

- This exam is to be completed in the allotted time period of 2 hours.
- There will only be 20 problems on the actual final exam.
- Read each question carefully.
- Credit may not be given without sufficient supporting work.
- Simplify answers when possible.
- The use of cell phones, books, or notes are not permitted while taking this exam.
- Approved calculators are allowed.

1. [5 points] Find the area and perimeter of a rectangle whose length is 7 meters and whose width is 14 meters. Be sure to include the correct units in your answers.

2. [5 points] A rectangle has a length of 14 inches and an area of 84 square inches. Find the **width** and the **perimeter** of the rectangle. Be sure to include the correct units in each answer.

3. [5 points] Simplify $-4\{x^2 - 2[x - (x - 3x^2)]\}$.

4. [5 points] Simplify $-4[3x^2 - (7x^2 - 2y)]$.

5. [5 points] Solve for m . Simplify your answer.

$$6m + 8 - 3m = 11 - 12m - 13$$

6. [5 points] Solve for c . Simplify answers.

$$4(3c + 2) - 7 = -8c - 4$$

7. [4 points] Solve the following equation for y .

$$5 - \frac{1}{3}y = \frac{1}{12}y$$

8. [4 points] Solve the following equation for y .

$$\frac{1}{3}(y - 9) = \frac{1}{6}y + 2$$

9. [5 points] Write the following verbal statement in algebraic form. “ x plus 5 equals three times the quantity of five times x minus 2”
10. [5 points] Write an algebraic expression for the quantities being compared. “The price of a share of AT&T stock is \$15 less than triple the price of a share of Comcast stock.”
11. [5 points] The average weekday high temperature last week was 43° . The high temperatures on Monday through Thursday were 35° , 38° , 44° , and 47° . What was the high temperature on Friday?
12. [5 points] Henry buys a new lawnmower from Amazon. The original price of the lawnmower was \$300, but they are having their end of the year sale, so all lawnmowers are 25% off. The delivery charge on the lawnmower is \$18. What was the cost of the lawnmower, including shipping? Define your variable, solve, and give your answer in a sentence.

13. [5 points] Solve the inequality for y .

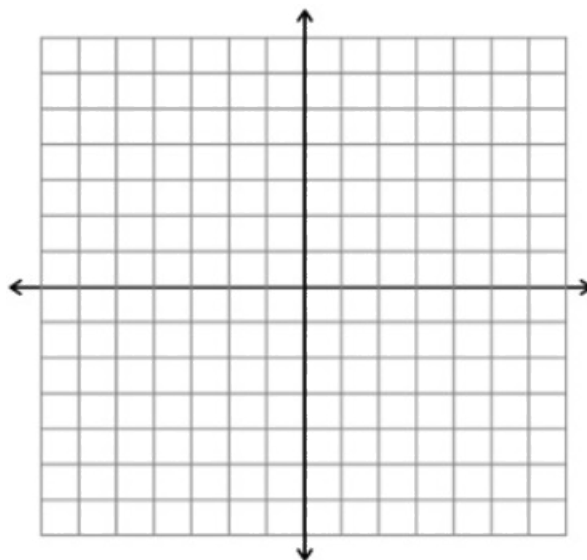
$$-8y + 5 \leq -2y - 7$$

14. [5 points] Solve and graph on the number line.

$$\frac{7}{6}x - 3 \geq \frac{2}{3}x$$



15. [5 points] Graph the line with slope $-\frac{2}{3}$ that passes through the point $(2, -1)$. Be sure to label axes with x , y , and with numbers. Identify at least three points on your line.

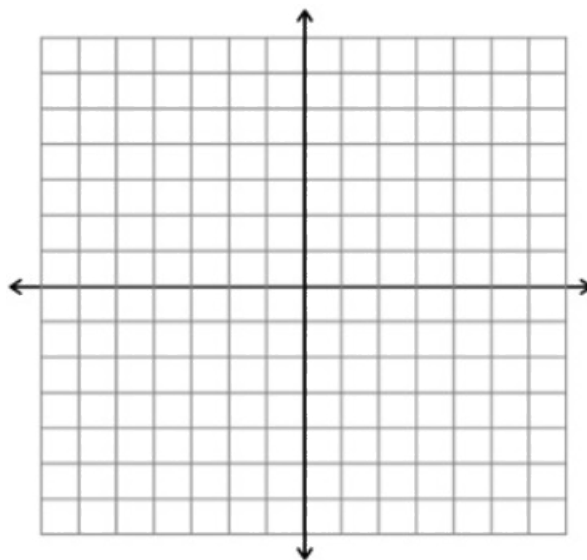


16. [5 points] Find the slope, y -intercept, and x -intercept of the line.

