Name and section: _____

Instructor's name: _____

- **Please do not open exam until instructed to begin.**

- This exam is to be completed in the allotted time period of 2 hours.

- There are 24 problems which appear on the fronts and backs of the pages of this exam.

- You may earn a total of 100 points.

- Read each question carefully.

- Credit may not be given without sufficient supporting work.

- Simplify answers when possible.

- The use of cell phones, books, or notes are not permitted while taking this exam.

- Calculators are allowed.

1. [4 points] Factor $12x^2 - 16x - 3$.

2. [4 points] Factor $5ax + 10bx - ay - 2by$.

3. [4 points] Simplify $\dfrac{x^3 + 3x^2}{x^2 - 2x - 15}$.

4. [4 points] Simplify $\dfrac{24x^3}{4x^2 - 16} \div \dfrac{8x^2}{x^2 - 4x + 4}$.
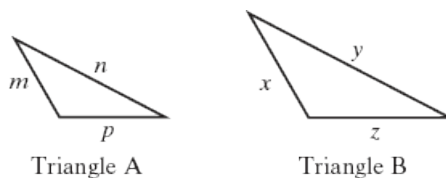
5. [4 points] Simplify $\dfrac{3}{x-7} - \dfrac{x}{x+3}$

6. [4 points] Simplify $\dfrac{3x+5}{x^2+4x+3} + \dfrac{-x+5}{x^2+2x-3}.$

7. [4 points] Simplify $\dfrac{\frac{a}{4b} - \frac{1}{3}}{\frac{5}{4b} - \frac{4}{a}}.$

8. [4 points] Solve $\dfrac{x-1}{x^2-4} = \dfrac{2}{x+2} + \dfrac{4}{x-2}.$

9. [4 points] Triangles A and B are similar.

Triangle A   Triangle B

If $z = 18$ in., $y = 25$ in., and $n = 9$ in., find the length of side p. Leave your answer as a fraction.

10. [4 points] Combine. Assume that all variables represent nonnegative real numbers. $\sqrt{45x} - \sqrt{128x} + \sqrt{72x}$.

11. [4 points] Simplify $\dfrac{\sqrt{3} + 3}{\sqrt{3} - 3}$.

12. [4 points] Solve $\sqrt{x + 8} - 8 = x$.

13. [4 points] Simplify $\sqrt{-25}$.

14. [4 points] y varies directly as x and inversely as the square of z. If $y = 64$ when $x = 32$ and $z = 4$, find y when $x = 75$ and $z = 5$.

15. [4 points] Solve by using the square root property. $(2x + 5)^2 = 81$.

16. [4 points] A company that manufactures bikes makes a daily profit, P, according to the equation $P(x) = -100x^2 + 4200x - 43371$ where P is measured in dollars and x is the number of mountain bikes made per day. Find the number of mountain bikes that must be made each day to produce a zero profit for the company. Round your answer to the nearest whole number.

