

# Computação Gráfica

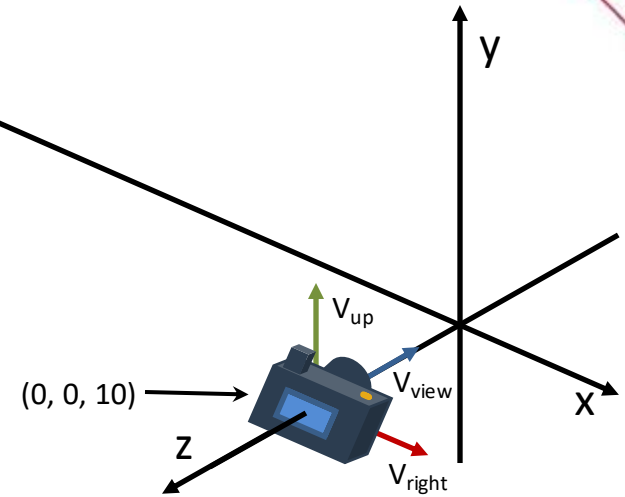
Aula 7: Revisão

## Exemplo Completo:

- Transformações Geométricas
- Quatérnios
- Transformação Look-at
- Transformação Perspectiva
- Coordenadas normalizadas (NDC)
- Divisão Homogênea
- Transformação de tela
- Supersampling

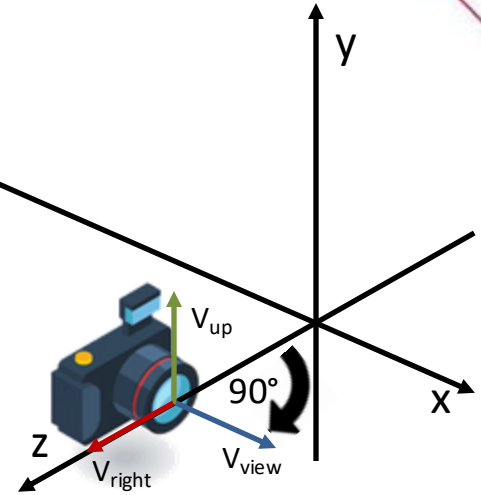
# Definindo posição da câmera virtual

```
<Scene>  
<Viewpoint/>  
</Scene>
```



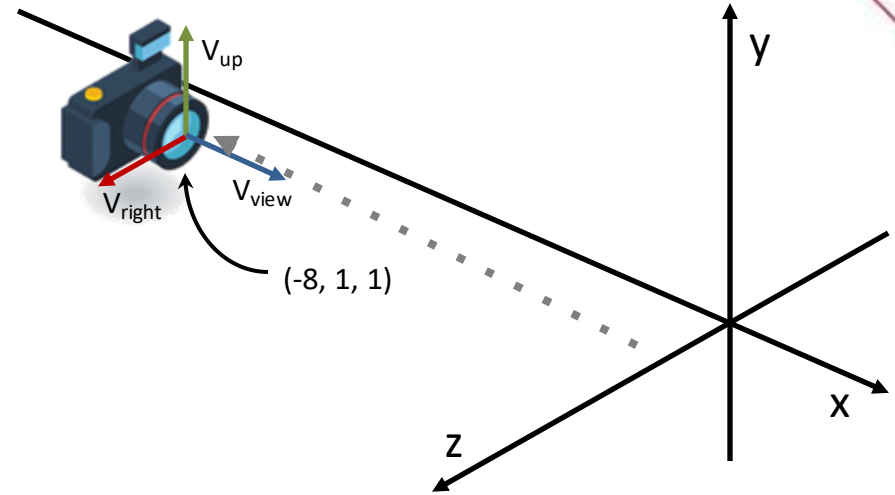
# Definindo posição da câmera virtual

```
<Scene>  
<Viewpoint orientation="0 1 0 -1.57"/>  
</Scene>
```



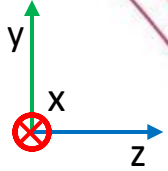
# Definindo posição da câmera virtual

```
<Scene>  
<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>  
</Scene>
```



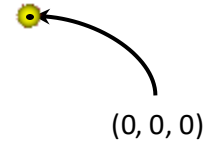
# Criando uma esfera no centro do mundo

Visualização do ponto de vista da câmera mas no sistema de coordenadas do mundo.



```
<Scene>
<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
<Transform>
  <Shape>
    <Sphere radius='0.1'/>
    <Appearance>
      <Material diffuseColor="1 1 0"/>
    </Appearance>
  </Shape>
</Transform>
</Scene>
```

amarelo



centro da esfera :  $[0 \ 0 \ 0 \ 1]^T$

ou

centro da esfera :  $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

# Criando um triângulo no centro do mundo

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```

vermelho

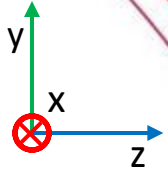
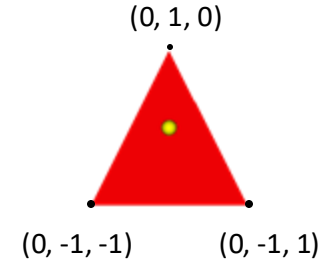


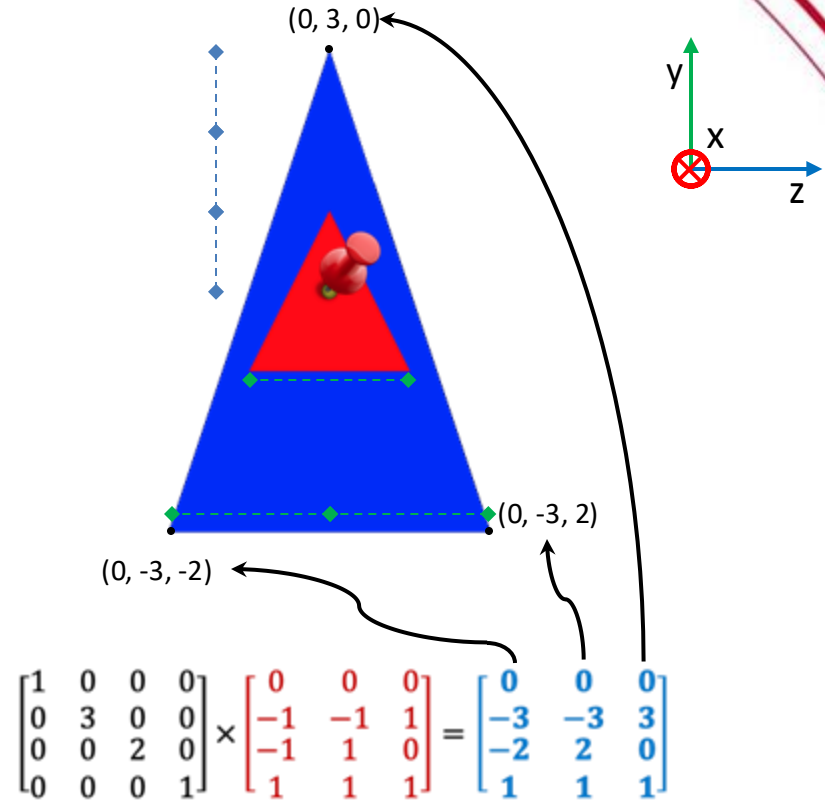
Diagram illustrating the vertices of a triangle in a 3D coordinate system, labeled  $P_0$ ,  $P_1$ , and  $P_2$ .