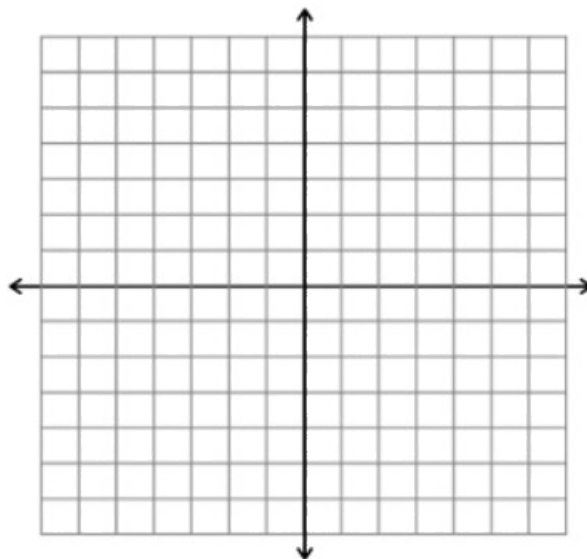
$$10x + 7y = -5$$

17. [5 points] Find the equation of the line that passes through $(-8, 4)$ and is perpendicular to the line $y = 4x + 12$.
18. [5 points] Find an equation of the line that passes through $(-10, 9)$ and $(-8, 25)$.
19. [5 points] Solve by graphing the given system of equations. Be sure to label axes with x , y , and with numbers. Identify and label the intersection point.

$$\begin{cases} 3x - y = 5 \\ 2x - 3y = -6 \end{cases}$$



20. [5 points] Solve the system of equations by graphing. Be sure to label axes with x , y , and numbers. Identify and label the point of intersection.

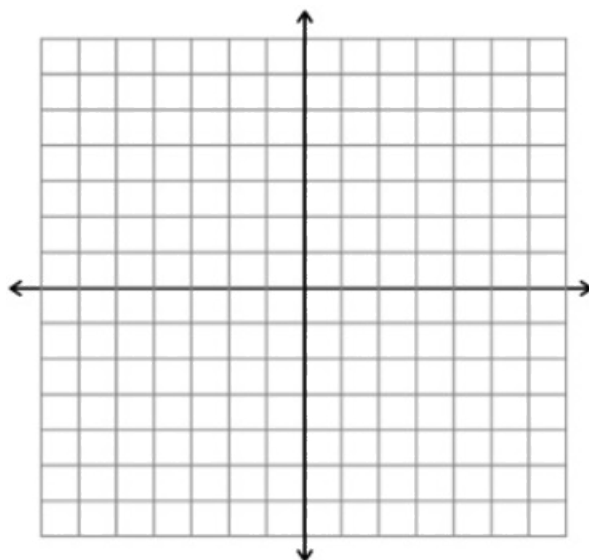


$$\begin{cases} y = \frac{1}{3}x - \frac{13}{3} \\ y = -x + 1 \end{cases}$$

21. [6 points] Ninety-eight passengers rode in an Amtrak train from Boston to Denver. Tickets for regular coach seats cost \$120. Tickets for sleeper car seats cost \$290. The receipts for the trip totaled \$19,750. How many passengers purchased regular coach seats? How many passengers purchased sleeper seats?

22. [6 points] On Monday, Tiana picked up 9 muffins and 18 cups of tea for the office staff and paid a total of \$44.73. On Saturday, Tiana picked up 10 muffins and 30 cups of tea (from the same coffee shop) and paid a total of \$64.60. How much does the coffee shop charge for one muffin? How much do they charge for one cup of tea?

