Name and section: _____

Instructor's name: _____

- **Please do not open exam until instructed to begin.**
- This exam is to be completed in the allotted time period of 2 hours.
- There are 24 problems which appear on the fronts and backs of the pages of this exam.
- You may earn a total of 100 points.
- Read each question carefully.
- Credit may not be given without sufficient supporting work.
- Simplify answers when possible.
- The use of cell phones, books, or notes are not permitted while taking this exam.
- Calculators are allowed.

1. [4 points] Factor $4x^2 - 8x - 5$.

2. [4 points] Factor $6ax + 48bx - ay - 8by$.

3. [4 points] Simplify $\dfrac{x^3 + 5x^2}{x^2 - 2x - 35}$.

4. [4 points] Simplify $\dfrac{16x^3}{12x^2 - 12} \div \dfrac{8x^2}{x^2 - 2x + 1}$.

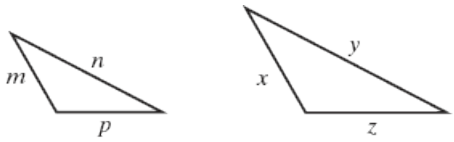
5. [4 points] Simplify $\dfrac{2}{x-8} - \dfrac{x}{x+3}$

6. [4 points] Simplify $\dfrac{3x+5}{x^2+4x+3} + \dfrac{-x+5}{x^2+2x-3}$.

7. [4 points] Simplify $\dfrac{\frac{a}{3b} - \frac{1}{2}}{\frac{7}{3b} - \frac{4}{a}}$.

8. [4 points] Solve $\dfrac{2}{x^2-1} + \dfrac{5}{x+1} = \dfrac{3}{x-1}$.

9. [4 points] Triangles A and B are similar.



Triangle A Triangle B

If $z = 18$ in., $y = 25$ in., and $n = 9$ in., find the length of side p. Leave your answer as a fraction.

10. [4 points] Combine. Assume that all variables represent nonnegative real numbers. $\sqrt{28x} - \sqrt{147x} + \sqrt{75x}$

11. [4 points] Simplify $\dfrac{\sqrt{5}+3}{\sqrt{5}-3}$.

12. [4 points] Solve $\sqrt{x+10} - 10 = x$.

13. [4 points] Simplify $\sqrt{-25}$.

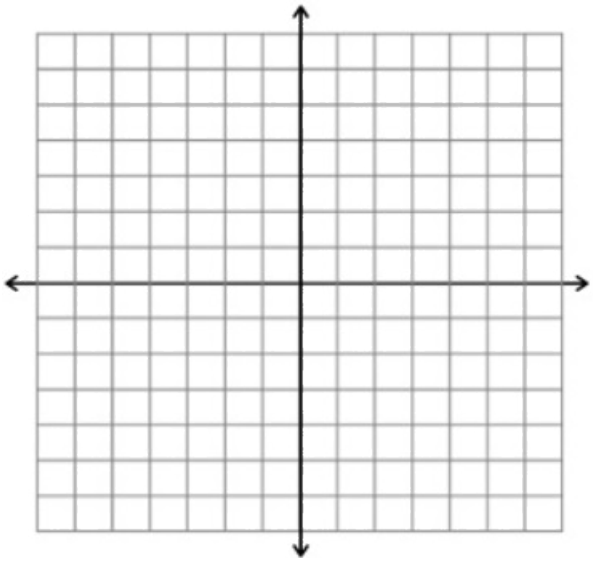
14. [4 points] y varies directly as x and inversely as the square of z. If $y = 64$ when $x = 32$ and $z = 4$, find y when $x = 75$ and $z = 5$.

15. [4 points] Solve by using the square root property. $(2x + 5)^2 = 81$.

16. [4 points] A company that manufactures bikes makes a daily profit, P, according to the equation $P(x) = -100x^2 + 4700x - 49449$ where P is measured in dollars and x is the number of mountain bikes made per day. Find the number of mountain bikes that must be made each day to produce a zero profit for the company. Round your answer to the nearest whole number.