$$\text{vértices do triângulo} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

# Novo triângulo ampliado

```
<Scene>
<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
<Transform>
  <Shape>
    <Sphere radius='0.1'/>
    <Appearance>
      <Material diffuseColor='1 1 0'/>
    </Appearance>
  </Shape>
</Transform>
<Transform>
  <Shape>
    <TriangleSet>
      <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor='1 0 0'/>
    </Appearance>
  </Shape>
</Transform>
<Transform scale="1 3 2">
  <Shape>
    <TriangleSet>
      <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor='0 0 1'/>
    </Appearance>
  </Shape>
</Transform>
</Scene>
```

azul





# Novo triângulo rotacionado

<Scene>

<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>

<Transform>

<Shape>

<Sphere radius='0.1'/>

<Appearance>

<Material diffuseColor='1 1 0'/>

</Appearance>

</Shape>

</Transform>

<Transform>

<Shape>

<TriangleSet>

<Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>

</TriangleSet>

<Appearance>

<Material diffuseColor='1 0 0'/>

</Appearance>

</Shape>

</Transform>

<Transform scale="1 3 2"

rotation="1 0 0 -1.57">

<Shape>

<TriangleSet>

<Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>

</TriangleSet>

<Appearance>

<Material diffuseColor='0 0 1'/>

</Appearance>

</Shape>

</Transform>

</Scene>

$$\mathbf{R}_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & \sin \theta & 0 \\ 0 & \sin \theta & -\cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$q = \cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)u_x i + \sin\left(\frac{\theta}{2}\right)u_y j + \sin\left(\frac{\theta}{2}\right)u_z k$$

$$q = \cos(-0.79) + \sin(-0.79)1i + \sin(-0.79)0j + \sin(-0.79)0k$$

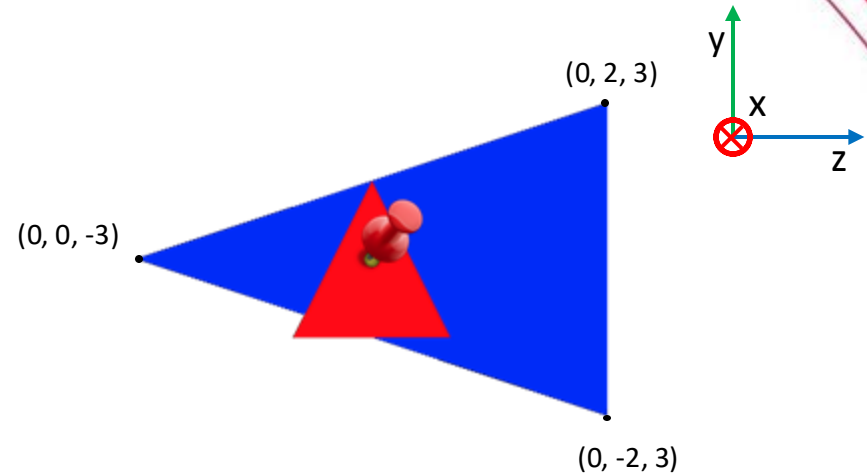
$$q = 0.71 - 0.71i$$

$$R = \begin{bmatrix} 1 - 2(q_j^2 + q_k^2) & 2(q_i q_j - q_k q_r) & 2(q_i q_k + q_j q_r) & 0 \\ 2(q_i q_j + q_k q_r) & 1 - 2(q_i^2 + q_k^2) & 2(q_j q_k - q_i q_r) & 0 \\ 2(q_i q_k - q_j q_r) & 2(q_j q_k + q_i q_r) & 1 - 2(q_i^2 + q_j^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Novo triângulo rotacionado

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius="0.1"/>
      <Appearance>
        <Material diffuseColor="1 1 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="1 0 0"/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57">
    <Shape>
      <TriangleSet>
        <Coordinate point="0 -1 -1 0 -1 1 0 1 0"/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor="0 0 1"/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



$$\begin{matrix} \text{rotação} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix} \times \begin{matrix} \text{escala} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

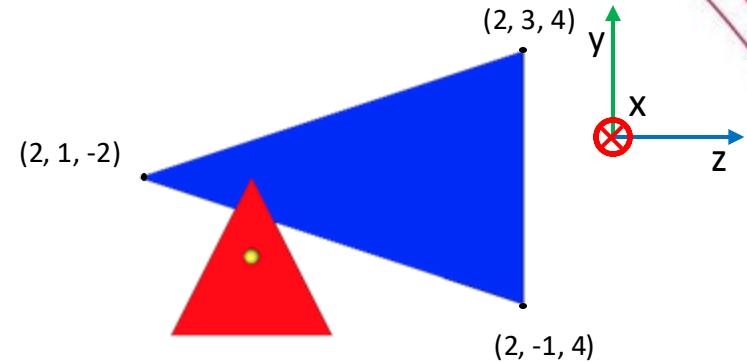
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ -2 & 2 & 0 \\ 3 & 3 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

# Novo triângulo transladando

```

<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <Triangle Set>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>

```



$$\begin{matrix} \text{translação} & \text{rotação} & \text{escala} \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

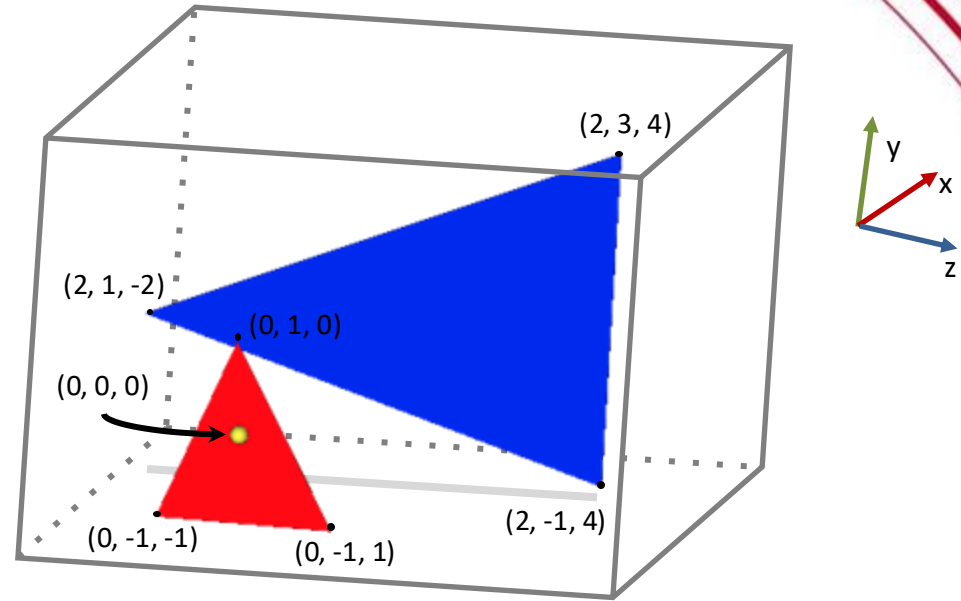
$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ -1 & 3 & 1 \\ 4 & 4 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