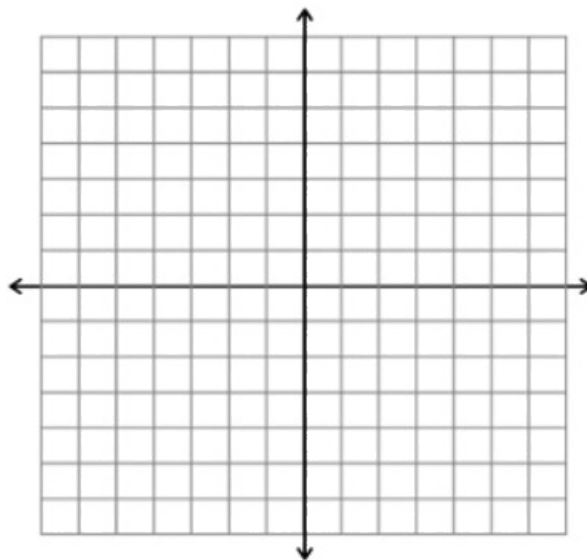
23. [5 points] Graph the solution to the following system of inequalities. Be sure to label axes with x , y , and with numbers. Identify and label the intersection.



$$\begin{cases} y < -x + 5 \\ y \geq 2x - 1 \end{cases}$$

24. [5 points] Graph the solution to the system of inequalities. Be sure to label the x and y axes.

$$\begin{cases} y \leq -3x - 3 \\ y > 2 \end{cases}$$



25. [5 points] Multiply and simplify your answer.

$$6x^{-3}y^6z^{-4} \cdot 4x^5y^{-4}z^5 \cdot 3x^5$$

26. [5 points] Multiply and simplify your answer.

$$4x^8y^6z^5 \cdot 6x^{12}y^7z^4 \cdot 3z^6$$

27. [5 points]

(a) Rewrite without an exponent: $(-12)^{-2}$

(b) Rewrite without using a negative exponent: $-8y^{-10}$

28. [5 points] Simplify. Express your answer with positive exponents. Assume that all variables are nonzero.

$$\frac{x^4 y^{-3}}{x^{-2} z^{-5}}.$$

29. [5 points]

(a) Write 8,540,200,000 in scientific notation.

(b) Write 0.000016403 in scientific notation.

30. [5 points]

(a) Write 5.412×10^{-6} in decimal notation.

(b) Write 8.31×10^7 in decimal notation.

31. [5 points] Identify the degree and leading coefficient of the polynomial.

$$5 + 6x^9 + 4x^8 + 3x^7 - 2x$$

Degree: _____

Leading Coefficient: _____

32. [5 points] Identify the degree and leading coefficient of the polynomial.

$$8x^7 - 3x^5 + 2x^3 - 2x^2$$

Degree: _____

Leading Coefficient: _____

33. [5 points] Simplify $(-10u^2 + 4u + 5) - (-12u - 3u^2 - 13) + (6 - 3u - 2u^2)$.

34. [5 points] Simplify $(-9r^2 + 5r - 16) - 2(12r - 15r^2 - 8)$.

35. [5 points] Multiply and simplify $(2x - 3)(3x - 5)$.

36. [5 points] Multiply and simplify $-6xy^2(7x^2 - 5y + 4y^2)$.

37. [5 points] Simplify. Express your answer with only positive exponents.

$$\frac{18a^3b^8c^2}{12b^4c^8}$$

38. [5 points] Simplify. Express your answer with only positive exponents.

$$\frac{(2ab^8c^2)^3}{20c^{12}}$$

39. [5 points] Divide. Write your answer in standard form, $Q(x) + \frac{R}{3x}$.

$$(18x^3 - 9x^2 + 3x + 5) \div (3x)$$

40. [5 points] Using long division, divide.

$$(12x^3 + 5x^2 + 15x - 6) \div (3x - 1)$$

Solutions

1. Find the area and perimeter of a rectangle whose length is 7 meters and whose width is 14 meters. Be sure to include the correct units in your answers.

$$\begin{array}{ll} \text{Area} = 7 \times 14 = 98 \text{ m}^2 & 2.5 \text{ pts} \\ \text{If missing units take off 0.5 pt} & \\ \text{Perimeter} = 2(7 + 14) = 42 \text{ meters} & 2.5 \text{ pts} \\ \text{If missing units take off 0.5 pt} & \end{array}$$

