

Name and section: _____

Instructor's name: _____

- **Please do not open exam until instructed to begin.**
- This exam is to be completed in the allotted time period of 2 hours.
- There are 24 problems which appear on the fronts and backs of the pages of this exam.
- You may earn a total of 100 points.
- Read each question carefully.
- Credit may not be given without sufficient supporting work.
- Simplify answers when possible.
- The use of cell phones, books, or notes are not permitted while taking this exam.
- Calculators are allowed.

1. [4 points] Factor $3x^2 - 2x - 8$.
2. [4 points] Factor $6ax + 48bx - ay - 8by$.
3. [4 points] Simplify $\frac{x^3 + 3x^2}{x^2 - 2x - 15}$.
4. [4 points] Simplify $\frac{24x^3}{4x^2 - 16} \div \frac{8x^2}{x^2 - 4x + 4}$.

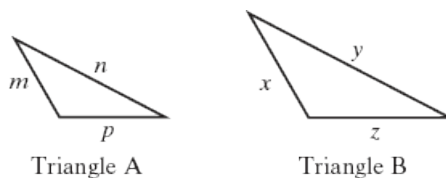
5. [4 points] Simplify $\frac{3}{x-7} - \frac{x}{x+3}$

6. [4 points] Simplify $\frac{3x+5}{x^2+4x+3} + \frac{-x+5}{x^2+2x-3}$.

7. [4 points] Simplify $\frac{\frac{a}{3b} - \frac{1}{2}}{\frac{7}{3b} - \frac{4}{a}}$.

8. [4 points] Solve $\frac{2}{x^2-4} + \frac{5}{x+2} = \frac{2}{x-2}$.

9. [4 points] Triangles A and B are similar.



If $z = 18$ in., $y = 25$ in., and $n = 9$ in., find the length of side p . Leave your answer as a fraction.

10. [4 points] Combine. Assume that all variables represent nonnegative real numbers.
- $$\sqrt{44x} - \sqrt{63x} + \sqrt{112x}$$

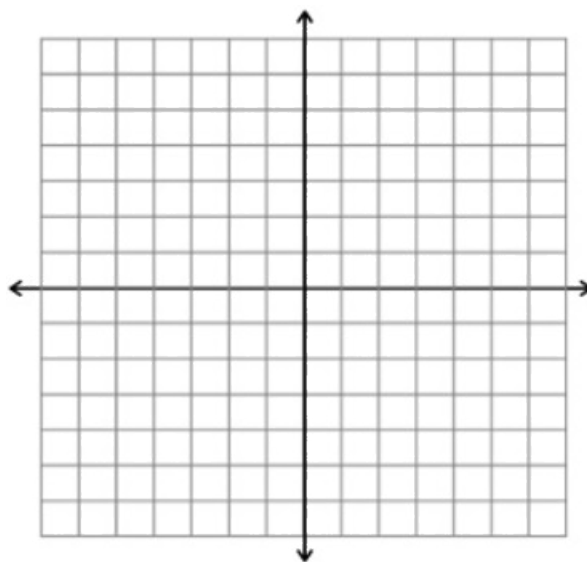
11. [4 points] Simplify $\frac{\sqrt{3} + 3}{\sqrt{3} - 3}$.

12. [4 points] Solve $\sqrt{x + 10} - 10 = x$.

13. [4 points] Simplify $\sqrt{-16}$.
14. [4 points] y varies directly as x and inversely as the square of z . If $y = 56$ when $x = 175$ and $z = 5$, find y when $x = 98$ and $z = 7$.
15. [4 points] Solve by using the square root property. $(2x + 7)^2 = 121$.
16. [4 points] A company that manufactures bikes makes a daily profit, P , according to the equation $P(x) = -100x^2 - 4500x - 48562$ where P is measured in dollars and x is the number of mountain bikes made per day. Find the number of mountain bikes that must be made each day to produce a zero profit for the company. Round your answer to the nearest whole number.