# Visualizando de outro ângulo

```

<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <Triangle Set>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>

```



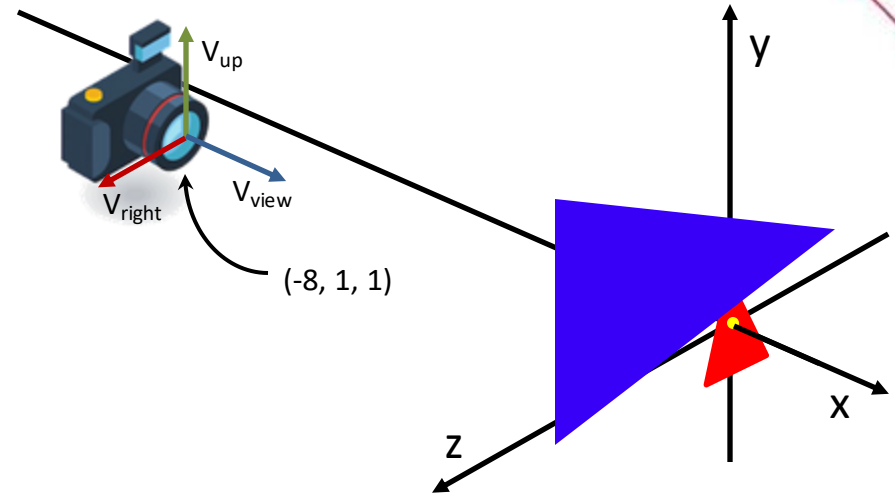
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 2 & 2 & 2 \\ -1 & 3 & 1 \\ 4 & 4 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

# Transformação Look-at

```

<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <Triangle Set>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>

```



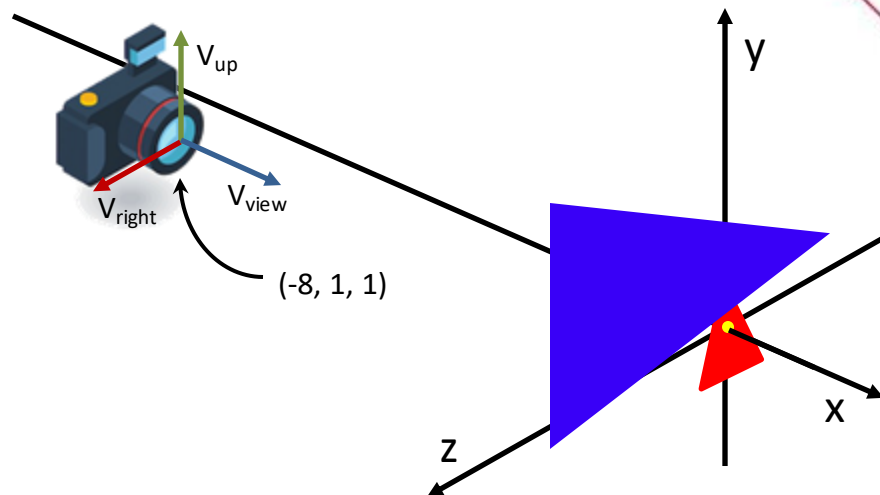
$$\begin{aligned} \mathbf{u} &= \mathbf{V}_{\text{right}} \\ \mathbf{v} &= \mathbf{V}_{\text{up}} \\ \mathbf{w} &= \mathbf{V}_{\text{view}} \end{aligned}$$

$$\begin{bmatrix} u_x & v_x & -w_x & e_x \\ u_y & v_y & -w_y & e_y \\ u_z & v_z & -w_z & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$(T \cdot R)^{-1} = R^{-1} \cdot T^{-1}$$

# Transformação Look-at (Rotação)

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <Triangle Set>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



## Rotação:

Como temos uma câmera que estava na base da cena (ou seja identidade), podemos usar diretamente a rotação realizada para calcular a matriz de look-at.

$$q = \cos(-0.79) + \sin(-0.79)0i + \sin(-0.79)1j + \sin(-0.79)0k$$

$$q = 0.71 - 0.71j$$

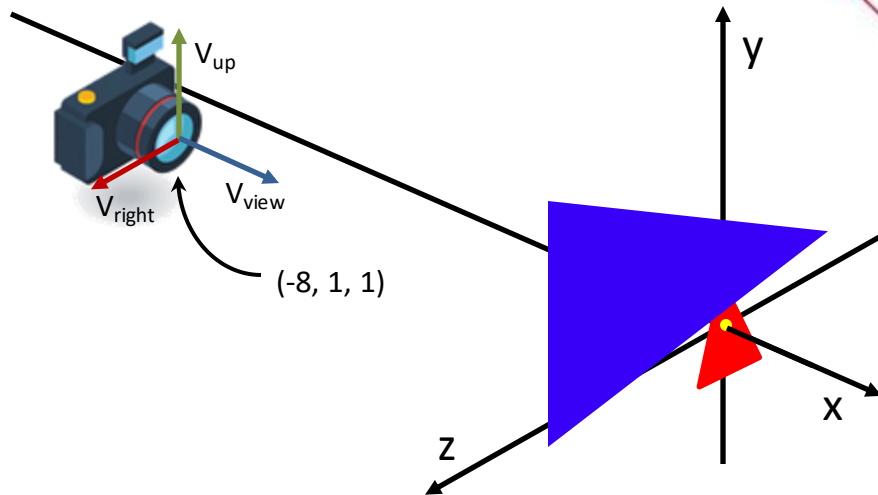
$$R = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Transformação Look-at (Translação)

```

<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <Triangle Set>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>

```



Translação:

$$T = \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

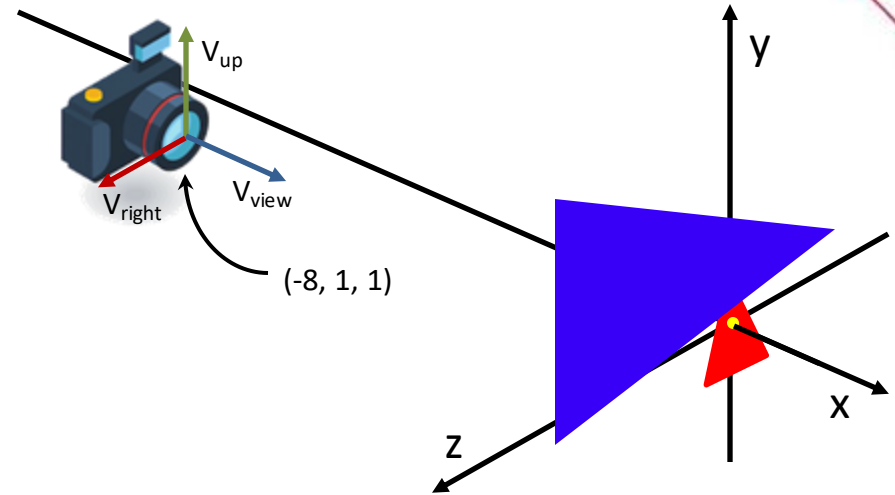
$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Transformação Look-at

```

<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <Triangle Set>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>