2. A rectangle has a length of 14 inches and an area of 84 square inches. Find the **width** and the **perimeter** of the rectangle. Be sure to include the correct units in each answer.

$$\begin{array}{ll} \text{Area} = \ell \times w & \\ 84 = 14w & \\ \text{Width} = \frac{84}{14} = 6 \text{ inches} & 2.5 \text{ pts; if missing units, deduct 0.5 pt.} \\ \text{Perimeter} = 2(14 + 6) = 40 \text{ inches} & 2.5 \text{ pts; if missing units, deduct 0.5 pt.} \end{array}$$

3. Simplify $-4\{x^2 - 2[x - (x - 3x^2)]\}$.

$$\begin{array}{ll} -4\{x^2 - 2[x - x + 3x^2]\} & 1 \text{ pt} \\ -4\{x^2 - 2[3x^2]\} & 2 \text{ pts to here} \\ -4\{x^2 - 6x^2\} & 3 \text{ pts to here} \\ -4\{-5x^2\} & 4 \text{ pts to here} \\ 20x^2 & 5 \text{ pts to here} \end{array}$$

4. Simplify $-4[3x^2 - (7x^2 - 2y)]$.

$$\begin{array}{ll} -4[3x^2 - 7x^2 + 2y] & 2 \text{ pts to here} \\ -4[-4x^2 + 2y] & 3 \text{ pts to here} \\ 16x^2 - 8y & 5 \text{ pts to here} \end{array}$$

5. Solve for m . Simplify your answer.

$$6m + 8 - 3m = 11 - 12m - 13$$

$$\begin{array}{ll} 3m + 8 = -12m - 2 & 1 \text{ pt} \\ 3m + 12m = -2 - 8 & 2 \text{ pts to here} \\ 15m = -10 & 3 \text{ pts to here} \\ m = - = 10/15 & 4 \text{ pts to here} \\ m = -2/3 & 5 \text{ pts total} \end{array}$$

6. Solve for c . Simplify answers.

$$4(3c + 2) - 7 = -8c - 4$$

$12c + 8 - 7 = -8c - 4$	1 pt to here
$12c + 8c = -8 + 7 - 4$	2 pts to here
$20c = -5$	3 pts to here
$c = -\frac{5}{20}$	4 pts to here
$c = -\frac{1}{4}$	5 pts total

7. Solve the following equation for y .

$$5 - \frac{1}{3}y = \frac{1}{12}y$$

$12(5 - \frac{1}{3}y) = 12(\frac{1}{12})y$	1 pts to here
$60 - 4y = y$	2 pts to here
$60 = 5y$	3 pts to here
$12 = y$	4 pts total

8. Solve the following equation for y .

$$\frac{1}{3}(y - 9) = \frac{1}{6}y + 2$$

$\frac{1}{3}y - 3 = \frac{1}{6}y + 2$	1 pt to here
$6(\frac{1}{3}y - 3) = 6(\frac{1}{6}y + 2)$	2 pts to here
$2y - 18 = y + 12$	3 pts to here
$y = 30$	4 pts total

9. Write the following verbal statement in algebraic form. “ x plus 5 equals three times the quantity of five times x minus 2”

$x + 5 = 3(5x - 2)$	5 pts
No partial credit.	

10. Write an algebraic expression for the quantities being compared. “The price of a share of AT&T stock is \$15 less than triple the price of a share of Comcast stock.”

$C = \text{price of a share of Comcast stock (dollars)}$	2 pts
$3C - 15 = \text{price of a share of AT\&T stock (dollars)}$	3 pts
Deduct 1 point if units are missing.	

11. The average weekday high temperature last week was 43° . The high temperatures on Monday through Thursday were 35° , 38° , 44° , and 47° . What was the high temperature on Friday?

$\frac{35+38+44+47+x}{5} = 43$	2 pts to here
$164 + x = 215$	3 pts to here
$x = 215 - 164 = 51$	4 pts to here
It was 51° on Friday.	5 pts total

12. Henry buys a new lawnmower from Amazon. The original price of the lawnmower was \$300, but they are having their end of the year sale, so all lawnmowers are 25% off. The delivery charge on the lawnmower is \$18. What was the cost of the lawnmower, including shipping? Define your variable, solve, and give your answer in a sentence.

