

Park-Clarke & Space Vector Cheat Sheet (Lipo-Novotny)

1) Conventions (UW Lipo-Novotny)

- q-axis $\parallel \alpha$ at $\theta=0$; d is -90° from q (clockwise).
- θ : electrical angle $\alpha \rightarrow q$, positive CCW; $\omega_e = d\theta/dt$.
- β plotted as $-\text{Imag}\{\cdot\}$; $q = \text{Real}\{\cdot\}$, $d = -\text{Imag}\{\cdot\}$.

2) Frames

- abc: $\{f_a, f_b, f_c\}$ phase quantities (stator).
- $\alpha\beta 0$: $\{f_\alpha, f_\beta, f_0\}$ stationary orthogonal frame (Clarke).
- qd0: $\{f_q, f_d, f_0\}$ rotor-synchronous frame (Park).

3) Clarke ($abc \rightarrow \alpha\beta 0$) — balanced 3ϕ

$$f_\alpha = (2/3)[f_a - 0.5 f_b - 0.5 f_c]$$
$$f_\beta = (2/3)[(\sqrt{3}/2)(f_b - f_c)]$$
$$f_0 = (1/3)(f_a + f_b + f_c)$$

4) Park ($\alpha\beta 0 \rightarrow qd0$) and reverse

Let $c\theta = \cos\theta$, $s\theta = \sin\theta$.

$$f_q = c\theta f_\alpha - s\theta f_\beta; \quad f_d = s\theta f_\alpha + c\theta f_\beta$$
$$f_\alpha = c\theta f_q + s\theta f_d; \quad f_\beta = -s\theta f_q + c\theta f_d$$

5) Complex space vectors

$$a = e^{j2\pi/3}$$
$$f_{\{\alpha\beta\}} = (2/3)(f_a + a f_b + a^2 f_c)$$

Park: $f_{\{qd\}} = f_{\{\alpha\beta\}} e^{-j\theta} \Rightarrow q = \text{Real}\{f_{\{qd\}}\}, d = -\text{Imag}\{f_{\{qd\}}\}$

Back-EMF: $e_{\{\alpha\beta\}} = d\lambda_{\{\alpha\beta\}}/dt = j \omega_e \lambda_{\{\alpha\beta\}}$ (EMF leads flux by 90°)

6) Symbols & meanings

α, β : Clarke axes (stationary)	v : voltage
q, d : Quadrature/Direct axes (rotating)	p : pole pairs
θ : electrical angle $\alpha \rightarrow q$ [rad]	f_0 : zero-sequence component
ω_e : electrical speed [rad/s]	$\text{Real}\{\cdot\}, \text{Imag}\{\cdot\}$: real/imag parts
λ, Λ : flux linkage	$j : \sqrt{-1}$ (90° rotation)
e : back-EMF	$a = e^{j2\pi/3}$: 120° operator
i : current	β plotting: often use $-\text{Imag}\{\cdot\}$
One-page quick ref — UW Lipo-Novotny convention. $e = d\lambda/dt \Rightarrow$ EMF \perp flux; $q \parallel \alpha$ at $\theta=0$.	

7) Common pitfalls

- Using angle-to-d (standard) instead of angle-to-q (Lipo-Novotny).
- Forgetting minus on β or on $d = -\text{Imag}\{\cdot\}$.
- Using mechanical speed ω_m when formulas need $\omega_e = p \cdot \omega_m$.