17. [4 points] A brace for a shelf has the shape of a right triangle. Its hypotenuse is 8 inches long and the two legs are equal in length. How long are the legs of the triangle? Keep answers in simplified radical form.

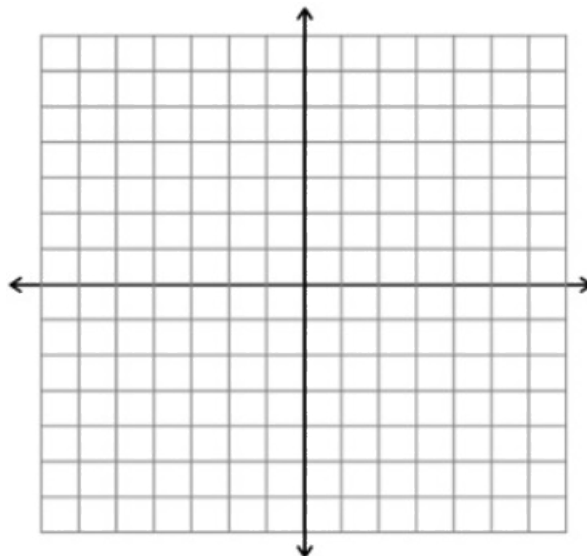
18. [8 points] Given $f(x) = x^2 + 2x - 3$. Identify the vertex, y-intercept, x-intercept(s), axis of symmetry, and graph the function on the graph paper and label your findings on it.



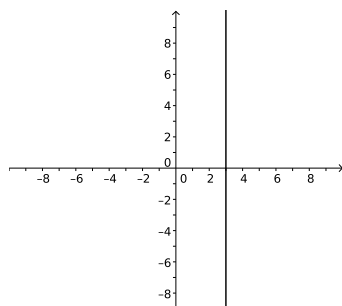
19. [4 points] Solve $|x - 5| + 4 = 12$.

20. [4 points] Find the distance between $(-7, 13)$ and $(-12, 1)$.

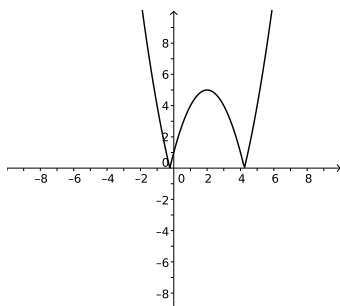
21. [4 points] Find the center and radius, and graph the circle $(x - 5)^2 + (y + 3)^2 = 4$.



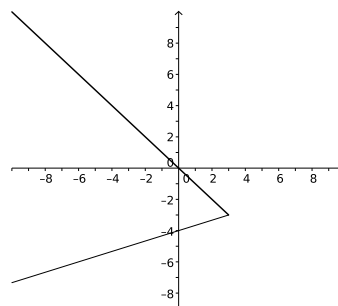
22. [4 points] Determine whether each graph represents a function.



Yes or No



Yes or No



Yes or No

23. [4 points] Given $f(x) = 4x - 7$, find $f(a - 6)$.

24. [4 points] Find the difference quotient of f ; that is find $\frac{f(x+h) - f(x)}{h}$. Assume $h \neq 0$. $f(x) = 3x + 5$.

Solutions

1. Factor $3x^2 - 2x - 8$.

$3x^2 - 6x + 4x - 8$	2 pts to here
$3x(x - 2) + 4(x - 2)$	3 pts to here
$(3x + 4)(x - 2)$	4 pts to here

2. Factor $6ax + 48bx - ay - 8by$.

$6x(a + 8b) - y(a + 8b)$	2 pts to here
$(a + 8b)(6x - y)$	4 pts to here

3. Simplify $\frac{x^3 + 3x^2}{x^2 - 2x - 15}$.

Partial factoring of only the numerator	1 pt
Partial factoring of only the denominator	2 pts
$\frac{x^2(x + 3)}{(x - 5)(x + 3)}$	3 pts to here
$\frac{x^2}{x - 5}$	4 pts to here

