- This exam is to be completed in the allotted time period of 2 hours.
- There will only be 20 problems on the actual final exam.
- Read each question carefully.
- Credit may not be given without sufficient supporting work.
- Simplify answers when possible.
- The use of cell phones, books, or notes are not permitted while taking this exam.
- Approved calculators are allowed.

1. [5 points] Find the area and perimeter of a rectangle whose length is 7 meters and whose width is 14 meters. Be sure to include the correct units in your answers.

2. [5 points] A rectangle has a length of 12 inches and an area of 84 square inches. Find the **width** and the **perimeter** of the rectangle. Be sure to include the correct units in each answer.

3. [5 points] Simplify $-4\{x^2 - 2[x - (x - 3x^2)]\}$.

4. [5 points] Simplify $-3[2x^2 - (4x^2 - y)]$.

5. [5 points] Solve for m. Simplify your answer.

$$4 - 7m - 13 = 8m - 3 - 5m$$

6. [5 points] Solve for c. Simplify answers.

$$4(3c + 2) - 7 = -8c - 4$$

7. [4 points] Solve the following equation for y.

$$5 - \frac{1}{3}y = \frac{1}{12}y$$

8. [4 points] Solve the following equation for y.

$$\frac{1}{2}(y-8) = \frac{1}{6}y + 5$$

9. [5 points] Write the following verbal statement in algebraic form. "x plus 5 equals three times the quantity of five times x minus 2"

10. [5 points] Write an algebraic expression for the quantities being compared. "The price of a share of AT&T stock is \$15 less than triple the price of a share of Comcast stock."

11. [5 points] The average weekday high temperature last week was 43°. The high temperatures on Monday through Thursday were 35°, 38°, 44°, and 47°. What was the high temperature on Friday?

12. [5 points] Henry buys a new lawnmower from Amazon. The original price of the lawnmower was \$300, but they are having their end of the year sale, so all lawnmowers are 25% off. The delivery charge on the lawnmower is \$18. What was the cost of the lawnmower, including shipping? Define your variable, solve, and give your answer in a sentence.

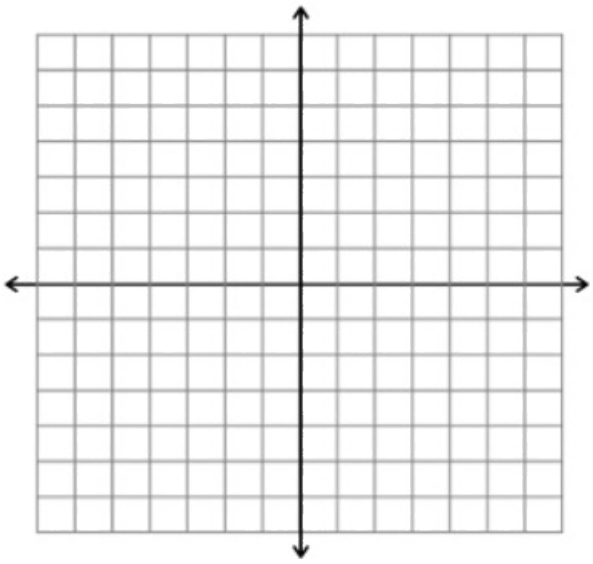
13. [5 points] Solve the inequality for y.

$$-8y + 5 \leq -2y - 7$$

14. [5 points] Solve and graph on the number line.

$$\frac{1}{3}x - 1 \geq \frac{5}{6}x$$

15. [5 points] Graph the line with a slope $\frac{-3}{4}$ that passes through the point $(-1, 2)$. Be sure to label axes with x, y, and with numbers. Identify at least three points on your line.



16. [5 points] Find the slope, y-intercept, and x-intercept of the line.