17. [4 points] A brace for a shelf has the shape of a right triangle. Its hypotenuse is 18 inches long and the two legs are equal in length. How long are the legs of the triangle? Keep answers in simplified radical form.

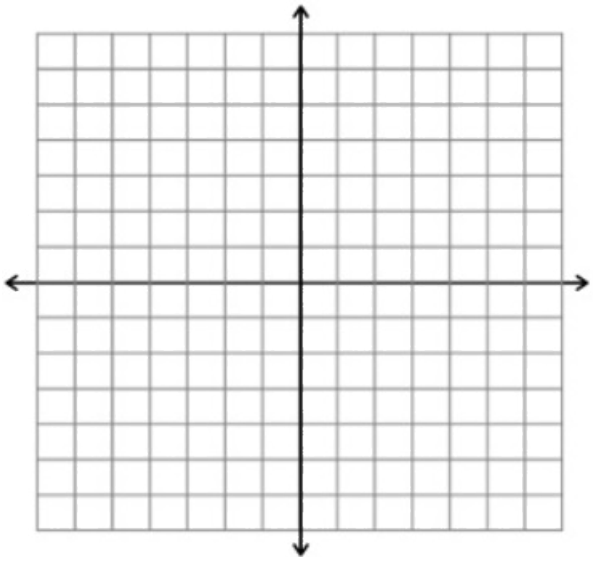
18. [8 points] Given $f(x) = x^2 + 2x - 3$. Identify the vertex, y-intercept, x-intercept(s), axis of symmetry, and graph the function on the graph paper and label your findings on it.



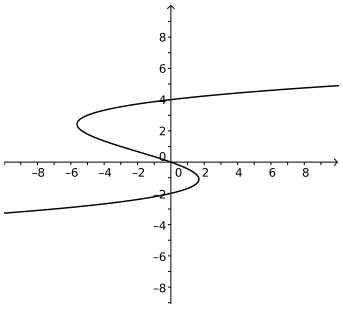
19. [4 points] Solve $|x - 5| + 4 = 12$.

20. [4 points] Find the distance between $(1, -3)$ and $(-11, -8)$.

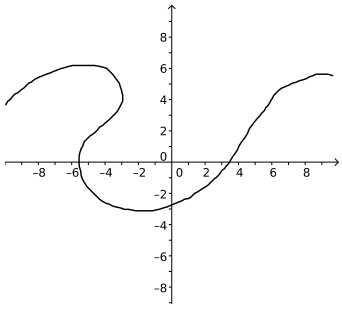
21. [4 points] Find the center and radius, and graph the circle $(x-5)^2 + (y+3)^2 = 4$.



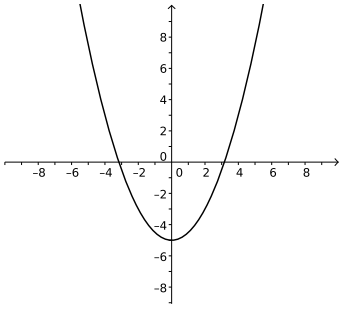
22. [4 points] Determine whether each graph represents a function.



Yes or No



Yes or No



Yes or No

23. [4 points] Given $f(x) = 4x - 7$, find $f(a-6)$.

24. [4 points] Find the difference quotient of f; that is find $\dfrac{f(x+h)-f(x)}{h}$. Assume $h \neq 0$. $f(x) = 5x + 8$.