17. [4 points] A brace for a shelf has the shape of a right triangle. Its hypotenuse is 8 inches long and the two legs are equal in length. How long are the legs of the triangle? Keep answers in simplified radical form.
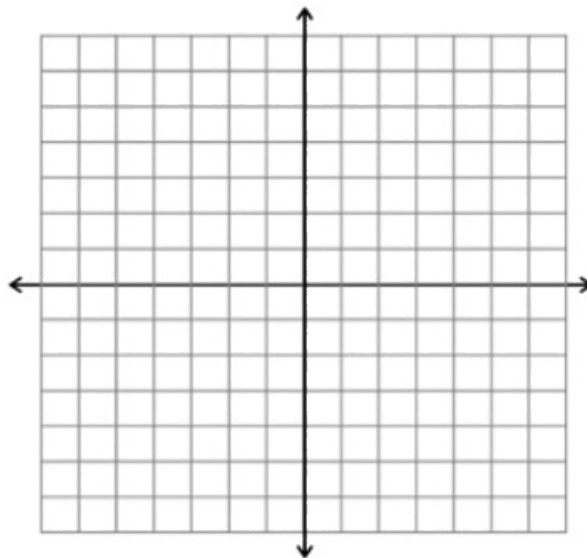
18. [8 points] Given $f(x) = x^2 + 2x - 3$. Identify the vertex, y-intercept, x-intercept(s), axis of symmetry, and graph the function on the graph paper and label your findings on it.
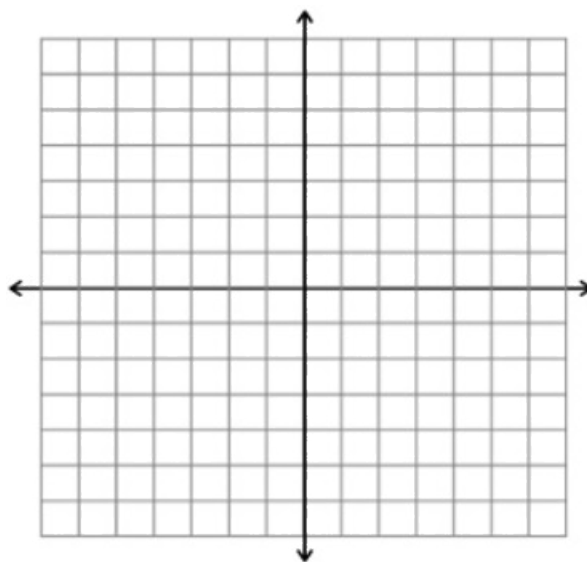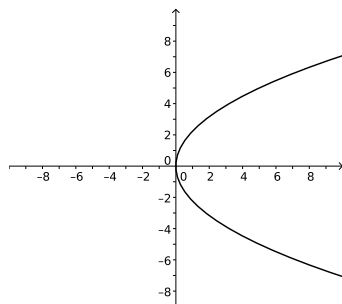

19. [4 points] Solve $|x + 2| - 1 = 6$.

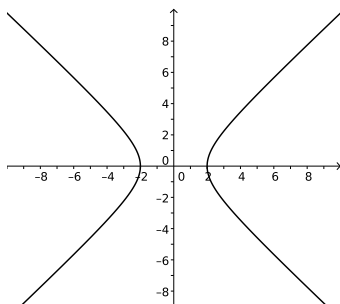20. [4 points] Find the distance between $(-7, 13)$ and $(-12, 1)$.

21. [4 points] Find the center and radius, and graph the circle $(x + 5)^2 + (y - 3)^2 = 4$.

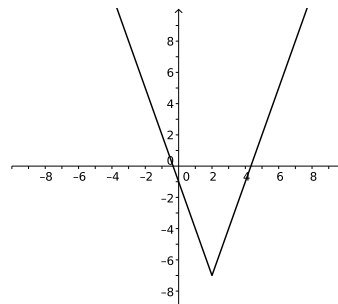22. [4 points] Determine whether each graph represents a function.

Yes or No   Yes or No   Yes or No

23. [4 points] Given $f(x) = 4x - 7$, find $f(a - 6)$.

24. [4 points] Find the difference quotient of f; that is find $\dfrac{f(x+h) - f(x)}{h}$. Assume $h \neq 0$. $f(x) = 4x + 3$.