```

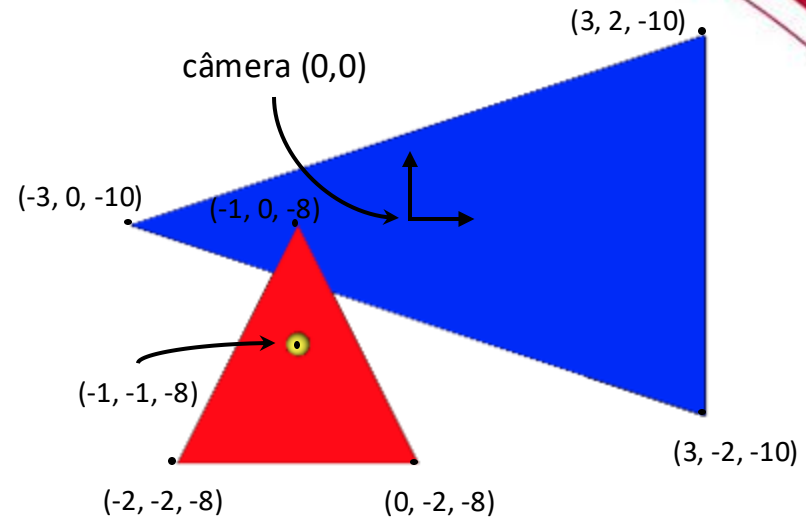


$$\begin{array}{ccc}
 \text{Orientação}^{-1} & \text{Translação}^{-1} & \text{Look-At} \\
 \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \times \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} & = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{array}$$



# Aplicando Look-At

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <Triangle Set>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



Look-At

$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -8 \\ 1 \end{bmatrix}$$

Look-At

$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -1 \\ -2 & -2 & 0 \\ -8 & -8 & -8 \\ 1 & 1 & 1 \end{bmatrix}$$

Look-At

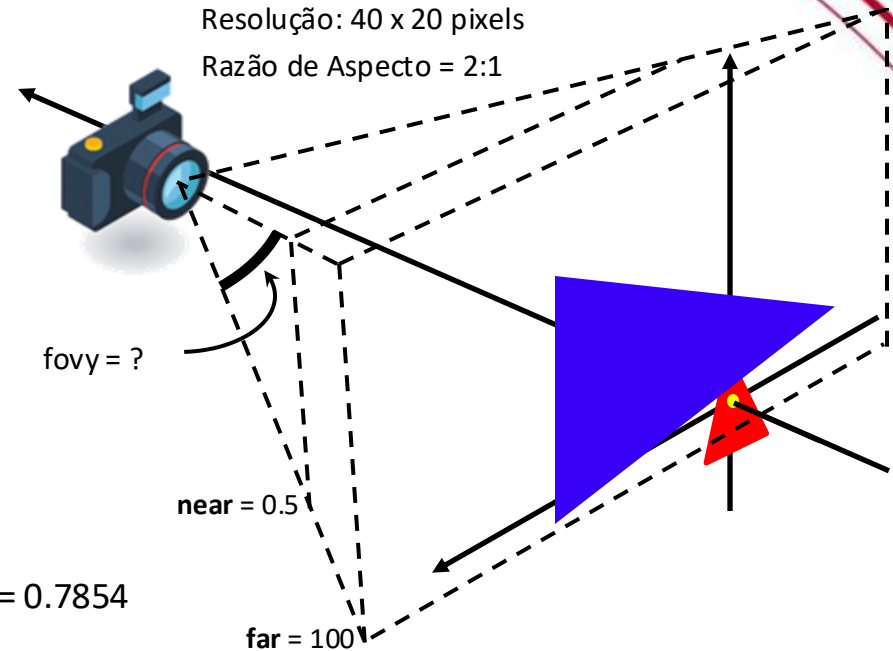
$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 2 & 2 \\ -1 & 3 & 1 \\ 4 & 4 & -2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -3 \\ -2 & 2 & 0 \\ -10 & -10 & -10 \\ 1 & 1 & 1 \end{bmatrix}$$

# Matriz Perspectiva

```

<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <Triangle Set>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
  
```

$$\text{fieldOfView} = \pi/4 = 0.7854$$



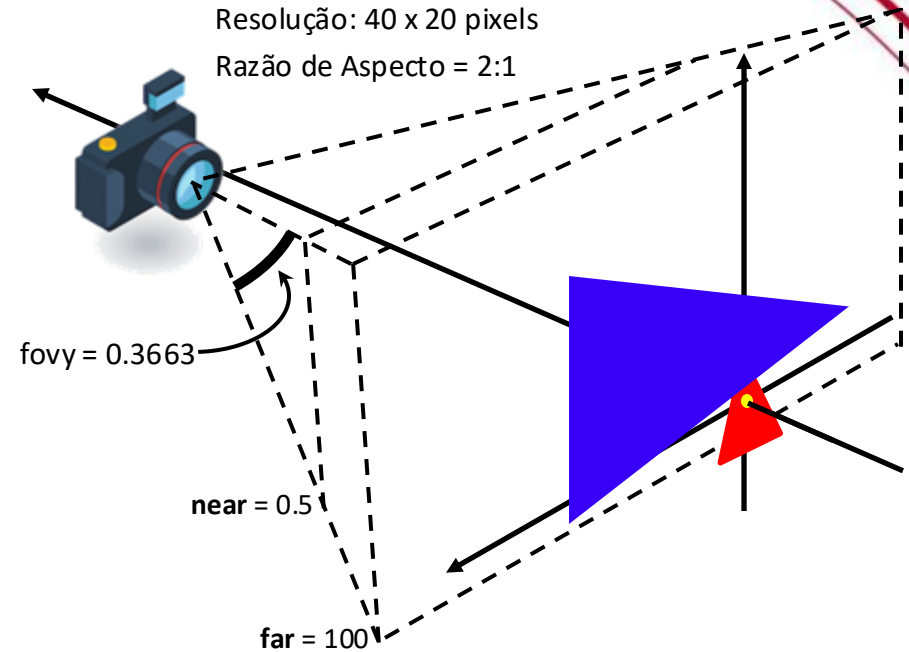
$$\text{FOV}_y = 2 \cdot \arctan \left( \tan \left( \frac{\text{FOV}_d}{2} \right) \cdot \frac{\text{Altura}}{\sqrt{\text{Altura}^2 + \text{Largura}^2}} \right)$$

$$\text{FOV}_y = 2 \cdot \arctan \left( \tan \left( \frac{0.7854}{2} \right) \cdot \frac{20}{\sqrt{20^2 + 40^2}} \right)$$

$$\text{FOV}_y = 0.3663$$

# Matriz Perspectiva

```
<Scene>
<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
<Transform>
  <Shape>
    <Sphere radius='0.1'/>
    <Appearance>
      <Material diffuseColor='1 1 0'/>
    </Appearance>
  </Shape>
</Transform>
<Transform>
  <Shape>
    <Triangle Set>
      <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
    </Triangle Set>
    <Appearance>
      <Material diffuseColor='1 0 0'/>
    </Appearance>
  </Shape>
</Transform>
<Transform scale="1 3 2"
  rotation="1 0 0 -1.57"
  translation="2 1 1">
  <Shape>
    <TriangleSet>
      <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
    </TriangleSet>
    <Appearance>
      <Material diffuseColor='0 0 1'/>
    </Appearance>
  </Shape>
</Transform>
</Scene>
```



$$P = \begin{bmatrix} \frac{near}{right} & 0 & 0 & 0 \\ 0 & \frac{near}{top} & 0 & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & \frac{-2far*near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