ℓ = the discounted price with the shipping charge	1 pt
$0.25(\$300) = \75 (the 25% discount)	1 pt
$\ell = \$300 - \$75 + \$18$	1 pt
$\ell = \$243$	1 pt
The cost of his lawnmower, including shipping, was \$243.	1 pt
Only take off 0.5 points if no dollar sign in the sentence.	

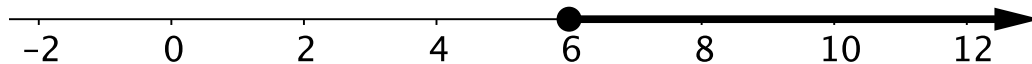
13. Solve the inequality for y .

$$-8y + 5 \leq -2y - 7$$

$-8y + 2y \leq -7 - 5$ (or $5 + 7 \leq -2y + 8y$)	2 pts to here
$-6y \leq -12$ (or $12 \leq 6y$)	4 pts to here
$y \geq 2$ (or $2 \leq y$)	5 pts total

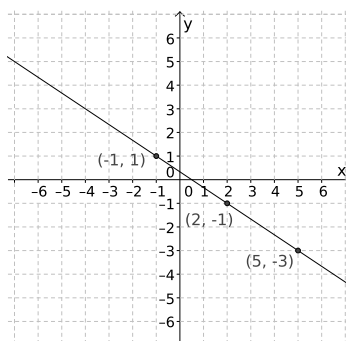
14. Solve and graph on the number line.

$$\frac{7}{6}x - 3 \geq \frac{2}{3}x$$



$\frac{6}{1} \left(\frac{7}{6}x - 3 \right) \geq \frac{6}{1} \left(\frac{2}{3}x \right)$	1 pt to here
$7x - 18 \geq 4x$	2 pts to here
$x \geq 6$ OR $6 \leq x$	3 pts to here
add 2 pts for correct number line.	

15. Graph the line with slope $-\frac{2}{3}$ that passes through the point $(2, -1)$. Be sure to label axes with x , y , and with numbers. Identify at least three points on your line.



1 point for correct labeling of axes and numbers on them.
 3 points for correctly identifying 3 pts.
 1 pt for the correct line.

16. Find the slope, y -intercept, and x -intercept of the line.

$$10x + 7y = -5$$

slope: $-\frac{10}{7}$ 1 pt
 y -intercept: $\left(0, -\frac{5}{7}\right)$ 2 pts
 x -intercept: $\left(-\frac{1}{2}, 0\right)$ 2 pts

17. Find the equation of the line that passes through $(-8, 4)$ and is perpendicular to the line $y = 4x + 12$.

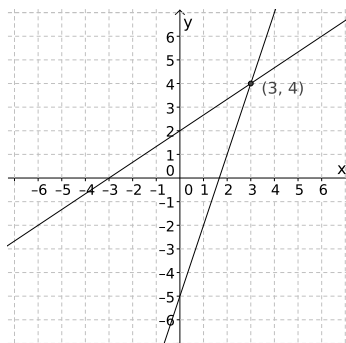
Slope of the line $y = 4x + 12$ is 4 1 pt to here
 Perpendicular slope: $m = -1/4$ 2 pts to here
 $4 = -\frac{1}{4}(-8) + b$ 3 pts to here
 $4 = 2 + b$ so $b = 2$ 4 pts to here
 $y = \frac{-1}{4}x + 2$ 5 pts total

18. Find an equation of the line that passes through $(-10, 9)$ and $(-8, 25)$.

$m = \frac{25 - 9}{-8 + 10}$ 1 pt
 $m = 8$ 1 pt
 $y = 8x + 89$ 3 pts for
 $y - 25 = 8(x + 8)$ correct equation
 $y - 9 = 8(x + 10)$ (any of these)

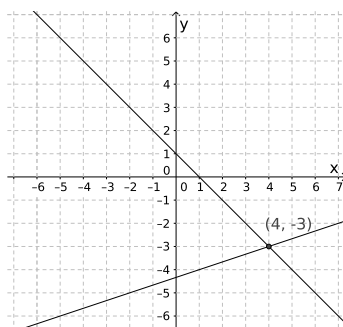
19. Solve by graphing the given system of equations. Be sure to label axes with x , y , and with numbers. Identify and label the intersection point.

