

ou

$$\gamma = 1 - \alpha - \beta \approx 0.138$$

Cor do pixel:

$$R = \alpha R_0 + \beta R_1 + \gamma R_2 = 0.668$$

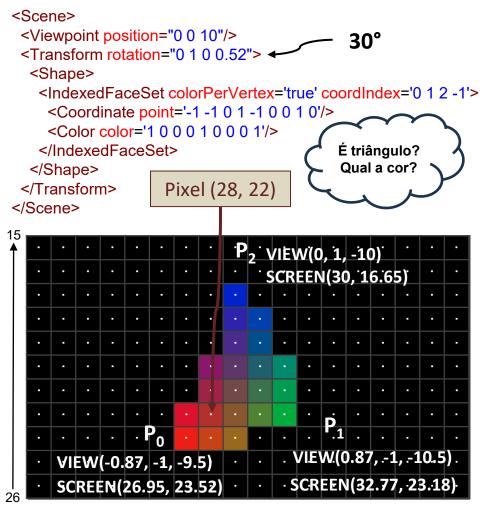
$$G = \alpha G_0 + \beta G_1 + \gamma G_2 = 0.194$$

$$B = \alpha B_0 + \beta B_1 + \gamma B_2 = 0.139$$

$$C = (0.668, 0.194, 0.139)$$

Percebeu que só usou coordenadas 2D da tela? Insper

Cores do Triângulo com Correção Perspectiva



Pesos
$$\alpha = 0.668$$
 $Z_0 = |-9.5| = 9.5$ $\beta = 0.194$ $Z_1 = |-10.5| = 10.5$ $Z_2 = |-10.0| = 10$

$$Z=rac{1}{lpharac{1}{Z_0}+etarac{1}{Z_1}+\gammarac{1}{Z_2}}$$

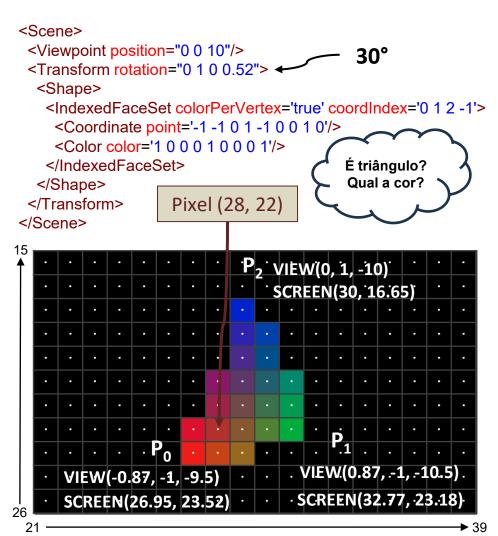
$$Z = \frac{1}{0.668 \frac{1}{9.5} + 0.194 \frac{1}{10.5} + 0.139 \frac{1}{10}}$$
$$Z = \frac{1}{0.0703 + 0.0185 + 0.0139}$$

$$Z = 9.74$$

Insper

→ 39

Cores do Triângulo com Correção Perspectiva



Pesos
$$Z_0 = |-9.5| = 9.5$$

 $α = 0.668$ $Z_1 = |-10.5| = 10.5$
 $β = 0.194$ $Z_2 = |-10.0| = 10$
 $Z = 9.74$

$$C = Z \cdot \left(lpha rac{C_0}{Z_0} + eta rac{C_1}{Z_1} + \gamma rac{C_2}{Z_2}
ight)$$

$$C_R = 9.74 \left(0.668 \frac{1}{9.5} + 0.194 \frac{0}{10.5} + 0.139 \frac{0}{10} \right) = 0.685$$

$$C_G = 9.74 \left(0.668 \frac{0}{9.5} + 0.194 \frac{1}{10.5} + 0.139 \frac{0}{10} \right) = 0.180$$

$$C_B = 9.74 \left(0.668 \frac{0}{9.5} + 0.194 \frac{0}{10.5} + 0.139 \frac{1}{10} \right) = 0.135$$

