Name and section: _____

Instructor's name: _____

- **Please do not open exam until instructed to begin.**

- This exam is to be completed in the allotted time period of 50 minutes.

- There are 17 problems which appear on the fronts and backs of the pages of this exam.

- You may earn a total of 96 points.

- Read each question carefully.

- Credit may not be given without sufficient supporting work.

- Simplify answers when possible.

- The use of cell phones, books, or notes are not permitted while taking this exam.

- Approved calculators are allowed.

1. Factor completely.

    $20x^2y - 32xy^2$

2. Factor completely.

    $x^2 - 3x - 28$

3. Factor completely.

    $9x^2 - 13x + 4$

4. Factor completely.

    $81a^2 - 49$

5. Factor completely.

   $16x^3y - 24x^2y + 9xy$

6. The height in feet that a model rocket attains is given by $h(t) = -5t^2 + 8t + 4$ where $t$ is the time measured in seconds. How many seconds will it take until the rocket finally reaches the ground? (Hint: ground level is $h = 0$)

7. Simplify.

   $$\frac{x^2 + 10x + 21}{2x^2 + 11x - 21}$$

8. Simplify.

   $$\frac{2x^2 - 7x - 15}{25 - x^2}$$

9. Multiply.

$$\frac{x^2 + x - 30}{2x - 10} \times \frac{5x + 15}{x^2 + 4x - 12}$$

10. Divide.

$$\frac{7x^2 + 28xy + 28y^2}{x^2 + 7xy + 6y^2} \div \frac{8x + 16y}{x + y}$$

11. Subtract.

$$\frac{5x}{x^2 - 64} - \frac{4}{x + 8}$$

12. Add.
$$\frac{4}{x^2 + 9x + 14} + \frac{3}{x^2 + 6x + 8}$$

13. Simplify.
$$\frac{\frac{14}{x^2 - 49}}{\frac{4}{x+7} + \frac{4}{x-7}}$$

14. Solve for $x$.
$$\frac{4x}{x^2 - 4} = \frac{5}{x + 2} - \frac{2}{x - 2}$$

15. Solve for $a$.
$$\frac{8a - 1}{6a + 8} = \frac{3}{4}$$

16. A 5-gallon can of paint covers 250 square feet. How many gallons of paint do you need to cover 400 square feet?

17. On a map the distance between two mountains is $5\frac{1}{2}$ inches. The actual distance is 116 miles. Russ is camped at a location that on the map is $\frac{3}{4}$ inch from the base of the mountain. How many miles is he from the base of the mountain? Round to the nearest tenth.

# Solutions

1. Factor completely.

   $20x^2y - 32xy^2$

   | | |
   |---|---|
   | $4xy$ | 2 pts to here |
   | $4xy(5x - 8y)$ | 4 pts to here |
   | Award 3 points if the correct GCF is found, | |
   | but there is a minor mistake in factoring. | |

2. Factor completely.

   $x^2 - 3x - 28$

   | | |
   |---|---|
   | $(x - 7)(x + 4)$ | 4 pts |
   | Reversed signs award 2 pts for the problem | |
   | One correct factor award 1 pt for the problem | |

3. Factor completely.

   $9x^2 - 13x + 4$

   | | |
   |---|---|
   | $9x^2 - 9x - 4x + 4$ | 1 pt to here |
   | $9x(x - 1) - 4(x - 1)$ | 2 pt to here |
   | $(9x - 4)(x - 1)$ | 4 pts |
   | Reversed signs award 3 pts for the problem | |
   | One correct factor award 2 pt for the problem | |

4. Factor completely.

   $81a^2 - 49$

   | | |
   |---|---|
   | $(9a - 7)(9a + 7)$ | 4 pts |
   | Incorrect signs, numbers are correct | 3 pts |
   | One correct factor award | 2 pts |

5. Factor completely.

   $16x^3y - 24x^2y + 9xy$

   | | |
   |---|---|
   | $xy(16x^2 - 24x + 9)$ | 2 pts to here |
   | $xy(4x - 3)^2$ | 4 pts |
   | Incorrect sign, numbers are correct | 3 pts |

6. The height in feet that a model rocket attains is given by $h(t) = -5t^2 + 8t + 4$ where $t$ is the time measured in seconds. How many seconds will it take until the rocket finally reaches the ground? (Hint: ground level is $h = 0$)

| | |
|---|---|
| $0 = -5t^2 + 8t + 4$ | 2 pts to here |
| $0 = -(5t + 2)(t - 2)$ | 4 pts to here |
| $t - 2 = 0$ | |
| $5t + 2 = 0$ | 6 pts to here |
| $t = -2/5$ second | 8 pts given both answers |
| $t = 2$ seconds | 10 pts if student shows |
| that 2 seconds is only answer | |
| (9 points if units are left off) | |

7. Simplify.

$$\frac{x^2 + 10x + 21}{2x^2 + 11x - 21}$$

| | |
|---|---|
| Partial factoring of only the numerator | 1 pt |
| Partial factoring of only the denominator | 2 pts to here |
| $\frac{(x+7)(x+3)}{(2x-3)(x+7)}$ | 3 pts to here |
| $\frac{x+3}{2x-3}$ | 4 pts to here |

8. Simplify.

$$\frac{2x^2 - 7x - 15}{25 - x^2}$$

| | |
|---|---|
| Partial factoring of only denominator or numerator | 1 pt |
| $\frac{(2x+3)(x-5)}{(5-x)(5+x)}$ | 2 pts to here |
| $\frac{-(2x+3)(5-x)}{(5-x)(5+x)}$ | 3 pts to here |
| $-\frac{2x+3}{5+x}$ | 4 pts to here |