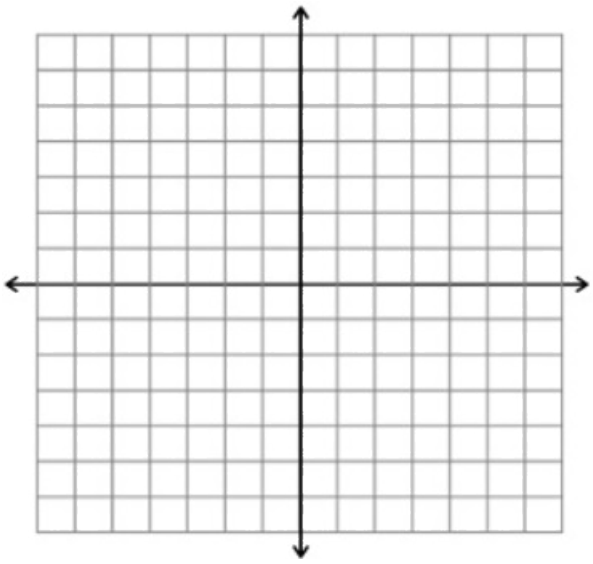
$$10x + 7y = -5$$

17. [5 points] Find the equation of the line that passes through $(-8, 4)$ and is perpendicular to the line $y = 4x + 12$.

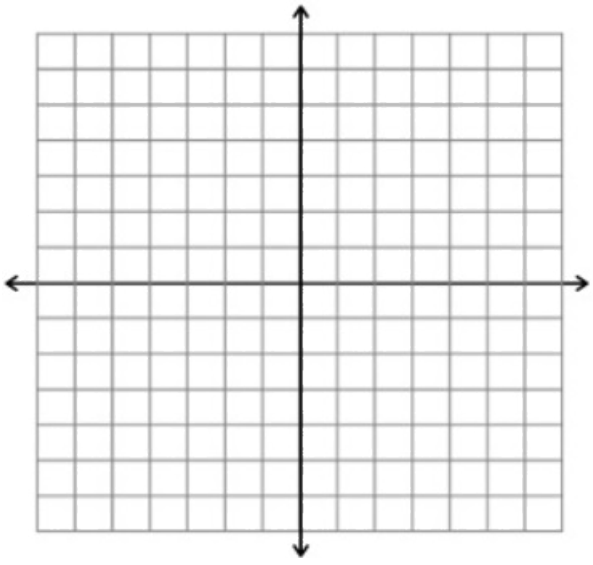
18. [5 points] Find an equation of the line that passes through $(-10, 9)$ and $(-8, 25)$.

19. [5 points] Solve by graphing the given system of equations.Be sure to label axes with x, y, and with numbers. Identify and label the intersection point.

$$\begin{cases} 3x - 2y = 6 \\ 4x + y = -3 \end{cases}$$



20. [5 points] Solve the system of equations by graphing. Be sure to label axes with x, y, and numbers. Identify and label the point of intersection.

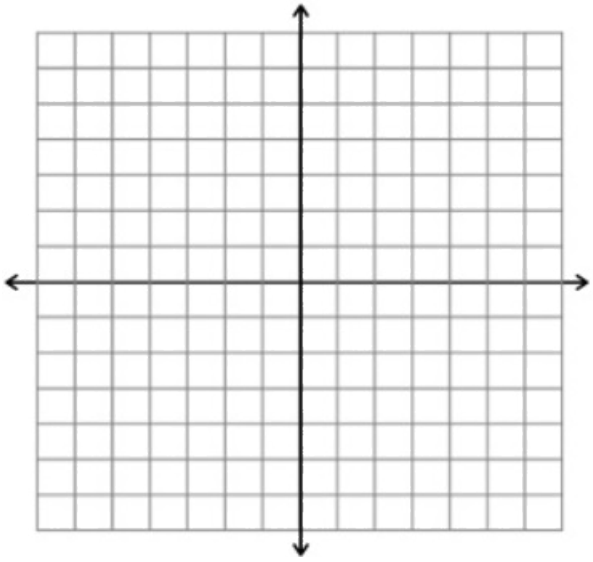


$$\begin{cases} y = \frac{1}{3}x - \frac{13}{3} \\ y = -x + 1 \end{cases}$$

21. [6 points] Ninety-eight passengers rode in an Amtrak train from Boston to Denver. Tickets for regular coach seats cost \$120. Tickets for sleeper car seats cost \$290. The receipts for the trip totaled \$19,750. How many passengers purchased regular coach seats? How many passengers purchased sleeper seats?

