(C) $$P(Y=1|X) = \frac{P(Y=1)\,P(X|Y=1)}{P(Y=1)P(X|Y=1) + P(Y=0)P(X|Y=0)}$$

$$= \frac{1}{1+\frac{P(Y=0)\,P(X|Y=0)}{P(Y=1)\,P(X|Y=1)}}$$

$$= \frac{1}{1+\exp\left(\ln\frac{P(Y=0)P(X|Y=0)}{P(Y=1)P(X|Y=1)}\right)}$$

suppose:
$P(Y=1)=\pi$
$P(Y=0)=1-\pi$

$$= \frac{1}{1+\exp\left(\ln\frac{P(Y=0)}{P(Y=1)} + \sum_i \ln\frac{P(X_i|Y=0)}{P(X_i|Y=1)}\right)}$$

$$= \frac{1}{1+\exp\left(\ln\frac{1-\pi}{\pi} + \sum_i \ln\frac{P(X_i|Y=0)}{P(X_i|Y=1)}\right)}$$

write $P(X_i=x|Y=1) = \theta_{i1}^{x}(1-\theta_{i1})^{1-x}$

$P(X_i=x|Y=0) = \theta_{i0}^{x}(1-\theta_{i0})^{1-x}$

where θ_{i1}: parameter of feature i ~~under class~~ when $Y=1$.

θ_{i0}: " " " " $Y=0$

Now consider just

$$\sum_i \ln\frac{P(X_i|Y=0)}{P(X_i|Y=1)} = \sum_i \ln\left(\frac{\theta_{i0}}{\theta_{i1}}\right)^{X_i}\cdot\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)^{1-X_i}$$

$$= \sum_i\left[\left(\ln\frac{\theta_{i0}}{\theta_{i1}} - \ln\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)X_i + \ln\frac{1-\theta_{i0}}{1-\theta_{i1}}\right]$$

Let $w_i = \ln\frac{\theta_{i0}}{\theta_{i1}} - \ln\frac{1-\theta_{i0}}{1-\theta_{i1}}$

then $$P(Y=1|X) = \frac{1}{1+\exp\left[\ln\frac{1-\pi}{\pi} + \sum_i\left(w_i X_i + \ln\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)\right]}$$

Let $w_0 = \ln\frac{1-\pi}{\pi} + \sum_i \ln\frac{1-\theta_{i0}}{1-\theta_{i1}}$

then $$P(Y=1|X) = \frac{1}{1+\exp\left(w_0 + \sum_{i=1}^{p} w_i X_i\right)}$$

So we can write $P(Y=1|X)$ in a form that matches the Logistic class distribution

(d) From (c), we show that the value of the weights w_i of LR can be provided in terms of the parameters estimated by the NB classifier. They have the same form. And when we optimize the conditional likelihood, ~~they~~ we get the same classifier.