Appendices

Day-Ahead Trading Mechanism of Green Hydrogen Based on Tullock Contest

APPENDIX A
PROOF OF THEOREM 2

According to Eq. (3) and Eq. (14), we can obtain that

$$\begin{aligned}
&\Delta u_i\left(b_i, b_i^{'}, \mathbf{b}_{-i}; v_i, \mathbf{v}_{-i}\right) \\
&= u_i\left(b_i, \mathbf{b}_{-i}; v_i, \mathbf{v}_{-i}\right) - u_i\left(b_i^{'}, \mathbf{b}_{-i}; v_i, \mathbf{v}_{-i}\right) \\
&= \left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i}{b_j} g\left(b_i\right) v_i Q_k - b_i Q_k\right) - \left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i^{'}}{b_j} g\left(b_i^{'}\right) v_i Q_k - b_i^{'} Q_k\right)
\end{aligned} \tag{A1}$$

$$\begin{aligned}
&\Delta u_i\left(b_i, b_i^{'}, \mathbf{b}_{-i}; v_i^{'}, \mathbf{v}_{-i}\right) \\
&= u_i\left(b_i, \mathbf{b}_{-i}; v_i, \mathbf{v}_{-i}\right) - u_i\left(b_i^{'}, \mathbf{b}_{-i}; v_i^{'}, \mathbf{v}_{-i}\right) \\
&= \left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i}{b_j} g\left(b_i\right) v_i Q_k - b_i Q_k\right) - \left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i^{'}}{b_j} g\left(b_i^{'}\right) v_i^{'} Q_k - b_i^{'} Q_k\right)
\end{aligned} \tag{A2}$$

Then, we can derive the following:

$$\begin{aligned}
&\Delta u_i\left(b_i, b_i^{'}, \mathbf{b}_{-i}; v_i, \mathbf{v}_{-i}\right) - \Delta u_i\left(b_i, b_i^{'}, \mathbf{b}_{-i}; v_i^{'}, \mathbf{v}_{-i}\right) \\
&= \left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i^{'}}{b_j} g\left(b_i^{'}\right) v_i^{'} Q_k - b_i^{'} Q_k\right) - \left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i^{'}}{b_j} g\left(b_i^{'}\right) v_i Q_k - b_i^{'} Q_k\right) \\
&= \left(v_i^{'} - v_i\right) \sum_{k=1}^{K} \frac{1}{N}\frac{b_i^{'}}{b_j} g\left(b_i^{'}\right) Q_k \\
&\geq \left(v_i^{'} - v_i\right) \left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i^{'}}{b_j} g\left(b_i^{'}\right) Q_k - \sum_{k=1}^{K} \frac{1}{N}\frac{b_i}{b_j} g\left(b_i\right) Q_k\right)
\end{aligned} \tag{A3}$$

Let

$$\varphi_i\left(b_i\right) = \sum_{k=1}^{K} \frac{1}{N}\frac{b_i}{b_j} g\left(b_i\right) Q_k \tag{A4}$$

The first and second order derivatives of $\varphi_i\left(b_i\right)$ with respect to b_i are provided as follows:

$$\frac{\partial \varphi_i\left(b_i\right)}{\partial b_i} = \sum_{k=1}^{K} \frac{1}{N}\frac{1}{b_j} g\left(b_i\right) Q_k + \sum_{k=1}^{K} \frac{1}{N}\frac{b_i}{b_j} g^{'}\left(b_i\right) Q_k \tag{A5}$$

$$\frac{\partial^2 \varphi_i\left(b_i\right)}{\partial \left(b_i\right)^2} = \sum_{k=1}^{K} \frac{2}{N}\frac{1}{b_j} g^{'}\left(b_i\right) Q_k + \sum_{k=1}^{K} \frac{1}{N}\frac{b_i}{b_j} g^{''}\left(b_i\right) Q_k < 0 \tag{A6}$$

Thus, $\varphi_i\left(b_i\right)$ is a concave function of b_i, and we have

$$\varphi_i\left(b_i\right) - \varphi_i\left(b_i^{'}\right) \geq \left(b_i - b_i^{'}\right) \frac{\partial \varphi_i\left(b_i\right)}{\partial b_i} \tag{A7}$$

Consequently, we can derive that

$$\begin{aligned}
&\Delta u_i\left(b_i, b_i^{'}, \mathbf{b}_{-i}; v_i, \mathbf{v}_{-i}\right) - \Delta u_i\left(b_i, b_i^{'}, \mathbf{b}_{-i}; v_i^{'}, \mathbf{v}_{-i}\right) \\
&\geq \left(\sum_{k=1}^{K} \left(\frac{1}{N}\frac{1}{b_j} g\left(b_i\right) + \frac{1}{N}\frac{b_i}{b_j} g^{'}\left(b_i\right)\right) Q_k\right) \left(b_i - b_i^{'}\right)\left(v_i - v_i^{'}\right) \\
&\geq \sum_{k=1}^{K} \frac{1}{N v_{\max}} \left(1 - \sum_{e=0}^{K-1} \frac{1}{N-e}\right) Q_k \left(b_i - b_i^{'}\right)\left(v_i - v_i^{'}\right)
\end{aligned} \tag{A8}$$

We have that

$$\Theta = \sum_{k=1}^{K} \frac{1}{N v_{\max}} \left(1 - \sum_{e=0}^{K-1} \frac{1}{N-e}\right) Q_k \tag{A9}$$

This confirms the condition in Eq. (12).