22. [6 points] On Friday, Leah picked up 8 cruellers and 16 cups of tea for the office staff and paid a total of \$36.56. On Saturday, Leah picked up 4 cruellers and 12 cups of tea (from the same coffee shop) and paid a total of \$22.64. How much does the coffee shop charge for one crueller? How much do they charge for one cup of tea?

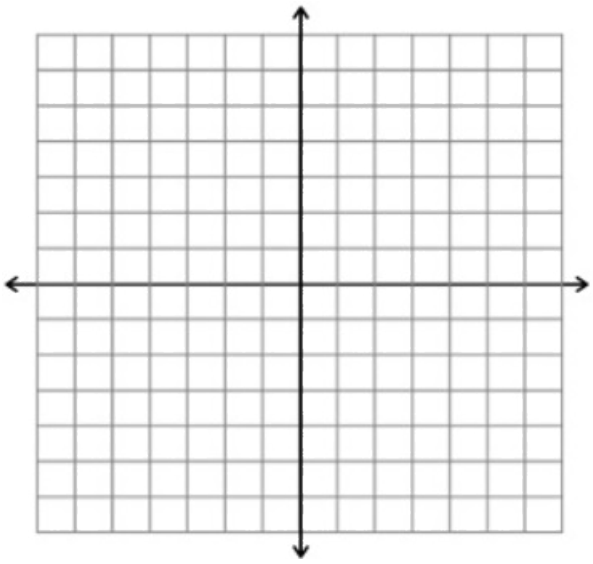
23. [5 points] Graph the solution to the following system of inequalities. Be sure to label axes with x, y, and with numbers. Identify and label the intersection.



$$\begin{cases} y < -x + 5 \\ y \geq 2x - 1 \end{cases}$$

24. [5 points] Graph the solution to the system of inequalities. Be sure to label the x and y axes.

$$\begin{cases} y \leq -x + 3 \\ x > 4 \end{cases}$$



25. [5 points] Multiply and simplify your answer.

$$6x^{-3}y^{6}z^{-4} \cdot 4x^{5}y^{-4}z^{5} \cdot 3x^{5}$$

26. [5 points] Multiply and simplify your answer.

$$6x^{3}y^{8}z^{4} \cdot 3x^{7}y^{9}z^{13} \cdot 5x^{10}$$

27. [5 points]

    (a) Rewrite without an exponent: $(-12)^{-2}$

    (b) Rewrite without using a negative exponent: $-8y^{-10}$

28. [5 points] Simplify. Express your answer with positive exponents. Assume that all variables are nonzero.

$$\frac{x^{-4}y^{-2}z^{4}}{z^{-5}}.$$

29. [5 points]

    (a) Write 8,540,200,000 in scientific notation.

    (b) Write 0.000016403 in scientific notation.

30. [5 points]

    (a) Write 5.412×10^{-6} in decimal notation.

    (b) Write 8.31×10^{7} in decimal notation.

31. [5 points] Identify the degree and leading coefficient of the polynomial.

$$2x^2 - 10 + 2x^6 - 5x^4 + 3x$$

Degree:_____
Leading Coefficient: _____

32. [5 points] Identify the degree and leading coefficient of the polynomial.

$$8x^7 - 3x^5 + 2x^3 - 2x^2$$

Degree:_____
Leading Coefficient: _____

33. [5 points] Simplify $(-10u^2 + 4u + 5) - (-12u - 3u^2 - 13) + (6 - 3u - 2u^2)$.

34. [5 points] Simplify $(-8r^2 + 9r - 14) - 4(7r - 9r^2 - 6)$.

35. [5 points] Multiply and simplify $(2x - 3)(3x - 5)$.

36. [5 points] Multiply and simplify $-6xy^2(7x^2 - 5y + 4y^2)$.

37. [5 points] Simplify. Express your answer with only positive exponents.

$$\frac{18a^3b^8c^2}{12b^4c^8}$$

38. [5 points] Simplify. Express your answer with only positive exponents.

$$\frac{(2ab^8c^2)^3}{20c^{12}}$$

39. [5 points] Divide. Write your answer in standard form, $Q(x) + \dfrac{R}{3x}$.

$$(24x^3 - 6x^2 - 12x + 8) \div (3x)$$

40. [5 points] Using long division, divide.

$$(12x^3 + 5x^2 + 15x - 6) \div (3x - 1)$$

Solutions

1. Find the area and perimeter of a rectangle whose length is 7 meters and whose width is 14 meters. Be sure to include the correct units in your answers.