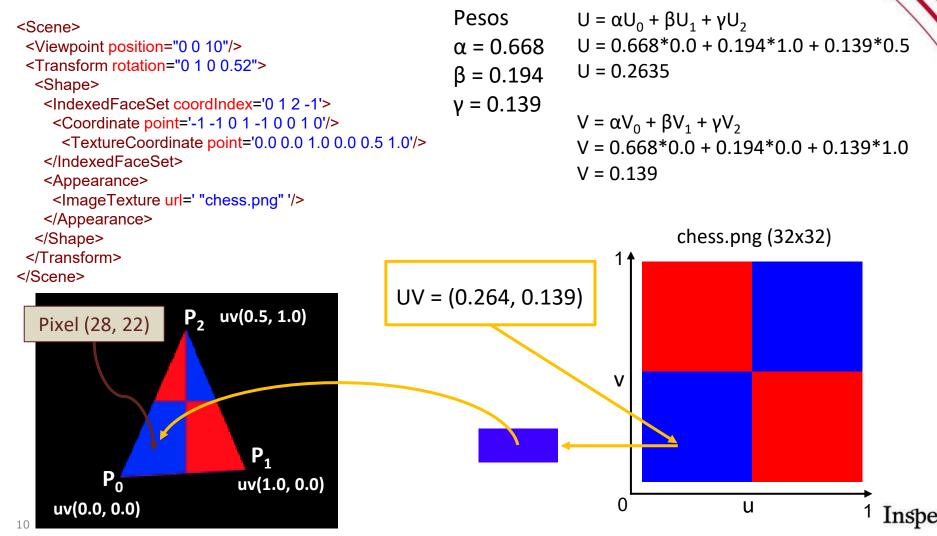
$$C = (0.685, 0.180, 0.135)$$

Antes C = (0.668, 0.194, 0.139)

Aplicando Texturas



Triângulo com Texturas



Triângulo com Texturas MipMap

```
<Scene>
 <Viewpoint position="0 0 10"/>
 <Transform rotation="0 1 0 0.52">
  <Shape>
   <IndexedFaceSet coordIndex='0 1 2 -1'>
    <Coordinate point='-1 -1 0 1 -1 0 0 1 0'/>
      <TextureCoordinate point='0.0 0.0 1.0 0.0 0.5 1.0'/>
   IndexedFaceSet>
   <Appearance>
    <lmageTexture url=' "chess.png" '/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