$$\begin{cases} 3x - y = 5 \\ 2x - 3y = -6 \end{cases}$$



Correct system is graphed	award 2 pts
Axes are labeled	award 1 pt
Intersection point (3, 4)	award 2 pts

20. Solve the system of equations by graphing. Be sure to label axes with x , y , and numbers. Identify and label the point of intersection.



$$\begin{cases} y = \frac{1}{3}x - \frac{13}{3} \\ y = -x + 1 \end{cases}$$

Correct system is graphed	award 2 pts
Axes are labeled	award 1 pt
Intersection point $(-5, -2)$	award 2 pts

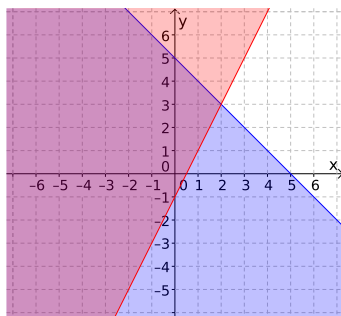
21. Ninety-eight passengers rode in an Amtrak train from Boston to Denver. Tickets for regular coach seats cost \$120. Tickets for sleeper car seats cost \$290. The receipts for the trip totaled \$19,750. How many passengers purchased regular coach seats? How many passengers purchased sleeper seats?

Let x = the number of regular coach seats	
and y = the number of sleeper seats	1 pt to here
$x + y = 98$ and $120x + 290y = 19750$	2 pts to here
$x = 98 - y$ and $120(98 - y) + 290y = 19750$	3 pts to here
$11760 - 120y + 290y = 19750$	
$11760 + 170y = 19750$	4 pts up to here
$170y = 7990$	
$y = 47$ so $x = 98 - 47 = 51$	5 pts up to here
There were 47 sleeper seats sold	
and 51 regular coach seats sold.	6 pts total

22. On Monday, Tiana picked up 9 muffins and 18 cups of tea for the office staff and paid a total of \$44.73. On Saturday, Tiana picked up 10 muffins and 30 cups of tea (from the same coffee shop) and paid a total of \$64.60. How much does the coffee shop charge for one muffin? How much do they charge for one cup of tea?

Let x = the price of a muffin (in dollars)	
and y = the price of a cup of tea (in dollars)	1 pt to here
$9x + 18y = 44.73$ and $10x + 30y = 64.60$	3 pts to here
$x = 1.99$ and $y = 1.49$	5 pts to here
One muffin costs \$1.99	
and one cup of tea costs \$1.49.	6 pts total

23. Graph the solution to the following system of inequalities. Be sure to label axes with x , y , and with numbers. Identify and label the intersection.

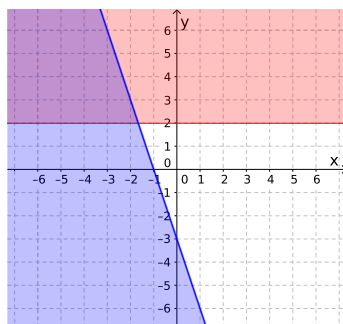


$$\begin{cases} y < -x + 5 \\ y \geq 2x - 1 \end{cases}$$

1 pt for each correct inequality (2 pts total)
2 pts for correct intersection
1 pt for the x and y axis labels

24. Graph the solution to the system of inequalities. Be sure to label the x and y axes.

$$\begin{cases} y \leq -3x - 3 \\ y > 2 \end{cases}$$



1 pt for each correct line
 1 pt for each correct shading
 1 pt for the x and y axis labels (5 pts total)

25. Multiply and simplify your answer.

$$6x^{-3}y^6z^{-4} \cdot 4x^5y^{-4}z^5 \cdot 3x^5$$

$72x^7y^2z$ 1 pt for each variable with correct exponent
 and 2 pts for the number 72

26. Multiply and simplify your answer.

$$4x^8y^6z^5 \cdot 6x^{12}y^7z^4 \cdot 3z^6$$

$72x^{20}y^{13}z^{15}$ 1 pt for each variable with correct exponent
 and 2 pts for the number 72

27. (a) Rewrite without an exponent: $(-12)^{-2}$

(b) Rewrite without using a negative exponent: $-8y^{-10}$

(a) $\frac{1}{144}$ 2.5 pts
 No partial credit.
 (b) $\frac{-8}{y^{10}}$ 2.5 pts
 No partial credit.