Area $= 7 \times 14 = 98\ m^2$	2.5 pts
If missing units take off 0.5 pt	
Perimeter $= 2(7 + 14) = 42$ meters	2.5 pts
If missing units take off 0.5 pt	

2. A rectangle has a length of 12 inches and an area of 84 square inches. Find the **width** and the **perimeter** of the rectangle. Be sure to include the correct units in each answer.

Area $= \ell \times w$	
$84 = 12w$	
Width $= \dfrac{84}{12} = 7$ inches	2.5 pts; if missing units, deduct 0.5 pt.
Perimeter $= 2(12 + 7) = 38$ inches	2.5 pts; if missing units, deduct 0.5 pt.

3. Simplify $-4\{x^2 - 2[x - (x - 3x^2)]\}$.

$-4\{x^2 - 2[x - x + 3x^2]\}$	1 pt
$-4\{x^2 - 2[3x^2]\}$	2 pts to here
$-4\{x^2 - 6x^2\}$	3 pts to here
$-4\{-5x^2\}$	4 pts to here
$20x^2$	5 pts to here

4. Simplify $-3[2x^2 - (4x^2 - y)]$.

$-3[2x^2 - 4x^2 + y]$	2 pts to here
$-3[-2x^2 + y]$	3 pts to here
$6x^2 - 3y$	5 pts to here

5. Solve for m. Simplify your answer.

$$4 - 7m - 13 = 8m - 3 - 5m$$

$7m - 9 = 3m - 3$	1 pt to here
$7m - 3m = -3 + 9$	2 pts to here
$4m = 6$	3 pts to here
$m = 6/4$	4 pts to here
$m = 3/2$ or $1\frac{1}{2}$	5 pts total

6. Solve for c. Simplify answers.

$$4(3c + 2) - 7 = -8c - 4$$

$12c + 8 - 7 = -8c - 4$	1 pt to here
$12c + 8c = -8 + 7 - 4$	2 pts to here
$20c = -5$	3 pts to here
$c = -\dfrac{5}{20}$	4 pts to here
$c = -\dfrac{1}{4}$	5 pts total

7. Solve the following equation for y.

$$5 - \frac{1}{3}y = \frac{1}{12}y$$

$12(5 - \frac{1}{3}y) = 12(\frac{1}{12})y$	1 pts to here
$60 - 4y = y$	2 pts to here
$60 = 5y$	3 pts to here
$12 = y$	4 pts total

8. Solve the following equation for y.

$$\frac{1}{2}(y - 8) = \frac{1}{6}y + 5$$

$\frac{1}{2}y - 4 = \frac{1}{6}y + 5$	1 pt to here
$6\left(\frac{1}{2}y - 4\right) = 6\left(\frac{1}{6}y + 5\right)$	2 pts to here
$3y - 24 = y + 30$	3 pts to here
$y = 27$	4 pts total

9. Write the following verbal statement in algebraic form. "x plus 5 equals three times the quantity of five times x minus 2"

$x + 5 = 3(5x - 2)$	5 pts
No partial credit.	

10. Write an algebraic expression for the quantities being compared. "The price of a share of AT&T stock is \$15 less than triple the price of a share of Comcast stock."

C = price of a share of Comcast stock (dollars)	2 pts
$3C - 15$ = price of a share of AT&T stock (dollars)	3 pts
Deduct 1 point if units are missing.	

11. The average weekday high temperature last week was 43°. The high temperatures on Monday through Thursday were 35°, 38°, 44°, and 47°. What was the high temperature on Friday?

$\frac{35+38+44+47+x}{5} = 43$	2 pts to here
$164 + x = 215$	3 pts to here
$x = 215 - 164 = 51$	4 pts to here
It was 51° on Friday.	5 pts total

12. Henry buys a new lawnmower from Amazon. The original price of the lawnmower was \$300, but they are having their end of the year sale, so all lawnmowers are 25% off. The delivery charge on the lawnmower is \$18. What was the cost of the lawnmower, including shipping? Define your variable, solve, and give your answer in a sentence.