```
pixel_{00}(28, 22) => UV(0.264, 0.139)
```

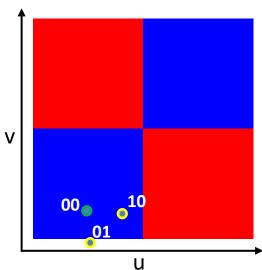
Coordenadas (u,v) Vizinhas: pixel₁₀(29, 22) => UV(0.414, 0.129) pixel₀₁(28, 23) => UV(0.248, -0.010)

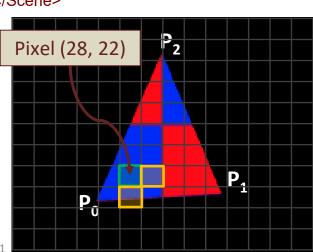
$$rac{\partial u}{\partial x} = rac{\mathrm{u}_{10} - \mathrm{u}_{00}}{\mathrm{x}_{10} - \mathrm{x}_{00}}$$

$$\frac{\partial u}{\partial y} = \frac{\mathbf{u}_{01} - \mathbf{u}_{00}}{\mathbf{y}_{01} - \mathbf{y}_{00}}$$

$$\frac{\partial v}{\partial x} = \frac{\mathbf{v}_{10} - \mathbf{v}_{00}}{\mathbf{x}_{10} - \mathbf{x}_{00}}$$

$$rac{\partial v}{\partial y} = rac{\mathrm{v}_{01} - \mathrm{v}_{00}}{\mathrm{y}_{01} - \mathrm{y}_{00}}$$





Triângulo com Texturas MipMap

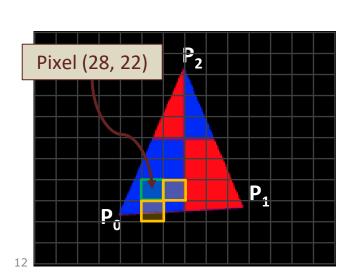
$$U = 0.264$$
 pixel₁₀(29, 22) => UV(0.414, 0.129)
 $V = 0.139$ pixel₀₁(28, 23) => UV(0.248, -0.010)

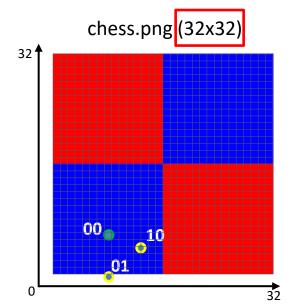
$$\frac{\partial u}{\partial x} = \frac{\mathbf{u}_{10} - \mathbf{u}_{00}}{29 - 28} = 32 \left(\frac{0.414 - 0.264}{1} \right) = 4.8$$

$$\frac{\partial u}{\partial y} = \frac{\mathbf{u}_{01} - \mathbf{u}_{00}}{23 - 22} = 32 \left(\frac{0.248 - 0.264}{1} \right) = -0.56$$

$$\frac{\partial v}{\partial x} = \frac{\mathbf{v}_{10} - \mathbf{v}_{00}}{29 - 28} = 32 \left(\frac{0.129 - 0.139}{1} \right) = -0.32$$

$$\frac{\partial v}{\partial y} = \frac{v_{01} - v_{00}}{23 - 22} = 32\left(\frac{-0.010 - 0.139}{1}\right) = -4.8$$



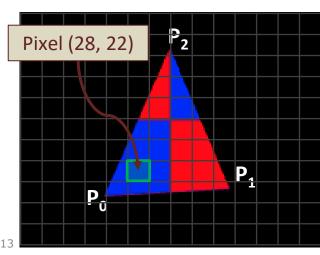


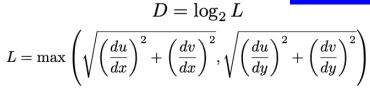
Triângulo com Texturas MipMap

$$U = 0.264$$
 pixel₁₀(29, 22) => UV(0.414, 0.129)
 $V = 0.139$ pixel₀₁(28, 23) => UV(0.248, -0.010)

$$\frac{du}{dx} = 4.8 \qquad \qquad \frac{dv}{dx} = -0.32$$

$$\frac{du}{dy} = -0.56 \qquad \frac{dv}{dy} = -4.8$$





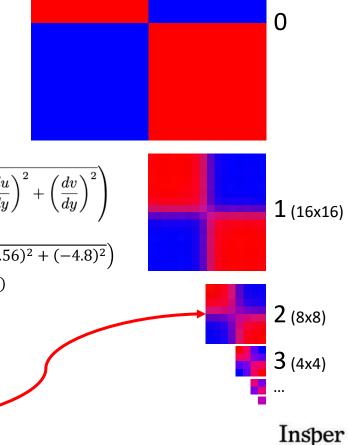
$$L = \max(\sqrt{(4.8)^2 + (-0.32)^2}, \sqrt{(-0.56)^2 + (-4.8)^2})$$

$$L = \max(4.81, 4.83)$$

L = 4.83

D = 2.27

D = 2



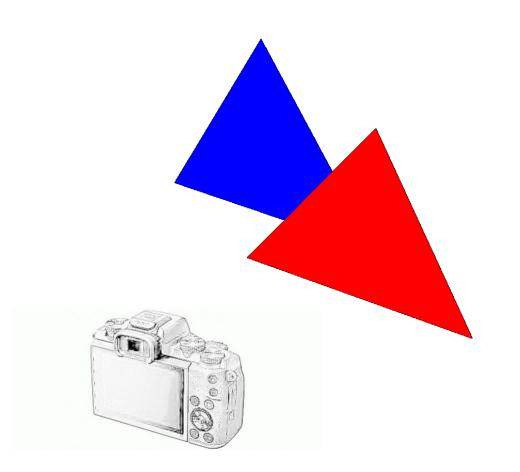
chess.png (32x32)

Z-Buffer



Desenhando Dois Triângulos

```
<Scene>
 <Viewpoint position="0 0 8">
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='1 0 0'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

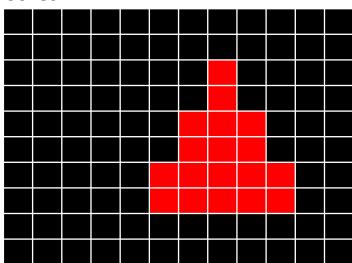




Desenhando SEM Z-Buffer

```
<Scene>
 <Viewpoint position="0 0 8">
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='1 0 0'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

Cores

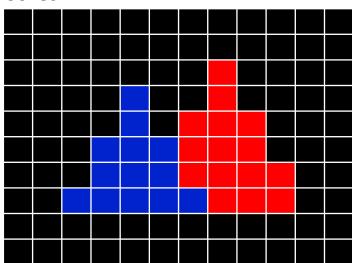




Desenhando SEM Z-Buffer

```
<Scene>
 <Viewpoint position="0 0 8">
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='1 0 0'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

Cores



Isso está Certo?