Solutions

1. Factor $4x^2 - 8x - 5$.

$4x^2 - 10x + 2x - 5$	2 pts to here
$2x(2x - 5) + (2x - 5)$	3 pts to here
$(2x - 5)(2x + 1)$	4 pts to here

2. Factor $6ax + 48bx - ay - 8by$.

$6x(a + 8b) - y(a + 8b)$	2 pts to here
$(a + 8b)(6x - y)$	4 pts to here

3. Simplify $\dfrac{x^3 + 5x^2}{x^2 - 2x - 35}$.

Partial factoring of only the numerator	1 pt
Partial factoring of only the denominator	2 pts
$\dfrac{x^2(x + 5)}{(x - 7)(x + 5)}$	3 pts to here
$\dfrac{x^2}{x - 7}$	4 pts to here

4. Simplify $\dfrac{16x^3}{12x^2 - 12} \div \dfrac{8x^2}{x^2 - 2x + 1}$.

Factoring of equivalent of one rational expression	1 pt
$\frac{16x^3}{12(x+1)(x-1)} \div \frac{8x^2}{(x-1)^2}$	2 pts to here
$\frac{16x^3}{12(x+1)(x-1)} \times \frac{(x-1)^2}{8x^2}$	3 pts to here
$\frac{x(x-1)}{6(x+1)}$	4 pts to here

5. Simplify $\dfrac{2}{x - 8} - \dfrac{x}{x + 3}$

$\frac{2(x+3)}{(x-8)(x+3)} - \frac{x(x-8)}{(x-8)(x+3)}$	1 pts to here
$\frac{2(x+3)-x(x-8)}{(x-8)(x+3)}$	2 pts to here
$\frac{2x+6-x^2+8x)}{(x-8)(x+3)}$	3 pts to here
$\frac{-x^2+10x+6}{(x-8)(x+3)}$	4 pts to here

6. Simplify $\dfrac{3x + 5}{x^2 + 4x + 3} + \dfrac{-x + 5}{x^2 + 2x - 3}$.

$\frac{(3x+5)(x-1)}{(x+1)(x+3)(x-1)} + \frac{(-x+5)(x+1)}{(x+3)(x-1)(x+1)}$	1 pt
$\frac{3x^2+2x-5-x^2+4x+5}{(x+3)(x-1)(x+1)}$	2 pts to here
$\frac{2x^2+6x}{(x+3)(x-1)(x+1)}$	3 pts to here
$\frac{2x}{(x-1)(x+1)}$	4 pts to here

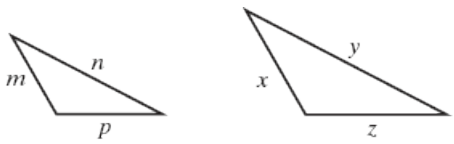
7. Simplify $\dfrac{\frac{a}{3b}-\frac{1}{2}}{\frac{7}{3b}-\frac{4}{a}}$.

Method 1	
$\frac{6ab}{6ab}(\frac{\frac{a}{3b}-\frac{1}{2}}{\frac{7}{3b}-\frac{4}{a}})$	2 pts to here
$\frac{2a^2-3ab}{14a-24b}$	4 pts to here
Method 2	
$\frac{\frac{2a}{6b}-\frac{3b}{6b}}{\frac{7a}{3ab}-\frac{12b}{3ab}}$	1 pt to here
$\frac{\frac{2a-3b}{6b}}{\frac{7a-12b}{3ab}}$	2 pts to here
$\frac{2a-3b}{6b} \times \frac{3ab}{7a-12b}$	3 pts to here
$\frac{a(2a-3b)}{2(7a-12b)}$ or $\frac{2a^2-3ab}{14a-24b}$	4 pts to here

8. Solve $\dfrac{2}{x^2-1}+\dfrac{5}{x+1}=\dfrac{3}{x-1}$.

$2+5(x-1)=3(x+1)$	2 pts to here
$5x-3=3x+3$	3 pts to here
$x=3$	4 pts to here

9. Triangles A and B are similar.



Triangle A Triangle B

If $z = 18$ in., $y = 25$ in., and $n = 9$ in., find the length of side p. Leave your answer as a fraction.