**top** = near \* tan (fovy) = 0.5 \* 0.3663 = 0.1832

**bottom** = -top = -top = -0.1832

**right** = top \* aspect = top \* aspect = 0.3663

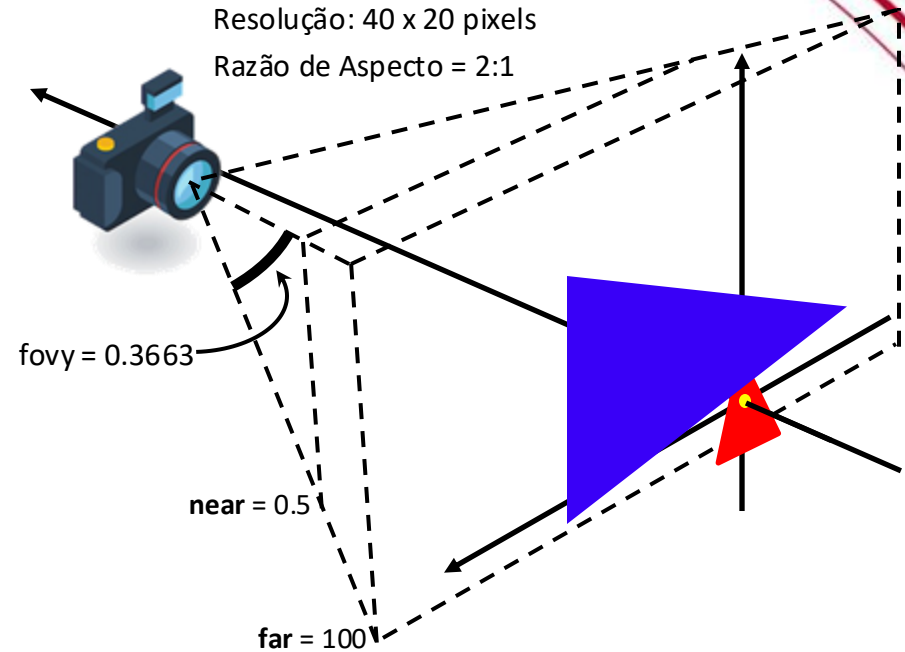
**left** = -right = -right = -0.3663

# Matriz Perspectiva

```

<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <Triangle Set>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>

```



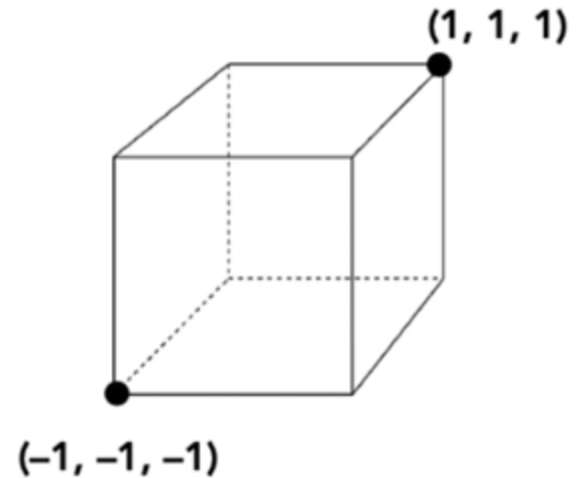
$$P = \begin{bmatrix} \frac{\text{near}}{\text{right}} & 0 & 0 & 0 \\ 0 & \frac{\text{near}}{\text{top}} & 0 & 0 \\ 0 & 0 & -\frac{\text{far} + \text{near}}{\text{far} - \text{near}} & -\frac{2\text{far} * \text{near}}{\text{far} - \text{near}} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

top = 0.1832  
bottom = -0.1832  
right = 0.3663  
left = -0.3663

$$P = \begin{bmatrix} \frac{0.5}{0.3663} & 0 & 0 & 0 \\ 0 & \frac{0.5}{0.1832} & 0 & 0 \\ 0 & 0 & -\frac{100 + 0.5}{100 - 0.5} & -\frac{2 * 100 * 0.5}{100 - 0.5} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1.3649 & 0 & 0 & 0 \\ 0 & 2.7298 & 0 & 0 \\ 0 & 0 & -1.01 & -1.005 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

# Aplicando Matriz Perspectiva

Projeção em NDC  
(Normalized Device Coordinate)



Projeção Perspectiva

$$\begin{bmatrix} 1.3649 & 0 & 0 & 0 \\ 0 & 2.7298 & 0 & 0 \\ 0 & 0 & -1.01 & -1.005 \\ 0 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 \\ -1 \\ -8 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.3649 \\ -2.7298 \\ 7.075 \\ 8 \end{bmatrix}$$

Projeção Perspectiva

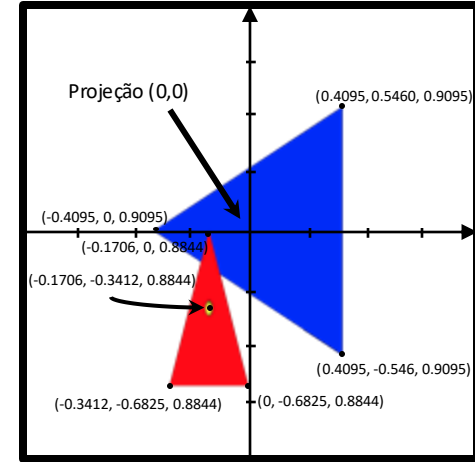
$$\begin{bmatrix} 1.3649 & 0 & 0 & 0 \\ 0 & 2.7298 & 0 & 0 \\ 0 & 0 & -1.01 & -1.005 \\ 0 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} -2 & 0 & -1 \\ -2 & -2 & 0 \\ -8 & -8 & -8 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2.7298 & 0 & -1.3649 \\ -5.4596 & -5.4596 & 0 \\ 7.075 & 7.075 & 7.075 \\ 8 & 8 & 8 \end{bmatrix}$$

Projeção Perspectiva

$$\begin{bmatrix} 1.3649 & 0 & 0 & 0 \\ 0 & 2.7298 & 0 & 0 \\ 0 & 0 & -1.01 & -1.005 \\ 0 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 3 & -3 \\ -2 & 2 & 0 \\ -10 & -10 & -10 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -4.0947 & 4.0947 & -4.0947 \\ -5.4596 & 5.4596 & 0 \\ 9.095 & 9.095 & 9.095 \\ 10 & 10 & 10 \end{bmatrix}$$