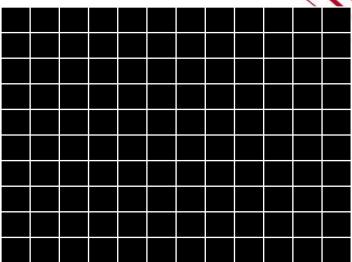


Desenhando com Z-Buffer

```
<Scene>
 <Viewpoint position="0 0 8">
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='1 0 0'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

Por praticidade estamos definindo o Z-buffer variando de 0 a 1. Sendo o 1 para valores distantes e 0 para próximos da câmera. Vamos mostrar na tabela só os dígitos que fazer diferença.

Cores



Profundidade

1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1

Desenhando com Z-Buffer

Z no NDC

0.99751998

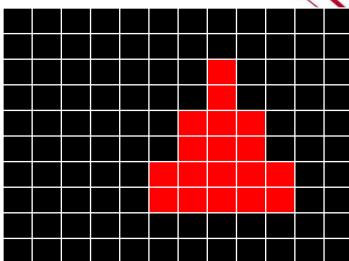
Normalizando

0.99875999

```
<Scene>
 <Viewpoint position="0 0 8">
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='1 0 0'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

Por praticidade estamos definindo o Z-buffer variando de 0 a 1. Sendo o 1 para valores distantes e 0 para próximos da câmera. Vamos mostrar na tabela só os dígitos que fazer diferença.

Cores



Profundidade (0.998X)

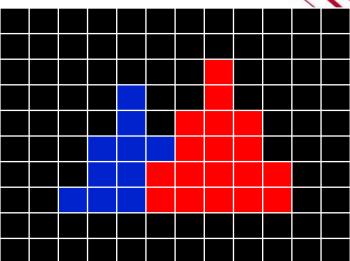
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	.7	1	1	1	1
1	1	1	1	1	1	1	.7	1	1	1	1
1	1	1	1	1	1	.7	.7	.7	1	1	1
1	1	1	1	1	1	.7	.7	.7	1	1	1
1	1	1	1	1	.7	.7	.7	.7	.7	1	1
1	1	1	1	1	.7	.7	.7	.7	.7	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1

Desenhando com Z-Buffer

```
<Scene>
 <Viewpoint position="0 0 8">
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='1 0 0'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="-1 0 0">
  <Shape>
                                                   Z no NDC
   <TriangleSet>
                                                  0.99779776
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
                                                 Normalizando
    <Material emissiveColor='0 0 1'/>
                                                  0.99889888
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

Por praticidade estamos definindo o Z-buffer variando de 0 a 1. Sendo o 1 para valores distantes e 0 para próximos da câmera. Vamos mostrar na tabela só os dígitos que fazer diferença.

Cores



Profundidade (0.998X)

				•			,				
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	.7	1	1	1	1
1	1	1	1	.8	1	1	.7	1	1	1	1
1	1	1	1	.8	1	.7	.7	.7	1	1	1
1	1	1	.8	.8	.8	.7	.7	.7	1	1	1
1	1	1	.8	.8	.7	.7	.7	.7	.7	1	1
1	1	.8	.8	.8	.7	.7	.7	.7	.7	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1



```
<Scene>
 <Viewpoint position="0 0 8">
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor ='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='0 1 0'
              transparency='0.4'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

Vamos deixar o triângulo transparente como último no processo para evitar ter de ordenar os triângulos.



Cores (Vermelho, Verde, Azul)

```
<Scene>
 <Viewpoint position="0 0 8">
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor ='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='0 1 0'
              transparency='0.4'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

```
cor_anterior = framebuffer[x,y] * transparência
cor_nova = rbg * (1 - transparência)
framebuffer[x,y] = cor_anterior + cor_nova
```

```
<Scene>
 <Viewpoint position="0 0 8">
 <Transform translation="1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 -1 2 -2 -1 0 2 -1'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor ='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
 <Transform translation="-1 0 0">
  <Shape>
   <TriangleSet>
    <Coordinate point='-2 -2 0 2 -2 0 0 2 0'/>
   </TriangleSet>
   <Appearance>
    <Material emissiveColor='0 1 0'
              transparency='0.4'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

Cores (Vermelho, Verde, Azul)

(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0.6,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0.6,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0)	(0,0,0)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0.4)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0)	(0,0.6,0.4)	(0,0.6,0.4)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

cor_anterior = framebuffer[x,y] * transparência
cor_nova = rbg * (1 - transparência)
framebuffer[x,y] = cor_anterior + cor_nova

Insper

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