28. Simplify. Express your answer with positive exponents. Assume that all variables are nonzero.

$$\frac{x^4y^{-3}}{x^{-2}z^{-5}}$$

$\frac{x^4x^2z^5}{y^3}$ 1 pt each for variables y and z (with positive exponent)

$$\frac{x^6z^5}{y^3}$$

2 pts for getting exponent of x correct
 and 1 pt extra for getting it all correct.

29. (a) Write 8,540,200,000 in scientific notation.
 (b) Write 0.000016403 in scientific notation.

(a) 8.5402×10^9	2.5 pts
No partial credit.	
(b) 1.6403×10^{-5}	2.5 pts
No partial credit.	

30. (a) Write 5.412×10^{-6} in decimal notation.
 (b) Write 8.31×10^7 in decimal notation.

(a) 0.000005412	2.5 pts
No partial credit.	
(b) 83,100,000	2.5 pts
No partial credit.	

31. Identify the degree and leading coefficient of the polynomial.

$$5 + 6x^9 + 4x^8 + 3x^7 - 2x$$

Degree: _____

Leading Coefficient: _____

Degree: 9	2.5 pts
Leading Coefficient: 6	2.5 pts
No partial credit.	

32. Identify the degree and leading coefficient of the polynomial.

$$8x^7 - 3x^5 + 2x^3 - 2x^2$$

Degree: _____

Leading Coefficient: _____

Degree: 7	2.5 pts
Leading Coefficient: 8	2.5 pts
No partial credit.	

33. Simplify $(-10u^2 + 4u + 5) - (-12u - 3u^2 - 13) + (6 - 3u - 2u^2)$.

$-10u^2 + 4u + 5 + 12u + 3u^2 + 13 + 6 - 3u - 2u^2$	2 pts to here
$= -9u^2 + 13u + 24$	5 pts total
Partial credit: 1.5 pt for each correct term	

34. Simplify $(-9r^2 + 5r - 16) - 2(12r - 15r^2 - 8)$.

$-9r^2 + 5r - 16 - 24r + 30r^2 + 16$	3 pts to here
$= 21r^2 - 19r$	5 pts total

35. Multiply and simplify
- $(2x - 3)(3x - 5)$
- .

$6x^2 - 10x - 9x + 15$ $= 6x^2 - 19x + 15$	3 pts to here 5 pts total
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36. Multiply and simplify
- $-6xy^2(7x^2 - 5y + 4y^2)$
- .

$-6xy^2(7x^2) - 6xy^2(-5y) - 6xy^2(4y^2)$ $= -42x^3y^2 + 30xy^3 - 24xy^4$	2 pts to here 1 pt for each correct term
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37. Simplify. Express your answer with only positive exponents.

$$\frac{18a^3b^8c^2}{12b^4c^8}$$

$\frac{3a^3b^4}{2c^6}$	1 pt for each variable with correct exponent 2 pts for the number $\frac{3}{2}$
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38. Simplify. Express your answer with only positive exponents.

$$\frac{(2ab^8c^2)^3}{20c^{12}}$$

$\frac{2a^3b^{24}}{5c^6}$	1 pt for each variable with correct exponent 2 pts for the number $\frac{2}{5}$ (or anything equal to $\frac{2}{5}$)
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39. Divide. Write your answer in standard form,
- $Q(x) + \frac{R}{3x}$
- .

$$(18x^3 - 9x^2 + 3x + 5) \div (3x)$$

$\frac{18x^3}{3x} - \frac{9x^2}{3x} - \frac{3x}{3x} + \frac{5}{3x}$	3 pts to here
$6x^2 - 3x + 1 + \frac{5}{3x}$	5 pts total

40. Using long division, divide.

$$(12x^3 + 5x^2 + 15x - 6) \div (3x - 1)$$

Set up long division properly $4x^2 + 3x + 6$	2 points 1 point for each correct term
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