# Aplicando Divisão Homogênea

Projeção em NDC  
(Normalized Device Coordinate)



$$\begin{bmatrix} -1.3649 \\ -2.7298 \\ 7.075 \\ 8 \end{bmatrix} \rightarrow \begin{bmatrix} -0.1706 \\ -0.3412 \\ 0.8844 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2.7298 & 0 & -1.3649 \\ -5.4596 & -5.4596 & 0 \\ 7.075 & 7.075 & 7.075 \\ 8 & 8 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} -0.3412 & 0 & -0.1706 \\ -0.6825 & -0.6825 & 0 \\ 0.8844 & 0.8844 & 0.8844 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -4.0947 & 4.0947 & -4.0947 \\ -5.4596 & 5.4596 & 0 \\ 9.095 & 9.095 & 9.095 \\ 10 & 10 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} -0.4095 & 0.4095 & -0.4095 \\ -0.5460 & 0.5460 & 0 \\ 0.9095 & 0.9095 & 0.9095 \\ 1 & 1 & 1 \end{bmatrix}$$

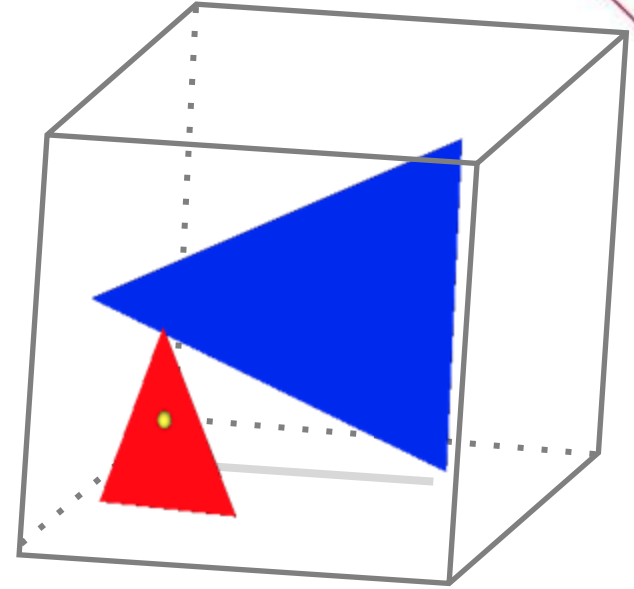
# Aplicando Divisão Homogênea

Projeção em NDC  
(Normalized Device Coordinate)

$$\begin{bmatrix} -1.3649 \\ -2.7298 \\ 7.075 \\ 8 \end{bmatrix} \rightarrow \begin{bmatrix} -0.1706 \\ -0.3412 \\ 0.8844 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2.7298 & 0 & -1.3649 \\ -5.4596 & -5.4596 & 0 \\ 7.075 & 7.075 & 7.075 \\ 8 & 8 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} -0.3412 & 0 & -0.1706 \\ -0.6825 & -0.6825 & 0 \\ 0.8844 & 0.8844 & 0.8844 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -4.0947 & 4.0947 & -4.0947 \\ -5.4596 & 5.4596 & 0 \\ 9.095 & 9.095 & 9.095 \\ 10 & 10 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} -0.4095 & 0.4095 & -0.4095 \\ -0.5460 & 0.5460 & 0 \\ 0.9095 & 0.9095 & 0.9095 \\ 1 & 1 & 1 \end{bmatrix}$$



# Mapeando coordenadas para tela

Escala para (W/2, H/2)

Translate por (1,1)

Espelha em Y

$$\begin{bmatrix} \frac{W}{2} & 0 & 0 & 1 \\ 0 & \frac{H}{2} & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{W}{2} & 0 & 0 & \frac{W}{2} \\ 0 & -\frac{H}{2} & 0 & \frac{H}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Resolução = 40 x 20

$$\text{Coordenadas da Tela} = S_{40,20} \cdot T_{1,1} \cdot E = \begin{bmatrix} \frac{40}{2} & 0 & 0 & 0 \\ 0 & \frac{20}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 0 & 0 & 20 \\ 0 & -10 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Obs: Não há necessidade da matriz ser 4x4.



# Coordenadas de Tela

<Scene>

<Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>

<Transform>

<Shape>

<Sphere radius='0.1'/>

<Appearance>

<Material diffuseColor='1 1 0'/>

</Appearance>

</Shape>

</Transform>

<Transform>

<Shape>

<Triangle Set>

<Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>

</Triangle Set>

<Appearance>

<Material diffuseColor='1 0 0'/>

</Appearance>

</Shape>

</Transform>

<Transform scale="1 3 2"

rotation="1 0 0 -1.57"

translation="2 1 1">

<Shape>

<TriangleSet>

<Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>

</TriangleSet>

<Appearance>

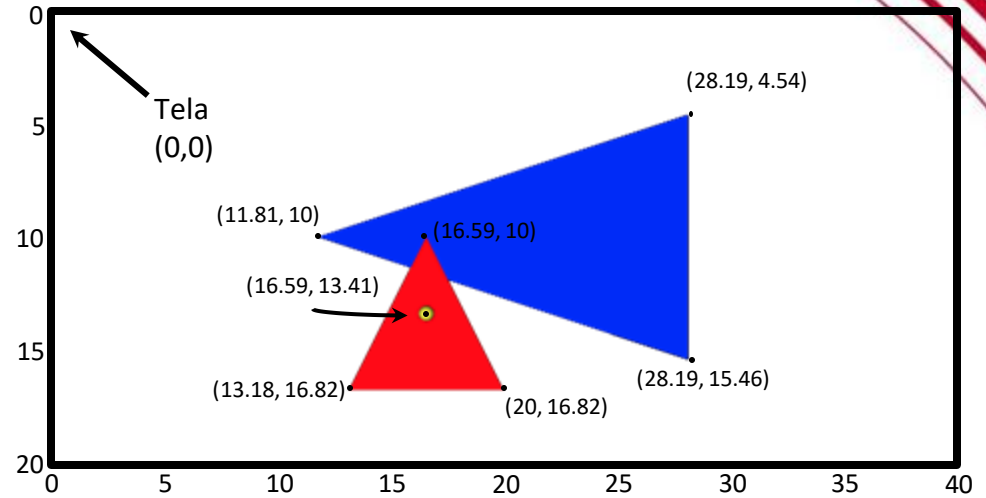
<Material diffuseColor='0 0 1'/>

</Appearance>

</Shape>

</Transform>

</Scene>



Tela

$$\begin{bmatrix} 20 & 0 & 0 & 20 \\ 0 & -10 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -0.1706 \\ -0.3412 \\ 0.8844 \\ 1 \end{bmatrix} = \begin{bmatrix} 16.59 \\ 13.41 \\ 0.88 \\ 1 \end{bmatrix}$$

Tela

$$\begin{bmatrix} 20 & 0 & 0 & 20 \\ 0 & -10 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -0.3412 & 0 & -0.1706 \\ -0.6825 & -0.6825 & 0 \\ 0.8844 & 0.8844 & 0.8844 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 13.18 & 20 & 16.59 \\ 16.82 & 16.82 & 10 \\ 0.88 & 0.88 & 0.88 \\ 1 & 1 & 1 \end{bmatrix}$$