$\ell =$ the discounted price with the shipping charge	1 pt
$0.25(\$300) = \$75$ the 25% discount	1 pt
$\ell = \$300 - \$75 + \$18$	1 pt
$\ell = \$243$	1 pt
The cost of his lawnmower, including shipping, was \$243.	1 pt
Only take off 0.5 points if no dollar sign in the sentence.	

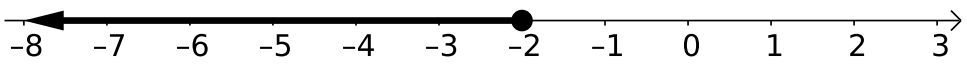
13. Solve the inequality for y.

$$-8y + 5 \leq -2y - 7$$

$-8y + 2y \leq -7 - 5$ (or $5 + 7 \leq -2y + 8y$)	2 pts to here
$-6y \leq -12$ (or $12 \leq 6y$)	4 pts to here
$y \geq 2$ (or $2 \leq y$)	5 pts total

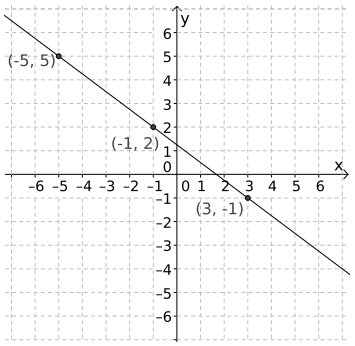
14. Solve and graph on the number line.

$$\frac{1}{3}x - 1 \geq \frac{5}{6}x$$



$\frac{6}{1}\left(\frac{1}{3}x - 1\right) \geq \frac{6}{1}\left(\frac{5}{6}x\right)$	1 pt to here
$2x - 6 \geq 5x$	2 pts to here
$x \leq -2$ OR $-2 \geq x$	3 pts to here
	add 2 pts for correct number line.

15. Graph the line with a slope $\frac{-3}{4}$ that passes through the point $(-1, 2)$. Be sure to label axes with x, y, and with numbers. Identify at least three points on your line.



1 point for correct labeling of axes and numbers on them. 3 points for correctly identifying 3 pts. 1 pt for the correct line.

16. Find the slope, y-intercept, and x-intercept of the line.

$$10x + 7y = -5$$

slope: $-\dfrac{10}{7}$	1 pt
y-intercept: $\left(0, -\dfrac{5}{7}\right)$	2 pts
x-intercept: $\left(-\dfrac{1}{2}, 0\right)$	2 pts

17. Find the equation of the line that passes through $(-8, 4)$ and is perpendicular to the line $y = 4x + 12$.

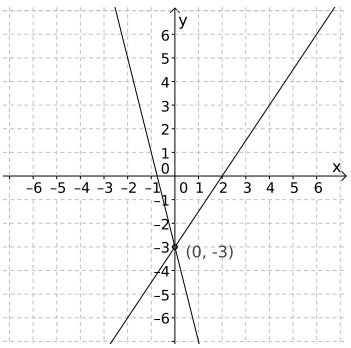
Slope of the line $y = 4x + 12$ is 4	1 pt to here
Perpendicular slope: $m = -1/4$	2 pts to here
$4 = -\dfrac{1}{4}(-8) + b$	3 pts to here
$4 = 2 + b$ so $b = 2$	4 pts to here
$y = \dfrac{-1}{4}x + 2$	5 pts total

18. Find an equation of the line that passes through $(-10, 9)$ and $(-8, 25)$.

$m = \dfrac{25 - 9}{-8 + 10}$	1 pt
$m = 8$	1 pt
$y = 8x + 89$	3 pts for
$y - 25 = 8(x + 8)$	correct equation
$y - 9 = 8(x + 10)$	(any of these)

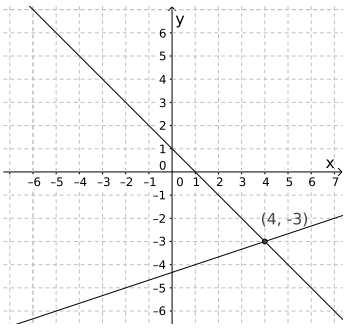
19. Solve by graphing the given system of equations.Be sure to label axes with x, y, and with numbers. Identify and label the intersection point.