$\frac{9}{25}=\frac{p}{18}$ or	
$\frac{9}{p}=\frac{25}{18}$ or	
$\frac{25}{9}=\frac{18}{p}$ or	
$\frac{p}{9}=\frac{18}{25}$	2 pts to here
$p=\frac{162}{25}$ inches or $p=6\frac{12}{25}$ inches	4 pts to here
(3 pts for correct solution, but no units are given)	

10. Combine. Assume that all variables represent nonnegative real numbers. $\sqrt{28x}-\sqrt{147x}+\sqrt{75x}$

$2\sqrt{7x}-7\sqrt{3x}+5\sqrt{3x}$	2 pts to here
$2\sqrt{7x}-2\sqrt{3x}$	4 pts to here

11. Simplify $\dfrac{\sqrt{5}+3}{\sqrt{5}-3}$.

$\frac{\sqrt{5}+3}{\sqrt{5}-3} \times \frac{\sqrt{5}+3}{\sqrt{5}+3}$	2 pts to here
$\frac{5+3\sqrt{5}+3\sqrt{5}+9}{5-9}$	3 pts to here
$\frac{14+6\sqrt{3}}{-4}$	
$\frac{-7-3\sqrt{5}}{-2}$	4 pts to here

12. Solve $\sqrt{x+10} - 10 = x$.

$\sqrt{x+10} = x+10$	
$x+10 = x^2+20x+100$	1 pt to here
$0 = x^2+19x+90$	2 pts to here
$0 = (x+9)(x+10)$	3 pts to here
$x = -9$ and $x = -10$	4 pts to here

13. Simplify $\sqrt{-25}$.

$\sqrt{-1}\sqrt{25}$	1 pts to here
$5\sqrt{-1}$	2 pts to here or
$i\sqrt{25}$	3 pts to here
$5i$	4 pts to here

14. y varies directly as x and inversely as the square of z. If $y = 64$ when $x = 32$ and $z = 4$, find y when $x = 75$ and $z = 5$.

$y = \frac{kx}{z^2}$	
$64 = \frac{k(32)}{4^2}$	1 pt to here
$64 = 2k$	
$32 = k$	2 pts to here
$y = \frac{32x}{z^2}$	
$y = \frac{32(75)}{5^2}$	3 pts to here
$y = 96$	4 pts to here

15. Solve by using the square root property. $(2x+5)^2 = 81$.

$2x+5 = \pm 9$	1 pt to here
$2x = 4$	
$x = 2$	2 pts to here
$2x = -14$	3 pts to here
$x = -7$	4 pts

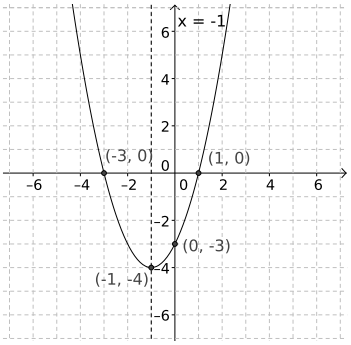
16. A company that manufactures bikes makes a daily profit, P, according to the equation $P(x) = -100x^2 + 4700x - 49449$ where P is measured in dollars and x is the number of mountain bikes made per day. Find the number of mountain bikes that must be made each day to produce a zero profit for the company. Round your answer to the nearest whole number.

$x = \frac{-4700 \pm \sqrt{(4700)^2 - 4(-100)(-49449)}}{2(-100)}$	1 pt to here
$x = \frac{-4700 \pm \sqrt{2310400}}{-200}$	2 pts to here
$x = \frac{-4700+1520}{-200} = 15.9 \approx 16$ bikes	3 pts to here
$x = \frac{-4700-1520}{-200} = 31.1 \approx 31$ bikes	4 pts to here (3 pts if no units)

17. A brace for a shelf has the shape of a right triangle. Its hypotenuse is 18 inches long and the two legs are equal in length. How long are the legs of the triangle? Keep answers in simplified radical form.