4. Simplify $\frac{24x^3}{4x^2 - 16} \div \frac{8x^2}{x^2 - 4x + 4}$.

Factoring of equivalent of one rational expression	1 pt
$\frac{24x^3}{4(x+2)(x-2)} \div \frac{8x^2}{(x-2)^2}$	2 pts to here
$\frac{24x^3}{4(x+2)(x-2)} \times \frac{(x-2)^2}{8x^2}$	3 pts to here
$\frac{3x(x-2)}{4(x+2)}$	4 pts to here

5. Simplify $\frac{3}{x-7} - \frac{x}{x+3}$

$\frac{3(x+3)}{(x-7)(x+3)} - \frac{x(x-7)}{(x-7)(x+3)}$	1 pts to here
$\frac{3(x+3)-x(x-7)}{(x-7)(x+3)}$	2 pts to here
$\frac{3x+9-x^2+7x}{(x-7)(x+3)}$	3 pts to here
$\frac{-x^2+10x+9}{(x-7)(x+3)}$	4 pts to here

6. Simplify $\frac{3x+5}{x^2+4x+3} + \frac{-x+5}{x^2+2x-3}$.

$\frac{(3x+5)(x-1)}{(x+1)(x+3)(x-1)} + \frac{(-x+5)(x+1)}{(x+3)(x-1)(x+1)}$	1 pt
$\frac{3x^2+2x-5-x^2+4x+5}{(x+3)(x-1)(x+1)}$	2 pts to here
$\frac{2x^2+6x}{(x+3)(x-1)(x+1)}$	3 pts to here
$\frac{2x}{(x-1)(x+1)}$	4 pts to here

7. Simplify $\frac{\frac{a}{3b} - \frac{1}{2}}{\frac{7}{3b} - \frac{4}{a}}$.

Method 1

$$\frac{6ab \left(\frac{\frac{a}{3b} - \frac{1}{2}}{\frac{7}{3b} - \frac{4}{a}} \right)}{6ab \left(\frac{7}{3b} - \frac{4}{a} \right)} \quad 2 \text{ pts to here}$$

$$\frac{2a^2 - 3ab}{14a - 24b} \quad 4 \text{ pts to here}$$

Method 2

$$\frac{\frac{2a}{6b} - \frac{3b}{6b}}{\frac{7a}{6b} - \frac{12b}{6b}} \quad 1 \text{ pt to here}$$

$$\frac{\frac{2a-3b}{6b}}{\frac{7a-12b}{6b}} \quad 2 \text{ pts to here}$$

$$\frac{2a-3b}{6b} \times \frac{3ab}{7a-12b} \quad 3 \text{ pts to here}$$

$$\frac{a(2a-3b)}{2(7a-12b)} \text{ or } \frac{2a^2-3ab}{14a-24b} \quad 4 \text{ pts to here}$$

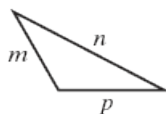
8. Solve $\frac{2}{x^2 - 4} + \frac{5}{x + 2} = \frac{2}{x - 2}$.

$$2 + 5(x - 2) = 2(x + 2) \quad 2 \text{ pts to here}$$

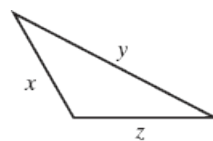
$$5x - 8 = 2x + 4 \quad 3 \text{ pts to here}$$

$$x = 4 \quad 4 \text{ pts to here}$$

9. Triangles A and B are similar.



Triangle A



Triangle B

If $z = 18$ in., $y = 25$ in., and $n = 9$ in., find the length of side p . Leave your answer as a fraction.

$$\frac{9}{25} = \frac{p}{18} \text{ or}$$

$$\frac{9}{p} = \frac{25}{18} \text{ or}$$

$$\frac{25}{9} = \frac{18}{p} \text{ or}$$

$$\frac{p}{9} = \frac{18}{25}$$

$$p = \frac{162}{25} \text{ inches or } p = 6\frac{12}{25} \text{ inches}$$

(3 pts for correct solution, but no units are given)

