

Park-Clarke & Space Vector Cheat Sheet (Lipo-Novotny)

1) Conventions (UW Lipo-Novotny)

- q-axis $\parallel \alpha$ at $\theta=0$; d is -90° from q (clockwise).
- θ : electrical angle $\alpha \rightarrow q$, positive CCW; $\omega_e = d\theta/dt$.
- β plotted as $-\text{Imag}\{\cdot\}$; q = Real $\{\cdot\}$, d = $-\text{Imag}\{\cdot\}$.

2) Frames

abc: {f_a,f_b,f_c} phase quantities (stator).

$\alpha\beta 0$: {f_α,f_β,f_0} stationary orthogonal frame (Clarke).

qd0: {f_q,f_d,f_0} rotor-synchronous frame (Park).

3) Clarke (abc → αβ0) — balanced 3φ

$$f_\alpha = (2/3)[f_a - 0.5 f_b - 0.5 f_c]$$

$$f_\beta = (2/3)[(\sqrt{3}/2)(f_b - f_c)]$$

$$f_0 = (1/3)(f_a + f_b + f_c)$$

4) Park ($\alpha\beta 0 \rightarrow qd0$) and reverse

Let $c\theta=\cos\theta$, $s\theta=\sin\theta$.

$$f_q = c\theta f_\alpha - s\theta f_\beta; \quad f_d = s\theta f_\alpha + c\theta f_\beta$$

$$f_\alpha = c\theta f_q + s\theta f_d; \quad f_\beta = -s\theta f_q + c\theta f_d$$

5) Complex space vectors

$$a = e^{j2\pi/3}$$

$$f_{\alpha\beta} = (2/3)(f_a + a f_b + a^2 f_c)$$

$$\text{Park: } f_{qd} = f_{\alpha\beta} e^{-j\theta} \Rightarrow q = \text{Real}\{f_{qd}\}, d = -\text{Imag}\{f_{qd}\}$$

$$\text{Back-EMF: } e_{\alpha\beta} = d\lambda_{\alpha\beta}/dt = j \omega_e \lambda_{\alpha\beta} \quad (\text{EMF leads flux by } 90^\circ)$$

6) Symbols & meanings

α, β : Clarke axes (stationary)

v : voltage

q, d : Quadrature/Direct axes (rotating)

p : pole pairs

θ : electrical angle $\alpha \rightarrow q$ [rad]

f_0 : zero-sequence component

ω_e : electrical speed [rad/s]

Real $\{\cdot\}$, Imag $\{\cdot\}$: real/imag parts

λ, Λ : flux linkage

j : $\sqrt{-1}$ (90° rotation)

e : back-EMF

a = $e^{j2\pi/3}$: 120° operator

i : current

β plotting: often use $-\text{Imag}\{\cdot\}$

One-page quick ref — UW Lipo-Novotny convention. $e = d\lambda/dt \Rightarrow \text{EMF} \perp \text{flux}; q \parallel \alpha$ at $\theta=0$.

7) Common pitfalls

- Using angle-to-d (standard) instead of angle-to-q (Lipo-Novotny).
- Forgetting minus on β or on $d = -\text{Imag}\{\cdot\}$.
- Using mechanical speed ω_m when formulas need $\omega_e = p \cdot \omega_m$.