6.1

1) take one pill from Bottle #1, two pills from Bottle #2, three pills from Bottle #3, and so on.

2) weight those pills on the scale.

3) If all pills were one gram each,the scale would read 210grams(1 + 2+...+20 = 20* 21/2 = 210). Any "overage" must come from the extra 0.1 gram pills.

This formula will tell you the bottle number:

(weight - 210 grams) / 0.1 grams.

So, if the set of pills weighed 211.3 grams,then Bottle #13 would have the heavy pills. Because

211.3 - 210 = 1.3 which is 0.1* 13

6.2

probability of winning Game 1:

p

probability of winning Game 2:

C(3,2) * (1-p) p^2 //probability of making 2 shots,C(3,2) means number ways of selecting 2 out of 3

+ p^3 // probability of making all three shots

Should play game 1 if P(Game1) > P(Game2)

p > 3*(1-p)*p^2 + p^3

from which we get p<0.5

So,

if 0<p<0.5,should play Game 1

if 0.5<p<1,should play Game 2

if p=0 OR 0.5 OR 1, it doesn't matter which game we play.

6.3

It's impossible. The board initially has 32 black and 32 write squares. By moving opposite corners , there only 30 of one color and 32 of the other color. For example, there are 30 black and 32 white squares.

Each domino we set on the board will always take up one white and one black square. Therefore, 31 dominos will take up 31 white squares and 31 black squares exactly. On this board, we only have 30 black squares and 32 white squares. Hence, it is impossible.

6.4

P(collide) = 1- P(not collide)

= 1 - P(same direction) = 1- 2*(1/2)^(n-1)

same direction has two directions: clockwise and counterclockwise

6.5

1,Filled 5-quart jug

2,Filled 3-quart with 5-quart's contents

3,Dumped 3 quart

4, Fill 3 quart with 5 quart contes

5, Filled 5 quart

6, Fill reminder of 3 quart with 5 quart,then get 4 quart

6.6

if x men have blue eyes, it will take c nights for the blue eyed men to leave. All leave on the same night.

6.7 the Apocalypse

G: girl

B: boy

probability of gender sequence:

P(G)=1/2;

P(BG)=1/4;

P(BBG)=1/8;

Expected number of boys each family has:

$$\sum_{i=0}^{\infty} \frac{i}{2^i}$$

when i goes to infinity, the expected number of boy will go to 1.

Since we know that each family has one girl. So this means gender ratio is even.

(The simulation code is on github.)

6.8 The egg drop problems

create a system for dropping Egg1 such that the number of drops is as consistent as possible, whether Egg 1 breaks on the first drop or the last drop.

1. A perfectly load-balanced system would be one in which Drops(Egg 1) , Drops(Egg 2) is always the same, regardless of where Egg 1 breaks.

2. each drop of Egg1 takes one more step,Egg2 is allowed one fewer step.

3. We must, therefore, reduce the number of steps potentially required by Egg 2 by one drop each time.

5. Solve for X :X + (X-l) + (X-2) + ... + 1 = 100

X (X + 1) / 2 = 100 -> X = 14.

We go to floor 14, then 27, then 39, ...This takes 14 steps in the worse case.

6.9 lockers

10 lockers are open at the end.

1) door n is toggled once for each factor of n

2) a door is open if the number of factors is odd (call count)

3) count is odd when n is a perfect square

4) there are 10 perfect squares within 100.

5) so there will be 10 lockers are open at the end.

6.10 poison

1) take each bottle number and look at its binary representation. If theres is a 1 in the i th digit, ten we willed a drip of this bottle’s contents to test script i. Since 2^10=1024 > 1000 so 10 test scripts will be enough to handle 1000 bottles.

2) after 7 days, read the results. If test strip i is positive, the set bit i of the result value. Reading all the test scripts will give us the ID of the poisoned bottle.

(The simulation code is on github.)