$x^2 + x^2 = 18^2$	1 pt to here
$x^2 = 162$	2 pts to here
$x = \sqrt{162}$ or $x = \pm\sqrt{162}$	3 pts to here
$x = 9\sqrt{2}$ in.	4 pts to here (3 pts no units)

18. Given $f(x) = x^2 + 2x - 3$. Identify the vertex, y-intercept, x-intercept(s), axis of symmetry, and graph the function on the graph paper and label your findings on it.



vertex $(-1, -4)$	Add 1 pt
x-intercepts $(-3, 0)$, $(1, 0)$	Add 1 pt for each
y-intercept $(0, -3)$	Add 1 pt
Axis of Symmetry $x = -1$	Add 1 pt
Correct graph	Add 1 pt
All of the points above marked on the graph	Add 2 pts

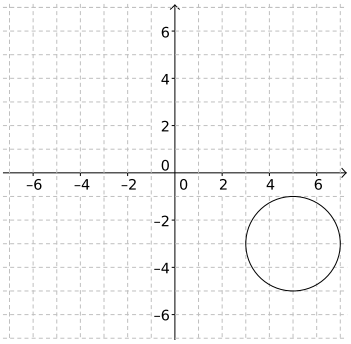
19. Solve $|x - 5| + 4 = 12$.

$\|x - 5\| = 8$	1 pt
$x - 5 = 8$	
$x = 13$	2 pt
$x - 5 = -8$	3 pt to here
$x = -3$	4 pt to here

20. Find the distance between $(1, -3)$ and $(-11, -8)$.

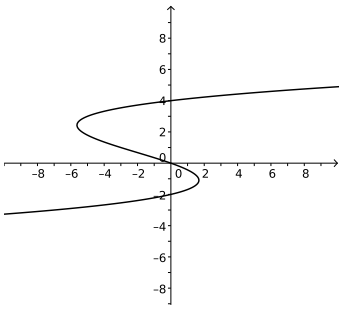
$d = \sqrt{(1-(-11))^2 + (-3-(-8))^2}$	
$d = \sqrt{(12)^2 + (5)^2}$	1 pts to here
$d = \sqrt{144 + 25}$	2 pts to here
$d = \sqrt{169}$	3 pts to here
$d = 13$	4 pts to here

21. Find the center and radius, and graph the circle $(x-5)^2 + (y+3)^2 = 4$.

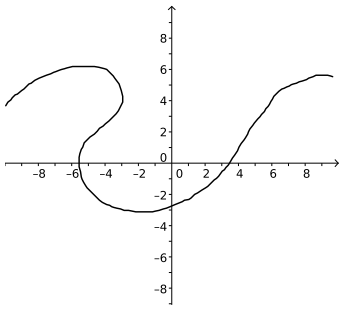


Correct graph	Add 2 points
Center $(5, -3)$	Add 1 point
Radius 2	Add 1 point

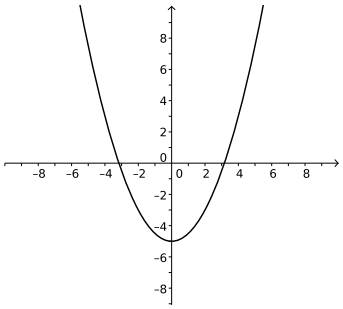
22. Determine whether each graph represents a function.



Yes or No



Yes or No



Yes or No

No (1 pt)	No (1 pt)	Yes (1 pt)
Add 1 point if all 3 correct		

23. Given $f(x) = 4x - 7$, find $f(a-6)$.

$4(a-6) - 7$	2 pt
$4a - 24 - 7$	3 pts to here
$4a - 31$	4 pts to here

24. Find the difference quotient of f; that is find $\dfrac{f(x+h)-f(x)}{h}$. Assume $h \neq 0$. $f(x) = 5x + 8$.

$f(x+h) = 5x + 5h + 8$	1 pt
$\frac{(5x+5h+8)-(5x+8)}{h}$	2 pts to here
$\frac{5h}{h}$	3 pts to here
5	4 pts to here