2 pts to here

4 pts to here

10. Combine. Assume that all variables represent nonnegative real numbers. $\sqrt{44x} - \sqrt{63x} + \sqrt{112x}$

$$2\sqrt{11x} - 3\sqrt{7x} + 4\sqrt{7x} \quad 2 \text{ pts to here}$$

$$2\sqrt{11x} + \sqrt{7x} \quad 4 \text{ pts to here}$$

11. Simplify $\frac{\sqrt{3} + 3}{\sqrt{3} - 3}$.

$\frac{\sqrt{3}+3}{\sqrt{3}-3} \times \frac{\sqrt{3}+3}{\sqrt{3}+3}$	2 pts to here
$\frac{3+3\sqrt{3}+3\sqrt{3}+9}{3-9}$	3 pts to here
$\frac{12+6\sqrt{3}}{-6}$	
$-2 - \sqrt{3}$	4 pts to here

12. Solve $\sqrt{x+10} - 10 = x$.

$\sqrt{x+10} = x+10$	
$x+10 = x^2 + 20x + 100$	1 pt to here
$0 = x^2 + 19x + 90$	2 pts to here
$0 = (x+9)(x+10)$	3 pts to here
$x = -9$ and $x = -10$	4 pts to here

13. Simplify $\sqrt{-16}$.

$\sqrt{-1}\sqrt{16}$	1 pts to here
$4\sqrt{-1}$	2 pts to here or
$i\sqrt{16}$	3 pts to here
$4i$	4 pts to here

14. y varies directly as x and inversely as the square of z . If $y = 56$ when $x = 175$ and $z = 5$, find y when $x = 98$ and $z = 7$.

$y = \frac{kx}{z^2}$	
$56 = \frac{k(175)}{5^2}$	1 pt to here
$56 = 7k$	
$8 = k$	2 pts to here
$y = \frac{8x}{z^2}$	
$y = \frac{8(98)}{7^2}$	3 pts to here
$y = 16$	4 pts to here

15. Solve by using the square root property. $(2x+7)^2 = 121$.

$2x+7 = \pm 11$	1 pt to here
$2x = 4$	
$x = 2$	2 pts to here
$2x = -18$	3 pts to here
$x = -9$	4 pts

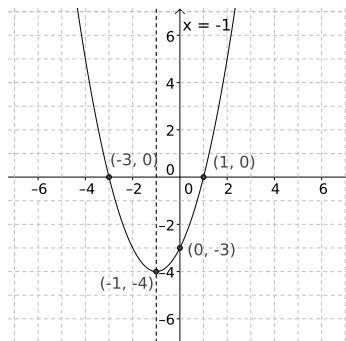
16. A company that manufactures bikes makes a daily profit, P , according to the equation $P(x) = -100x^2 - 4500x - 48562$ where P is measured in dollars and x is the number of mountain bikes made per day. Find the number of mountain bikes that must be made each day to produce a zero profit for the company. Round your answer to the nearest whole number.

$x = \frac{-4200 \pm \sqrt{(4200)^2 - 4(-100)(-43371)}}{2(-100)}$	1 pt to here
$x = \frac{-4200 \pm \sqrt{291600}}{-200}$	2 pts to here
$x = \frac{-4200 + 540}{-200} = 18.3 \approx 18$ bikes	3 pts to here
$x = \frac{-4200 - 540}{-200} = 23.7 \approx 24$ bikes	4 pts to here (3 pts if no units)

17. A brace for a shelf has the shape of a right triangle. Its hypotenuse is 8 inches long and the two legs are equal in length. How long are the legs of the triangle? Keep answers in simplified radical form.