Tela

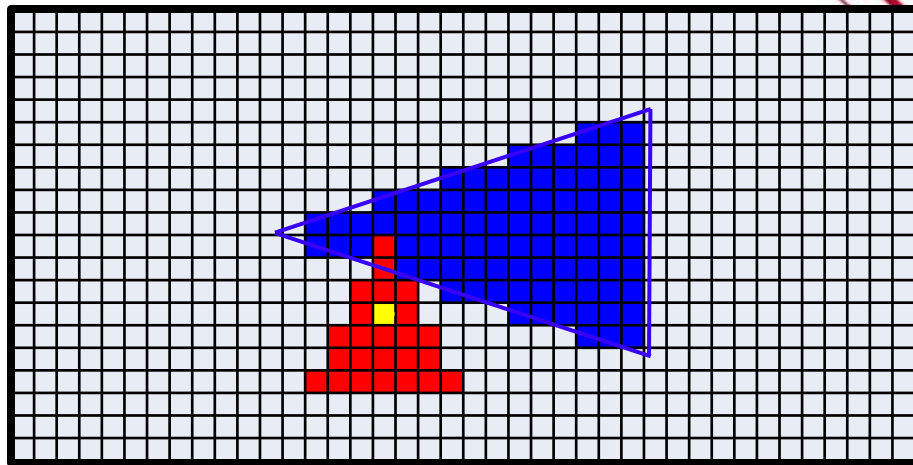
$$\begin{bmatrix} 20 & 0 & 0 & 20 \\ 0 & -10 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -0.4095 & 0.4095 & -0.4095 \\ -0.5460 & 0.5460 & 0 \\ 0.9095 & 0.9095 & 0.9095 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 28.19 & 28.19 & 11.81 \\ 15.46 & 4.54 & 10 \\ 0.91 & 0.91 & 0.91 \\ 1 & 1 & 1 \end{bmatrix}$$

# Renderização

```

<Scene>
  <Viewport position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <Triangle Set>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </Triangle Set>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
    rotation="1 0 0 -1.57"
    translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>

```



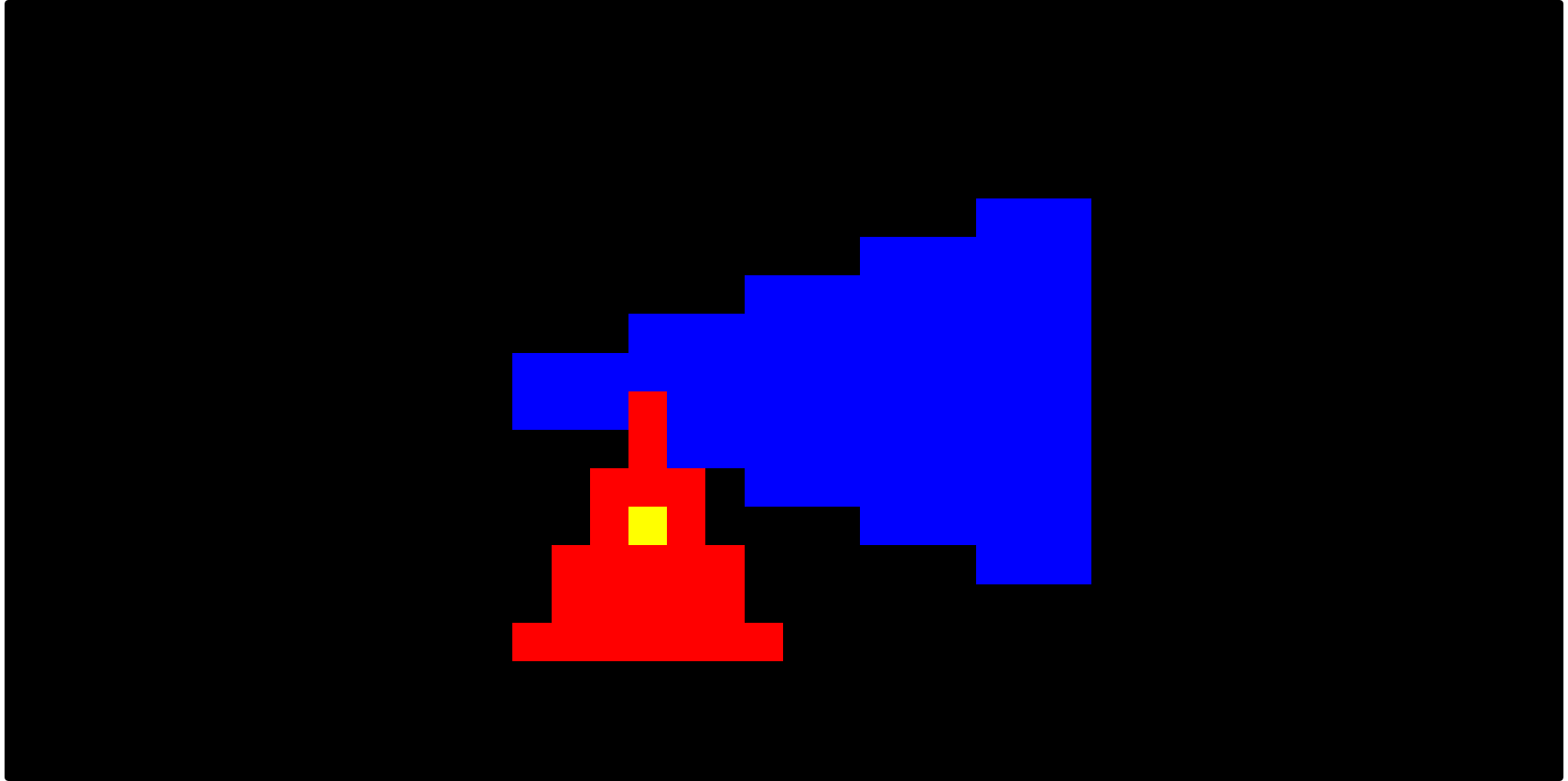
## Rasterizar os Triângulos

Esfera :  $\begin{bmatrix} 16.59 \\ 13.41 \end{bmatrix}$

Triângulo Vermelho :  $\begin{bmatrix} 13.18 & 20 & 16.59 \\ 16.82 & 16.82 & 10 \end{bmatrix}$

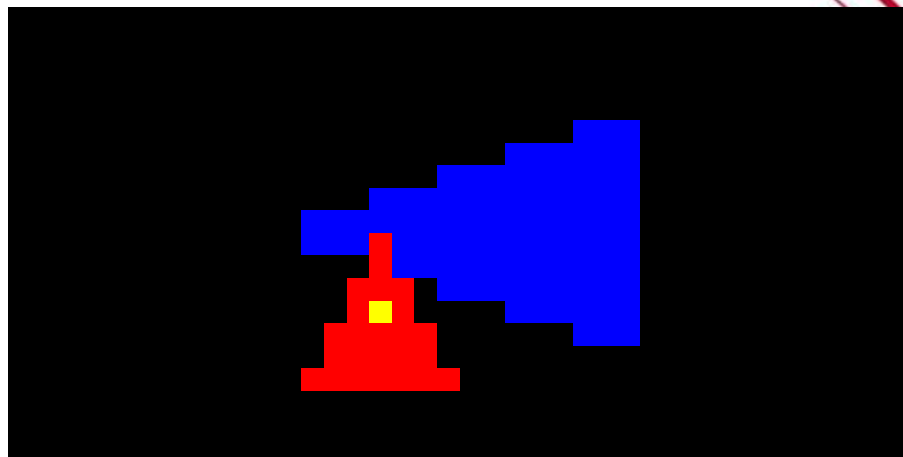
Triângulo Azul :  $\begin{bmatrix} 28.19 & 28.19 & 11.81 \\ 15.46 & 4.54 & 10 \end{bmatrix}$