$$\begin{cases} 3x - 2y = 6 \\ 4x + y = -3 \end{cases}$$



Correct system is graphed	award 2 pts
Axes are labeled	award 1 pt
Intersection point $(0, -3)$	award 2 pts

20. Solve the system of equations by graphing. Be sure to label axes with x, y, and numbers. Identify and label the point of intersection.



$$\begin{cases} y = \dfrac{1}{3}x - \dfrac{13}{3} \\ y = -x + 1 \end{cases}$$

Correct system is graphed	award 2 pts
Axes are labeled	award 1 pt
Intersection point $(-5, -2)$	award 2 pts

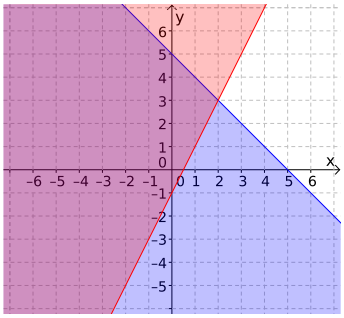
21. Ninety-eight passengers rode in an Amtrak train from Boston to Denver. Tickets for regular coach seats cost \$120. Tickets for sleeper car seats cost \$290. The receipts for the trip totaled \$19,750. How many passengers purchased regular coach seats? How many passengers purchased sleeper seats?

Let $x =$ the number of regular coach seats	
and $y =$ the number of sleeper seats	1 pt to here
$x + y = 98$ and $120x + 290y = 19750$	2 pts to here
$x = 98 - y$ and $120(98 - y) + 290y = 19750$	3 pts to here
$11760 - 120y + 290y = 19750$	
$11760 + 170y = 19750$	4 pts up to here
$170y = 7990$	
$y = 47$ so $x = 98 - 47 = 51$	5 pts up to here
There were 47 sleeper seats sold	
and 51 regular coach seats sold.	6 pts total

22. On Friday, Leah picked up 8 cruellers and 16 cups of tea for the office staff and paid a total of \$36.56. On Saturday, Leah picked up 4 cruellers and 12 cups of tea (from the same coffee shop) and paid a total of \$22.64. How much does the coffee shop charge for one crueller? How much do they charge for one cup of tea?

Let $x =$ the price of a crueller (in dollars)	
and $y =$ the price of a cup of tea (in dollars)	1 pt to here
$8x + 16y = 36.56$ and $4x + 12y = 22.64$	3 pts to here
$x = 2.39$ and $y = 1.09$	5 pts to here
One crueller costs \$2.39	
and one cup of tea costs \$1.09.	6 pts total

23. Graph the solution to the following system of inequalities. Be sure to label axes with x, y, and with numbers. Identify and label the intersection.

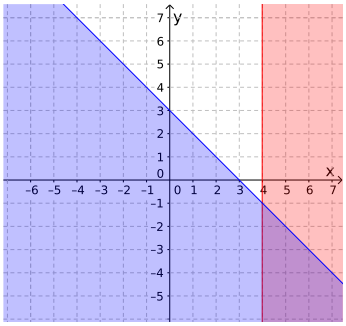


$$\begin{cases} y < -x + 5 \\ y \geq 2x - 1 \end{cases}$$

1 pt for each correct inequality (2 pts total)
2 pts for correct intersection
1 pt for the x and y axis labels

24. Graph the solution to the system of inequalities. Be sure to label the x and y axes.

$$\begin{cases} y \leq -x + 3 \\ x > 4 \end{cases}$$



1 pt for each correct line 1 pt for each correct shading 1 pts for the x and y axis labels (5 pts total)

25. Multiply and simplify your answer.

$$6x^{-3}y^{6}z^{-4} \cdot 4x^{5}y^{-4}z^{5} \cdot 3x^{5}$$

$72x^{7}y^{2}z$	1 pt for each variable with correct exponent and 2 pts for the number 72

26. Multiply and simplify your answer.

$$6x^{3}y^{8}z^{4} \cdot 3x^{7}y^{9}z^{13} \cdot 5x^{10}$$

$90x^{20}y^{17}z^{17}$	1 pt for each variable with correct exponent and 2 pts for the number 90

27. (a) Rewrite without an exponent: $(-12)^{-2}$

    (b) Rewrite without using a negative exponent: $-8y^{-10}$

(a) $\dfrac{1}{144}$	2.5 pts No partial credit.
(b) $\dfrac{-8}{y^{10}}$	2.5 pts No partial credit.