$x^2 + x^2 = 8^2$	1 pt to here
$x^2 = 32$	2 pts to here
$x = \sqrt{32}$ or $x = \pm\sqrt{32}$	3 pts to here
$x = 4\sqrt{2}$ in.	4 pts to here (3 pts no units)

18. Given $f(x) = x^2 + 2x - 3$. Identify the vertex, y-intercept, x-intercept(s), axis of symmetry, and graph the function on the graph paper and label your findings on it.



vertex $(-1, -4)$	Add 1 pt
x-intercepts $(-3, 0), (1, 0)$	Add 1 pt for each
y-intercept $(0, -3)$	Add 1 pt
Axis of Symmetry $x = -1$	Add 1 pt
Correct graph	Add 1 pt
All of the points above marked on the graph	Add 2 pts

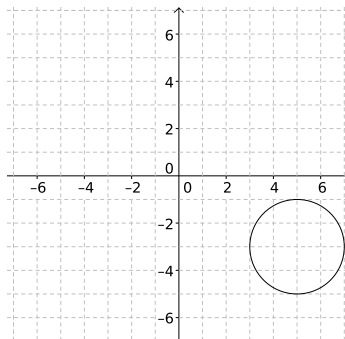
19. Solve $|x - 5| + 4 = 12$.

$ x - 5 = 8$	1 pt
$x - 5 = 8$	
$x = 13$	2 pt
$x - 5 = -8$	3 pt to here
$x = -3$	4 pt to here

20. Find the distance between $(-7, 13)$ and $(-12, 1)$.

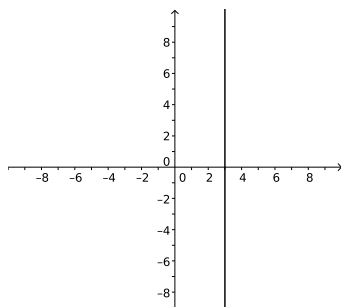
$d = \sqrt{(-7 - (-12))^2 + (13 - 1)^2}$	
$d = \sqrt{(5)^2 + (12)^2}$	1 pts to here
$d = \sqrt{25 + 144}$	2 pts to here
$d = \sqrt{169}$	3 pts to here
$d = 13$	4 pts to here

21. Find the center and radius, and graph the circle $(x - 5)^2 + (y + 3)^2 = 4$.

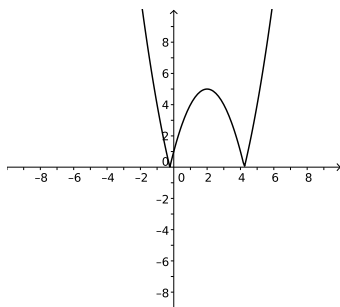


Correct graph	Add 2 points
Center $(5, -3)$	Add 1 point
Radius 2	Add 1 point

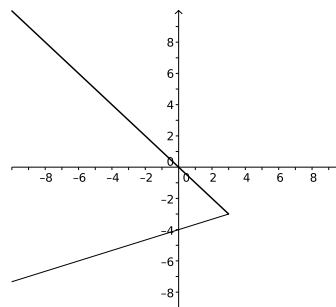
22. Determine whether each graph represents a function.



Yes or No



Yes or No



Yes or No

No (1 pt)	Yes (1 pt)	No (1 pt)
Add 1 point if all 3 correct		

23. Given $f(x) = 4x - 7$, find $f(a - 6)$.

$4(a - 6) - 7$	2 pt
$4a - 24 - 7$	3 pts to here
$4a - 31$	4 pts to here

24. Find the difference quotient of f ; that is find $\frac{f(x+h) - f(x)}{h}$. Assume $h \neq 0$. $f(x) = 3x + 5$.

$$f(x+h) = 3x + 3h + 5 \quad 1 \text{ pt}$$

$$\frac{(3x+3h+5)-(3x+5)}{h} \quad 2 \text{ pts to here}$$

$$\frac{3h}{h} \quad 3 \text{ pts to here}$$

$$3 \quad 4 \text{ pts to here}$$