# Resultado Final (sem anti-aliasing)



Resolução Final: 40 x 20 pixels

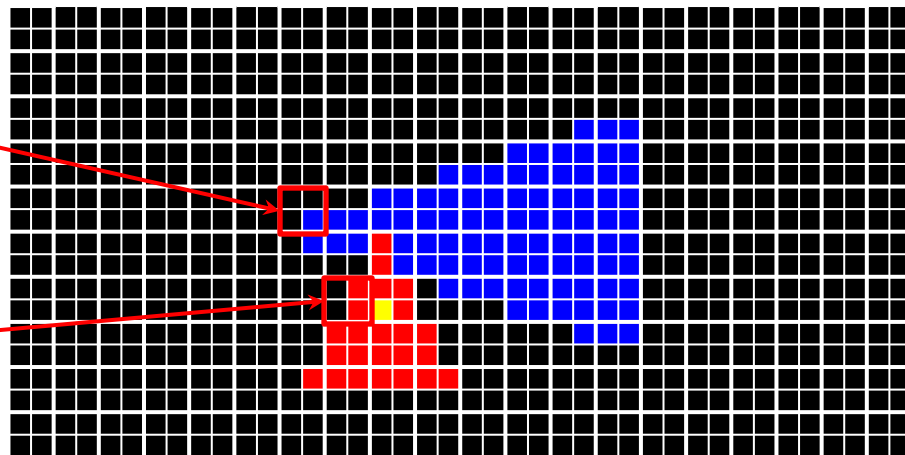
# Anti-Aliasing



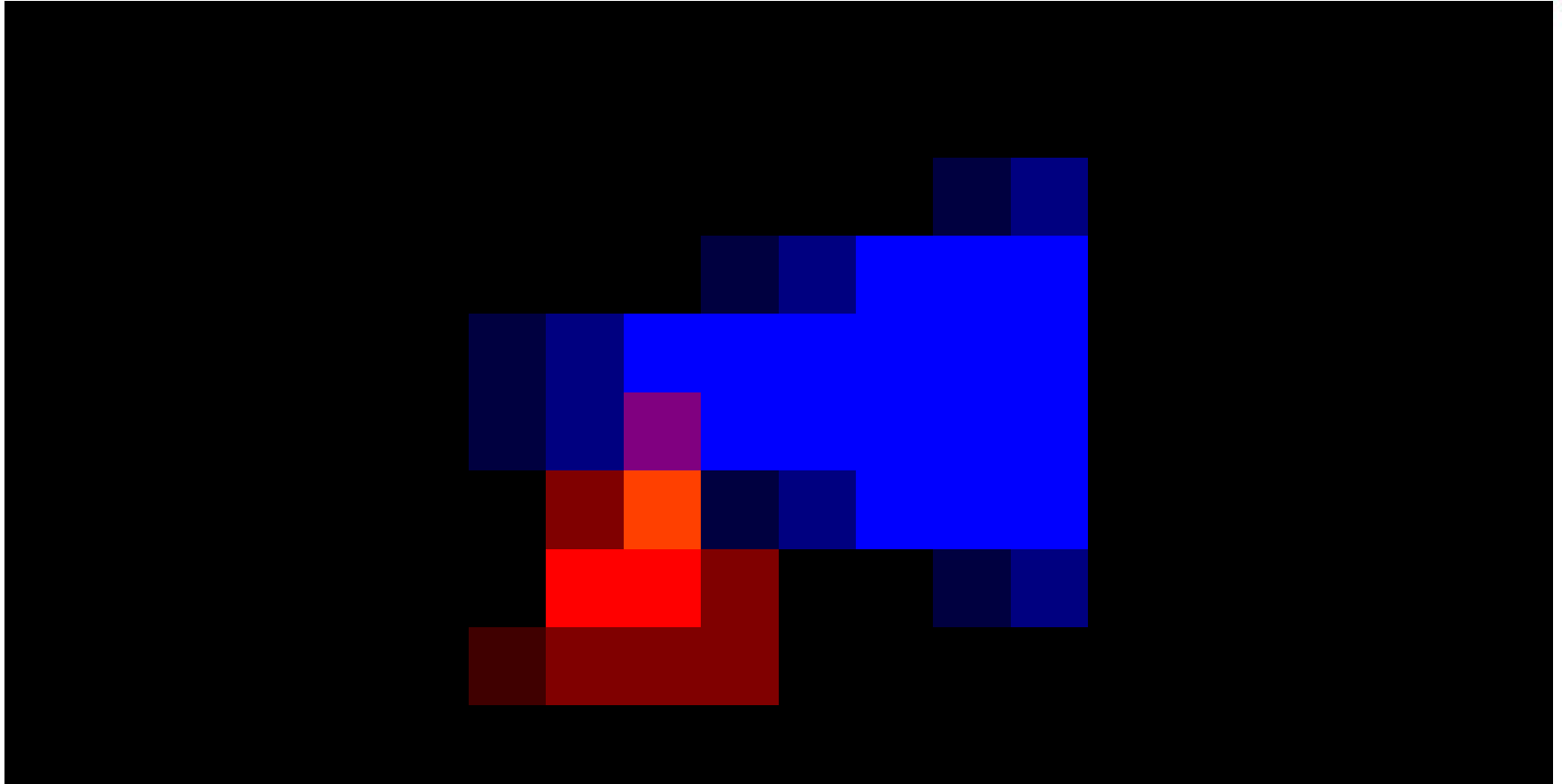
R:0, G:0, B:0  
R:0, G:0, B:0  
R:0, G:0, B:0  
R:0, G:0, B:255  
**R:0, G:0, B:64**

R:0, G:0, B:0  
R:255, G:0, B:0  
R:0, G:0, B:0  
R:255, G:0, B:0  
**R:128, G:0, B:0**

Supersampling 2x2



# Resultado Final (sem anti-aliasing)



Resolução Final: 20 x 10 pixels

# Revisão Numpy

- `numpy.array`: cria matrizes tipo numpy
- `numpy.matmul`: multiplica matrizes numpy
- `*` : multiplica todos os valores da matriz por um escalar
- `/` : divide todos os valores da matriz por um escalar
- `np.empty`: matriz iniciada com valores não iniciados
- `np.zeros`: matriz com todos os valores sendo zero
- `np.ones`: matriz com todos os valores sendo um
- `p.uint8`, `np.uint16`, `np.float32`: tipos de dados para numpy

# Computação Gráfica

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