Next, we will prove the condition in Eq. (13). Using Eq. (3) and Eq. (14), we can derive that

$$\begin{aligned}
&\Delta u_i\left(b_i, b_i^{'}, \mathbf{b}_{-i}; v_i, \mathbf{v}_{-i}\right) - \Delta u_i\left(b_i, b_i^{'}, \mathbf{b}_{-i}^{'}; v_i, \mathbf{v}_{-i}\right) \\
&= \left[\left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i}{b_j} g\left(b_i\right) v_i Q_k - b_i Q_k\right) - \left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i^{'}}{b_j} g\left(b_i^{'}\right) v_i Q_k - b_i^{'} Q_k\right)\right] \\
&\quad - \left[\left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i}{b_j} g\left(b_i\right) v_i Q_k - b_i Q_k\right) - \left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i^{'}}{b_j^{'}} g\left(b_{i,j}^{'}\right) v_i Q_k - b_i^{'} Q_k\right)\right]
\end{aligned} \tag{A10}$$

Define a function as follows:

$$\psi_i\left(\mathbf{b}_{-i}\right) = \left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i}{b_j} g\left(b_i\right) v_i Q_k - b_i Q_k\right) - \left(\sum_{k=1}^{K} \frac{1}{N}\frac{b_i^{'}}{b_j} g\left(b_i^{'}\right) v_i Q_k - b_i^{'} Q_k\right) \tag{A11}$$

Taking the second-order partial derivative of $\psi_i\left(\mathbf{b}_{-i}\right)$ with respect to b_j, we find that it is less than zero. This indicates that $\psi_i\left(\mathbf{b}_{-i}\right)$ is a concave function in terms of $\mathbf{b}_{-i}$. By applying Jensen's inequality, we obtain

$$\psi_i\left(\mathbf{b}_{-i}^{'}\right) \geq \psi_i\left(\mathbf{b}_{-i}\right) + \left(\mathbf{b}_{-i}^{'} - \mathbf{b}_{-i}\right)^T \frac{\partial \psi_i\left(\mathbf{b}_{-i}\right)}{\partial \mathbf{b}_{-i}} \tag{A12}$$

$$\psi_i\left(\mathbf{b}_{-i}\right) \geq \psi_i\left(\mathbf{b}_{-i}^{'}\right) + \left(\mathbf{b}_{-i} - \mathbf{b}_{-i}^{'}\right)^T \frac{\partial \psi_i\left(\mathbf{b}_{-i}^{'}\right)}{\partial \mathbf{b}_{-i}} \tag{A13}$$

From Eq. (A12) and Eq. (A13), we conclude that

$$\begin{aligned}
&\left|\psi_i\left(\mathbf{b}_{-i}\right) - \psi_i\left(\mathbf{b}_{-i}^{'}\right)\right| \leq \\
&\max\left(\left|\left(\mathbf{b}_{-i} - \mathbf{b}_{-i}^{'}\right)^T \frac{\partial \psi_i\left(\mathbf{b}_{-i}\right)}{\partial \mathbf{b}_{-i}}\right|, \left|\left(\mathbf{b}_{-i} - \mathbf{p}_{-i}^{'}\right)^T \frac{\partial \psi_i\left(\mathbf{b}_{-i}^{'}\right)}{\partial \mathbf{b}_{-i}}\right|\right)
\end{aligned} \tag{A14}$$

Then

$$\begin{aligned}
&\left|\Delta u_i\left(b_i, b_i^{'}, \mathbf{b}_{-i}; v_i, \mathbf{v}_{-i}\right) - \Delta u_i\left(b_i, b_i^{'}, \mathbf{b}_{-i}^{'}; v_i, \mathbf{v}_{-i}\right)\right| \\
&\leq \left|\left(b_j - b_j^{'}\right) \sum_{k=1}^{K} Q_k \left(\begin{array}{l} \frac{1}{N}\frac{-b_i}{b_j^2} g\left(b_i\right) + \frac{1}{N}\frac{b_i}{b_j} g^{'}\left(b_i\right)\frac{-b_i}{b_j} \\ + \frac{1}{N}\frac{b_i^{'}}{b_j^2} g\left(b_i^{'}\right) - \frac{1}{N}\frac{b_i^{'}}{b_j} g^{'}\left(b_i^{'}\right)\frac{-b_i}{b_j} \end{array}\right)\right| \\
&\leq \sum_{k=1}^{K} Q_k \left(\frac{1}{N}\frac{1}{b_j^2}\left(b_i^{'} - b_i\right) - \frac{1}{N}\frac{1}{b_j^2} \sum_{e=0}^{K-1} \frac{1}{N-e}\left(b_i - b_i^{'}\right)\right) \left\|\mathbf{b}_{-i} - \mathbf{b}_{-i}^{'}\right\| \\
&\leq \sum_{k=1}^{K} \frac{1}{N}\frac{1}{v_{\max}^2} \left(1 - \sum_{e=0}^{K-1} \frac{1}{N-e}\right) Q_k \left(b_i - b_i^{'}\right) \left\|\mathbf{b}_{-i} - \mathbf{b}_{-i}^{'}\right\|
\end{aligned} \tag{A15}$$

We conclude that

$$\Phi = \sum_{k=1}^{K} \frac{1}{N v_{\max}^{2}} \left(1 - \sum_{e=0}^{K-1} \frac{1}{N-e}\right) Q_k \tag{A16}$$

Thus, the condition in Eq. (13) is verified.

$$\frac{\Theta}{\Phi} = \frac{1}{v_{\max}} \leq 1 \tag{A17}$$

Therefore, we conclude that the proposed contest model has a unique BNE solution, and the proof is completed. ■