Solutions

1. Factor $12x^2 - 16x - 3$.

$12x^2 - 18x + 2x - 3$	2 pts to here
$6x(2x - 3) + (2x - 3)$	3 pts to here
$(2x - 3)(6x + 1)$	4 pts to here

2. Factor $5ax + 10bx - ay - 2by$.

$5x(a + 2b) - y(a + 2b)$	2 pts to here
$(a + 2b)(5x - y)$	4 pts to here

3. Simplify $\dfrac{x^3 + 3x^2}{x^2 - 2x - 15}$.

Partial factoring of only the numerator	1 pt
Partial factoring of only the denominator	2 pts
$\dfrac{x^2(x + 3)}{(x - 5)(x + 3)}$	3 pts to here
$\dfrac{x^2}{x - 5}$	4 pts to here

4. Simplify $\dfrac{24x^3}{4x^2 - 16} \div \dfrac{8x^2}{x^2 - 4x + 4}$.

Factoring of equivalent of one rational expression	1 pt
$\dfrac{24x^3}{4(x+2)(x-2)} \div \dfrac{8x^2}{(x-2)^2}$	2 pts to here
$\dfrac{24x^3}{4(x+2)(x-2)} \times \dfrac{(x-2)^2}{8x^2}$	3 pts to here
$\dfrac{3x(x-2)}{4(x+2)}$	4 pts to here

5. Simplify $\dfrac{3}{x - 7} - \dfrac{x}{x + 3}$

$\dfrac{3(x+3)}{(x-7)(x+3)} - \dfrac{x(x-7)}{(x-7)(x+3)}$	1 pts to here
$\dfrac{3(x+3) - x(x-7)}{(x-7)(x+3)}$	2 pts to here
$\dfrac{3x+9-x^2+7x)}{(x-7)(x+3)}$	3 pts to here
$\dfrac{-x^2+10x+9}{(x-7)(x+3)}$	4 pts to here

6. Simplify $\dfrac{3x + 5}{x^2 + 4x + 3} + \dfrac{-x + 5}{x^2 + 2x - 3}$.

$\dfrac{(3x+5)(x-1)}{(x+1)(x+3)(x-1)} + \dfrac{(-x+5)(x+1)}{(x+3)(x-1)(x+1)}$	1 pt
$\dfrac{3x^2+2x-5-x^2+4x+5}{(x+3)(x-1)(x+1)}$	2 pts to here
$\dfrac{2x^2+6x}{(x+3)(x-1)(x+1)}$	3 pts to here
$\dfrac{2x}{(x-1)(x+1)}$	4 pts to here

7. Simplify $\dfrac{\frac{a}{4b} - \frac{1}{3}}{\frac{5}{4b} - \frac{4}{a}}$.

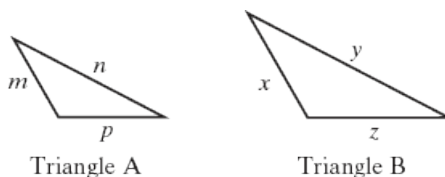| Method 1	
$\frac{12ab}{12ab}\left(\frac{\frac{a}{4b} - \frac{1}{3}}{\frac{5}{4b} - \frac{4}{a}}\right)$	2 pts to here
$\frac{3a^2 - 4ab}{15a - 48b}$	4 pts to here
Method 2	
$\frac{\frac{3a}{12b} - \frac{4b}{12b}}{\frac{5a}{4ab} - \frac{4ab}{16b}}$	1 pt to here
$\frac{\frac{3a-4b}{12b}}{\frac{5a-16b}{4ab}}$	2 pts to here
$\frac{3a-4b}{12b} \times \frac{4ab}{5a-16b}$	3 pts to here
$\frac{a(3a-4b)}{3(5a-16b)}$ or $\frac{3a^2-4ab}{15a-48b}$	4 pts to here

8. Solve $\dfrac{x-1}{x^2-4} = \dfrac{2}{x+2} + \dfrac{4}{x-2}$.

$x - 1 = 2(x-2) + 4(x+2)$	2 pts to here
$x - 1 = 6x + 4$	3 pts to here
$x = -1$	4 pts to here

9. Triangles A and B are similar.

Triangle A   Triangle B

If $z = 18$ in., $y = 25$ in., and $n = 9$ in., find the length of side p. Leave your answer as a fraction.

