AA 274A: Principles of Robot Autonomy I
Problem Set 4

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Problem 1: EKF Localization

(i) (code). Although this is not required by the pset, I'll setup the problem here since this is way too much for comments in code, and the derivation is neither in the notes or slides.

Given: A unicycle model with generalized coordinates and instantaneous control vector:

$$\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \end{bmatrix} \quad , \quad \mathbf{u}(t) = \begin{bmatrix} V(t) \\ \omega(t) \end{bmatrix} \tag{1}$$

Given: Continuous unicycle model dynamics:

$$\begin{aligned} \dot{x}(t) &= V(t)\cos(\theta(t)) \\ \dot{y}(t) &= V(t)\sin(\theta(t)) \\ \dot{\theta}(t) &= \omega(t) \end{aligned} \tag{2}$$

For clarity, we denote the value of a variable at discrete time step using subscript t from now on.

To find: Discrete-time state transition model

$$\mathbf{x}_t = g(\mathbf{x}_{t-1}, \mathbf{u}_t) \tag{3}$$

g can be interpreted as our belief of the state variables after taking control $\mathbf{u}$ from state $\mathbf{x}_{t-1}$. $\mathbf{x}_t$ is not directly observable due to uncertainty, but assuming g is well-behaved i.e. continuous etc., for small time steps Δt, we may rely on local similarity in order to approximate it. Let $\tilde{\mathbf{x}}_{t-1}$ and $\tilde{\mathbf{u}}_t$ be small perturbations about $\mathbf{x}_{t-1}$ and $\mathbf{u}_t$. We can use Taylor series approximation up to first order terms:

$$\begin{aligned} \mathbf{x}_t = g(\mathbf{x}_{t-1}, \mathbf{u}_t) \approx \tilde{\mathbf{x}}_t &= g(\tilde{\mathbf{x}}_{t-1}, \tilde{\mathbf{u}}_t) \\ &\approx g(\mathbf{x}_{t-1}, \mathbf{u}_t) + G_x(\mathbf{x}_{t-1}, \mathbf{u}_t) \cdot (\tilde{\mathbf{x}}_{t-1} - \mathbf{x}_{t-1}) + G_u(\mathbf{x}_{t-1}, \mathbf{u}_t) \cdot (\tilde{\mathbf{u}}_t - \mathbf{u}_t) \end{aligned} \tag{4}$$

where G_x and G_u are Jacobians.

We also assume a zero-order hold on $\mathbf{u}$, i.e. $\mathbf{u}$ is constant over some time period Δt. For small Δt this is a good approximation. In order to find $\mathbf{x}_t$, first we find $\tilde{\mathbf{x}}_t$ by discretizing the continuous model using small Δt and the zero-order hold.

$$\begin{aligned} \mathbf{x_t} \approx \tilde{\mathbf{x}}_t &= \mathbf{x}_{t-1} + \Delta\mathbf{x} \\ &= \mathbf{x}_{t-1} + \int_0^{\Delta t} \dot{\mathbf{x}}_{t-1} d\tau \end{aligned} \tag{5}$$

Individually,

$$\begin{aligned}\tilde{\theta}_t &= \theta_{t-1} + \int_0^{\Delta t} \omega_t \, d\tau \ , \quad \omega_t \text{ constant} \\ &= \theta_{t-1} + \omega_t \Delta t\end{aligned} \tag{6}$$

$$\begin{aligned}\tilde{x}_t &= x_{t-1} + \int_0^{\Delta t} \dot{x}_{t-1} \, d\tau \\ &= x_{t-1} + \int_0^{\Delta t} V_t \cos(\theta_t) \, d\tau \ , \quad V_t \text{ constant} \\ &= x_{t-1} + V_t \int_0^{\Delta t} \cos(\theta_{t-1} + \omega_t \tau) \, d\tau \\ &= x_{t-1} + \frac{V_t}{\omega_t} \int_0^{\Delta t} \omega_t \cdot \cos(\theta_{t-1} + \omega_t \tau) \, d\tau \\ &= x_{t-1} + \frac{V_t}{\omega_t} \sin(\theta_{t-1} + \omega_t \tau) \Big|_0^{\Delta t} \\ &= x_{t-1} + \frac{V_t}{\omega_t} \Big[\sin(\theta_{t-1} + \omega_t \Delta t) - \sin(\theta_{t-1}) \Big]\end{aligned} \tag{7}$$

Likewise,

$$\begin{aligned}\tilde{y}_t &= y_{t-1} + \int_0^{\Delta t} \dot{y}_{t-1} \, d\tau \\ &= y_{t-1} + \int_0^{\Delta t} V_t \sin(\theta_t) \, d\tau \\ &= y_{t-1} + \frac{V_t}{\omega_t} \int_0^{\Delta t} \omega_t \cdot \sin(\theta_{t-1} + \omega_t \tau) \, d\tau \\ &= y_{t-1} - \frac{V_t}{\omega_t} \Big[\cos(\theta_{t-1} + \omega_t \Delta t) - \cos(\theta_{t-1}) \Big]\end{aligned} \tag{8}$$

The Jacobian G_x at time t is then

$$G_{x,t} = \begin{bmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial y} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial x} & \frac{\partial y}{\partial y} & \frac{\partial y}{\partial \theta} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} & \frac{\partial \theta}{\partial \theta} \end{bmatrix}_t = \begin{bmatrix} 1 & 0 & \frac{\partial x}{\partial \theta} \\ 0 & 1 & \frac{\partial y}{\partial \theta} \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

This is so that we can use the model in an EKF. Let g be our nonlinear, discrete-time state transition model. Since the EKF assumes g to be Markov, at any time $t > t_0$, the state of our unicycle $\mathbf{x}_t$ can be expressed as:

$$\mathbf{x}_t = g(\mathbf{x}_{t-1}, \mathbf{u}_t) \tag{10}$$

$$\mathbf{x}_t = G_{x,t}\mathbf{x}_{t-1} + G_{u,t}\mathbf{u}_t \tag{11}$$

(ii) (code)

(iii) (code)

(iv) (code)

(v) (code)

(vi) (code)

(vii) (code)

(viii) TODO

Problem 2: EKF SLAM

(i) (code)

(ii) (code)

(iii) TODO

Extra Credit: Monte Carlo Localization

(i) (code)

(ii) (code)

(iii) (code)

(iv) TODO

(v) TODO