

when $c' \neq c$

$$\begin{aligned}\frac{\partial^2(\omega)}{\partial w_c \partial w_{c'}} &= \left\{ \sum_{j=1}^n x_j \left[I(y_j = c) - P(y_j = c | x_j, w) \right] \right\}' \\&= \sum_{j=1}^n x_j \left[I(y_j = c) - \frac{e^{w_c^T x_j}}{\sum_{c'=1}^C e^{w_{c'}^T x_j}} \right]' \\&= \sum_{j=1}^n x_j (x_j)^T \cdot \frac{e^{w_c^T x_j} \cdot e^{w_{c'}^T x_j}}{\left(\sum_{c'=1}^C e^{w_{c'}^T x_j} \right)^2} \\&= \sum_{j=1}^n (P(y_j = c | x_j, w) \cdot P(y_j = c' | x_j, w) \cdot x_j (x_j)^T)\end{aligned}$$