$\frac{9}{25} = \frac{p}{18}$ or	
$\frac{9}{p} = \frac{25}{18}$ or	
$\frac{25}{9} = \frac{18}{p}$ or	
$\frac{p}{9} = \frac{18}{25}$	2 pts to here
$p = \frac{162}{25}$ inches or $p = 6\frac{12}{25}$ inches	4 pts to here
(3 pts for correct solution, but no units are given)	

10. Combine. Assume that all variables represent nonnegative real numbers. $\sqrt{45x} - \sqrt{128x} + \sqrt{72x}$.

$3\sqrt{5x} - 8\sqrt{2x} + 6\sqrt{2x}$	2 pts to here
$3\sqrt{5x} - 2\sqrt{2x}$	4 pts to here

11. Simplify $\dfrac{\sqrt{3}+3}{\sqrt{3}-3}$.

$\frac{\sqrt{3}+3}{\sqrt{3}-3} \times \frac{\sqrt{3}+3}{\sqrt{3}+3}$ 2 pts to here

$\frac{3+3\sqrt{3}+3\sqrt{3}+9}{3-9}$ 3 pts to here

$\frac{12+6\sqrt{3}}{-6}$

$-2-\sqrt{3}$ 4 pts to here

12. Solve $\sqrt{x+8}-8=x$.

$\sqrt{x+8}=x+8$

$x+8=x^2+16x+64$ 1 pt to here

$0=x^2+15x+56$ 2 pts to here

$0=(x+7)(x+8)$ 3 pts to here

$x=-7$ and $x=-8$ 4 pts to here

13. Simplify $\sqrt{-25}$.

$\sqrt{-1}\sqrt{25}$ 1 pts to here

$5\sqrt{-1}$ 2 pts to here or

$i\sqrt{25}$ 3 pts to here

$5i$ 4 pts to here

14. y varies directly as x and inversely as the square of z. If $y=64$ when $x=32$ and $z=4$, find y when $x=75$ and $z=5$.

$y=\frac{kx}{z^2}$

$64=\frac{k(32)}{4^2}$ 1 pt to here

$64=2k$

$32=k$ 2 pts to here

$y=\frac{32x}{z^2}$

$y=\frac{32(75)}{5^2}$ 3 pts to here

$y=96$ 4 pts to here

15. Solve by using the square root property. $(2x+5)^2=81$.

$2x+5=\pm9$ 1 pt to here

$2x=4$

$x=2$ 2 pts to here

$2x=-14$ 3 pts to here

$x=-7$ 4 pts

16. A company that manufactures bikes makes a daily profit, P, according to the equation $P(x)=-100x^2+4200x-43371$ where P is measured in dollars and x is the number of mountain bikes made per day. Find the number of mountain bikes that must be made each day to produce a zero profit for the company. Round your answer to the nearest whole number.

$x = \frac{-4200 \pm \sqrt{(4200)^2 - 4(-100)(-43371)}}{2(-100)}$	1 pt to here
$x = \frac{-4200 \pm \sqrt{291600}}{-200}$	2 pts to here
$x = \frac{-4200 + 540}{-200} = 18.3 \approx 18$ bikes	3 pts to here
$x = \frac{-4200 - 540}{-200} = 23.7 \approx 24$ bikes	4 pts to here (3 pts if no units)

17. A brace for a shelf has the shape of a right triangle. Its hypotenuse is 8 inches long and the two legs are equal in length. How long are the legs of the triangle? Keep answers in simplified radical form.