9. Multiply.

$$\frac{x^2 + x - 30}{2x - 10} \times \frac{5x + 15}{x^2 + 4x - 12}$$

| | |
|---|---|
| Factoring of equivalent of one rational expression | 2 pt |
| $\frac{(x-5)(x+6)}{2(x-5)} \times \frac{5(x+3)}{(x-2)(x+6)}$ | 4 pts to here |
| $\frac{5(x+3)}{2(x-2)}$ | 6 pts to here |

10. Divide.

$$\frac{7x^2 + 28xy + 28y^2}{x^2 + 7xy + 6y^2} \div \frac{8x + 16y}{x + y}$$

| | |
|---|---|
| Factoring of equivalent of one rational expression | 2 pt |
| $\frac{7(x+2y)(x+2y)}{(x+6y)(x+y)} \times \frac{x+y}{8(x+2y)}$ | 3 pts to here |
| $\frac{7(x+2y)}{8(x+6y)}$ | 6 pts to here |

11. Subtract.

$$\frac{5x}{x^2 - 64} - \frac{4}{x + 8}$$

| | |
|---|---|
| Factoring of equivalent of one rational expression | 2 pt |
| $\frac{5x}{(x+8)(x-8)} - \frac{4(x-8)}{(x+8)(x-8)}$ | 3 pts to here |
| $\frac{5x-4x+32}{(x+8)(x-8)}$ | 5 pts to here |
| (4 pts if $3x - 2x - 32$) | |
| $\frac{x+32}{(x+8)(x-8)}$ | 6 pts to here |
| 5 pts if $\frac{x-32}{(x+8)(x-8)}$ | |

12. Add.

$$\frac{4}{x^2 + 9x + 14} + \frac{3}{x^2 + 6x + 8}$$

| | |
|---|---|
| $\frac{4}{(x+7)(x+2)} + \frac{3}{(x+2)(x+4)}$ | 2 pts to here |
| $\frac{4(x+4)}{(x+7)(x+2)(x+4)} + \frac{3(x+7)}{(x+7)(x+2)(x+4)}$ | 3 pts to here |
| $\frac{4x+16+3x+21}{(x+4)(x+2)(x+7)}$ | 5 pts to here |
| $\frac{7x+37}{(x+4)(x+2)(x+7)}$ | 6 pts to here |

13. Simplify.

$$\frac{\frac{14}{x^2-49}}{\frac{4}{x+7} + \frac{4}{x-7}}$$

| | |
|---|---|
| **Method 1** | |
| $\frac{\frac{14}{x^2-49}}{\frac{4(x-7)}{(x+7)(x-7)}} + \frac{4(x+7)}{(x+7)(x-7)}$ | 2 pts to here |
| $\frac{\frac{14}{x^2-49}}{\frac{8x}{(x+7)(x-7)}}$ | 3 pts to here |
| $\frac{14}{(x+7)(x-7)} \times \frac{(x+7)(x-7)}{8x}$ | 4 pts to here |
| $\frac{14}{8x}$ | 5 pts to here |
| $\frac{7}{4x}$ | 6 pts to here |
| **Method 2** | |
| $\frac{(x+7)(x-7)}{(x+7)(x-7)} \times \frac{\frac{14}{x^2-49}}{\frac{4}{x+7} + \frac{4}{x-7}}$ | 2 pts to here |
| $\frac{14}{4(x-7)+4(x+7)}$ | 4 pts to here |
| $\frac{14}{8x}$ | 5 pts to here |
| $\frac{7}{4x}$ | 6 pts to here |

14. Solve for $x$.

$$\frac{4x}{x^2 - 4} = \frac{5}{x + 2} - \frac{2}{x - 2}$$

$$(x+2)(x-2)\frac{4x}{(x+2)(x-2)} = \frac{(x+2)(x-2)5}{x+2} - \frac{(x+2)(x-2)2}{x-2} \quad \text{2 pts to here}$$
$$4x = 5(x-2) - 2(x+2) \quad \text{3 pts to here}$$
$$4x = 5x - 10 - 2x - 4 \quad \text{4 pts to here}$$
$$4x = 3x - 14 \quad \text{5 pts to here}$$
$$x = -14 \quad \text{6 pts to here}$$

15. Solve for $a$.
$$\frac{8a-1}{6a+8} = \frac{3}{4}$$

| | |
|---|---|
| $4(8a-1) = 3(6a+8)$ | 3 pts to here |
| $32a - 4 = 18a + 24$ | 4 pts to here |
| $14a = 28$ | 5 pts to here |
| $a = 2$ | 6 pts to here |

16. A 5-gallon can of paint covers 250 square feet. How many gallons of paint do you need to cover 400 square feet?

| | |
|---|---|
| $\frac{5}{250} = \frac{x}{400}$ | 4 pts to here |
| $5 \cdot 400 = 250x$ | 6 pts to here |
| $x = 8 gallons$ | 8 pts to here |

17. On a map the distance between two mountains is $5\frac{1}{2}$ inches. The actual distance is 116 miles. Russ is camped at a location that on the map is $\frac{3}{4}$ inch from the base of the mountain. How many miles is he from the base of the mountain? Round to the nearest tenth.

| | |
|---|---|
| $\frac{5.5in}{116miles} = \frac{.75in}{xmiles}$ | 4 pts to here |
| $5.5x = (.75)(116)$ | 6 pts to here |
| $x = 15.8$ | 7 pts to here |
| $x = 15.8 miles$ | 8 pts to here |