ECE 411 — DC Machine & Control Systems Cheat Sheet

1) Electrical Model

$$V_a = R_a I_a + L_a \frac{dI_a}{dt} + e$$

$$e = K_e \omega_m \quad \text{(Ke = Kt in SI)}$$

V_a [V], R_a [\Omega], L_a [H], I_a [A], e [V], K_e [\mathrm{V\,s/rad}], ω_m [rad/s]

2) Mechanical Model

$$J_m \frac{d\omega_m}{dt} = T_e - T_L$$

$$T_e = K_t I_a$$

J_m [kg\,m^2], T_e [N\,m], T_L [N\,m], K_t [N\,m/A], ω_m [rad/s]

3) Time Constants

$$\tau_a = \frac{L_a}{R_a}, \qquad \tau_m = \frac{J_m R_a}{K_t^2}$$

τ_a, τ_m [s]

4) Characteristic Equation (open-loop speed dynamics)

$$p^2 + \frac{1}{\tau_a} p + \frac{1}{\tau_a \tau_m} = 0$$

5) Natural Frequency & Damping Ratio

$$\omega_n = \frac{1}{\sqrt{\tau_a \tau_m}}, \qquad \zeta = \frac{1}{2}\sqrt{\frac{\tau_m}{\tau_a}}$$

ω_n [rad/s], ζ [1]

6) Eigenvalues (roots)

$$p_{1,2} = -\frac{1}{2\tau_a} \pm \sqrt{\left(\frac{1}{2\tau_a}\right)^2 - \frac{1}{\tau_a \tau_m}}$$

7) Minimum Inertia for Real-Valued Roots

Require discriminant >= 0 for real roots: tau_m >= 4 tau_a

$$J_{m,\min} = \frac{4 L_a K_t^2}{R_a^2}$$

8) Performance Metrics (2% criterion)

Settling time: $\mathrm{ST} = \frac{4}{\omega_n \zeta}$

Percent overshoot: $\mathrm{OS} = 100 \exp\left(-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}\right)\%$

Assumes viscous friction B ≈ 0 in time-constant formulas; Ke = Kt in SI.

General solution: $x(t) = X e^{pt}$

9) Impedance Analogy (Electrical ↔ Rotational Mechanical)

$V \leftrightarrow T$, $I \leftrightarrow \omega$, $R \leftrightarrow B$, $L \leftrightarrow J$, $C \leftrightarrow 1/B$

10) Variable & Unit Quick Reference

Va : Armature voltage [V]

Ia : Armature current [A]

Ra : Armature resistance [Ω]

La : Armature inductance [H]

e : Back-EMF [V]

Kt : Torque constant [N·m/A]

Ke : Voltage constant [V·s/rad]

Te : Electromagnetic torque [N·m]

Jm : Inertia [kg·m²]

ωm : Angular speed [rad/s]

τa, τm : Time constants [s]

ζ : Damping ratio [–]

ωn : Natural frequency [rad/s]

p : Laplace variable [s⁻¹]