$x^2 + x^2 = 8^2$	1 pt to here
$x^2 = 32$	2 pts to here
$x = \sqrt{32}$ or $x = \pm\sqrt{32}$	3 pts to here
$x = 4\sqrt{2}$ in.	4 pts to here (3 pts no units)

18. Given $f(x) = x^2 + 2x - 3$. Identify the vertex, y-intercept, x-intercept(s), axis of symmetry, and graph the function on the graph paper and label your findings on it.
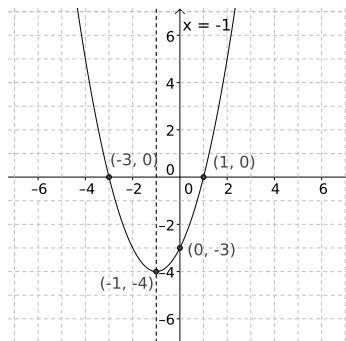
|	
vertex $(-1, -4)$	Add 1 pt
x-intercepts $(-3, 0)$, $(1, 0)$	Add 1 pt for each
y-intercept $(0, -3)$	Add 1 pt
Axis of Symmetry $x = -1$	Add 1 pt
Correct graph	Add 1 pt
All of the points above marked on the graph	Add 2 pts

19. Solve $|x + 2| - 1 = 6$.

$|x + 2| = 7$	1 pt
$x + 2 = 7$	
$x = 5$	2 pt
$x + 2 = -7$	3 pt to here
$x = -9$	4 pt to here

20. Find the distance between $(-7, 13)$ and $(-12, 1)$.
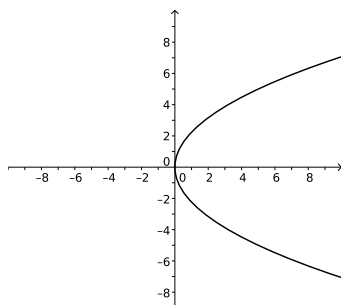
$d = \sqrt{(-7 - (-12))^2 + (13 - 1)^2}$	
$d = \sqrt{(5)^2 + (12)^2}$	1 pts to here
$d = \sqrt{25 + 144}$	2 pts to here
$d = \sqrt{169}$	3 pts to here
$d = 13$	4 pts to here

21. Find the center and radius, and graph the circle $(x + 5)^2 + (y - 3)^2 = 4$.
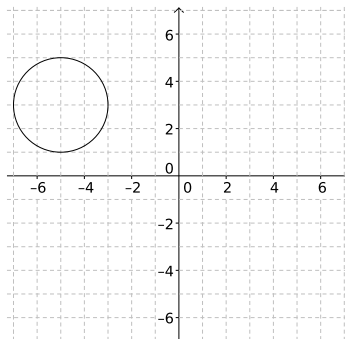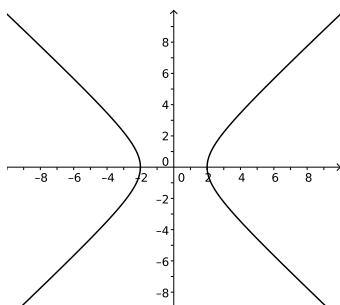

Correct graph	Add 2 points
Center $(-5, 3)$	Add 1 point
Radius 2	Add 1 point

22. Determine whether each graph represents a function.
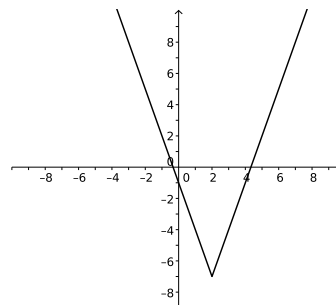
Yes or No   Yes or No   Yes or No

No (1 pt)	No (1 pt)	Yes (1 pt)
Add 1 point if all 3 correct		

23. Given $f(x) = 4x - 7$, find $f(a - 6)$.

$4(a - 6) - 7$	2 pt
$4a - 24 - 7$	3 pts to here
$4a - 31$	4 pts to here

24. Find the difference quotient of f; that is find $\dfrac{f(x+h)-f(x)}{h}$. Assume $h \neq 0$. $f(x) = 4x + 3$.

$f(x+h) = 4x + 4h + 3$	1 pt
$\frac{(4x+4h+3)-(4x+3)}{h}$	2 pts to here
$\frac{4h}{h}$	3 pts to here
4	4 pts to here