28. Simplify. Express your answer with positive exponents. Assume that all variables are nonzero.

$$\frac{x^{-4}y^{-2}z^{4}}{z^{-5}}.$$

$\dfrac{z^{5}z^{4}}{x^{4}y^{2}}$	1 pt each for variables x and y (with positive exponent)
$\dfrac{z^{9}}{x^{4}y^{2}}$	2 pts for getting exponent of z correct and 1 pt extra for getting it all correct.

29. (a) Write 8,540,200,000 in scientific notation.

    (b) Write 0.000016403 in scientific notation.

(a) 8.5402×10^9	2.5 pts
	No partial credit.
(b) 1.6403×10^{-5}	2.5 pts
	No partial credit.

30. (a) Write 5.412×10^{-6} in decimal notation.

    (b) Write 8.31×10^7 in decimal notation.

(a) 0.000005412	2.5 pts
	No partial credit.
(b) 83,100,000	2.5 pts
	No partial credit.

31. Identify the degree and leading coefficient of the polynomial.

$$2x^2 - 10 + 2x^6 - 5x^4 + 3x$$

Degree:_____
Leading Coefficient: _____

Degree: 6	2.5 pts
Leading Coefficient: 2	2.5 pts
No partial credit.	

32. Identify the degree and leading coefficient of the polynomial.

$$8x^7 - 3x^5 + 2x^3 - 2x^2$$

Degree:_____
Leading Coefficient: _____

Degree: 7	2.5 pts
Leading Coefficient: 8	2.5 pts
No partial credit.	

33. Simplify $(-10u^2 + 4u + 5) - (-12u - 3u^2 - 13) + (6 - 3u - 2u^2)$.

$-10u^2 + 4u + 5 + 12u + 3u^2 + 13 + 6 - 3u - 2u^2$	2 pts to here
$= -9u^2 + 13u + 24$	5 pts total
Partial credit: 1.5 pt for each correct term	

34. Simplify $(-8r^2 + 9r - 14) - 4(7r - 9r^2 - 6)$.

$-8r^2 + 9r - 14 - 28r + 36r^2 + 24$	3 pts to here
$= 28r^2 - 19r + 10$	5 pts total

35. Multiply and simplify $(2x-3)(3x-5)$.

$6x^2-10x-9x+15$	3 pts to here
$=6x^2-19x+15$	5 pts total

36. Multiply and simplify $-6xy^2(7x^2-5y+4y^2)$.

$-6xy^2(7x^2)-6xy^2(-5y)-6xy^2(4y^2)$	2 pts to here
$=-42x^3y^2+30xy^3-24xy^4$	1 pt for each correct term

37. Simplify. Express your answer with only positive exponents.

$$\frac{18a^3b^8c^2}{12b^4c^8}$$

$\frac{3a^3b^4}{2c^6}$	1 pt for each variable with correct exponent
	2 pts for the number $\frac{3}{2}$

38. Simplify. Express your answer with only positive exponents.

$$\frac{(2ab^8c^2)^3}{20c^{12}}$$

$\frac{2a^3b^{24}}{5c^6}$	1 pt for each variable with correct exponent
	2 pts for the number $\frac{2}{5}$ (or anything equal to $\frac{2}{5}$)

39. Divide. Write your answer in standard form, $Q(x)+\frac{R}{3x}$.

$$(24x^3-6x^2-12x+8)\div(3x)$$

$\frac{24x^3}{3x}-\frac{6x^2}{3x}-\frac{12x}{3x}+\frac{8}{3x}$	3 pts to here
$8x^2-2x-4+\frac{8}{3x}$	5 pts total

40. Using long division, divide.

$$(12x^3+5x^2+15x-6)\div(3x-1)$$

Set up long division properly	2 points
$4x^2+3x+6$	1 point for each correct term