AA 274A: Principles of Robot Autonomy I
Problem Set 2

Name: Li Quan Khoo
SUID: lqkhoo (06154100)

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Problem 1: A* Motion Planning

(i) (code)

(ii) (please see sim_astar section of merged pdf as requested)

Problem 2: Rapidly-Exploring Random Trees (RRT)

(i) (code)

(ii) (code)

(iii) (code)

(iv) (please see sim_rrt section of merged pdf as requested)

Problem 3: Geometric Planning to Trajectories and Control

(i) (code)

(ii) (code)

(iii) (code)

(iv) (please see sim_traj_planning section of merged pdf as requested)

Extra Problem: Bi-Directional Sampling-based Motion Planning

(i) (code)

(ii) (code)

(iii) (code)

(iv) (please see sim_bidirectional_rrt section of merged pdf as requested)

sim_astar

October 2, 2020

1 A* Motion Planning

```
In [108]: # The autoreload extension will automatically load in new code as you edit files,
          # so you don't need to restart the kernel every time
          %load_ext autoreload
          %autoreload 2
          import numpy as np
          import matplotlib.pyplot as plt
          from P1_astar import DetOccupancyGrid2D, AStar
          from utils import generate_planning_problem
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

1.1 Simple Environment

1.1.1 Workspace

(Try changing this and see what happens)

```
In [109]: width = 10
          height = 10
          obstacles = [((6,7),(8,8)),((2,2),(4,3)),((2,5),(4,7)),((6,3),(8,5))]
          occupancy = DetOccupancyGrid2D(width, height, obstacles)
```

1.1.2 Starting and final positions

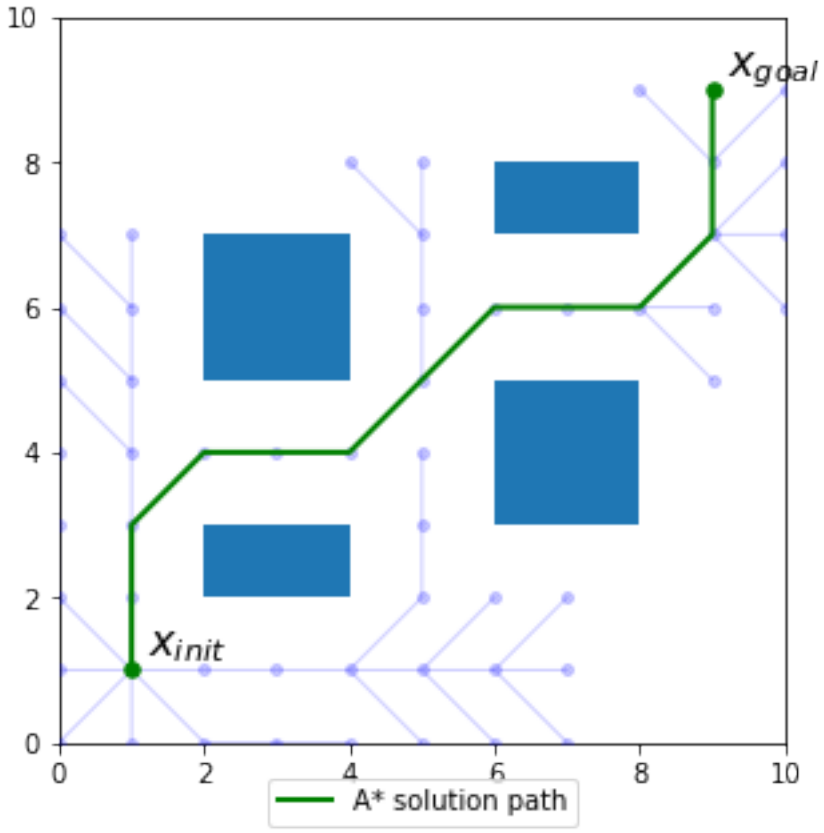
(Try changing these and see what happens)

```
In [110]: x_init = (1, 1)
          x_goal = (9, 9)
```

1.1.3 Run A* planning

```
In [111]: astar = AStar((0, 0), (width, height), x_init, x_goal, occupancy)
          if not astar.solve():
              print "No path found"
          else:
```

```
plt.rcParams['figure.figsize'] = [5, 5]
astar.plot_path()
astar.plot_tree()
```



1.2 Random Cluttered Environment

1.2.1 Generate workspace, start and goal positions

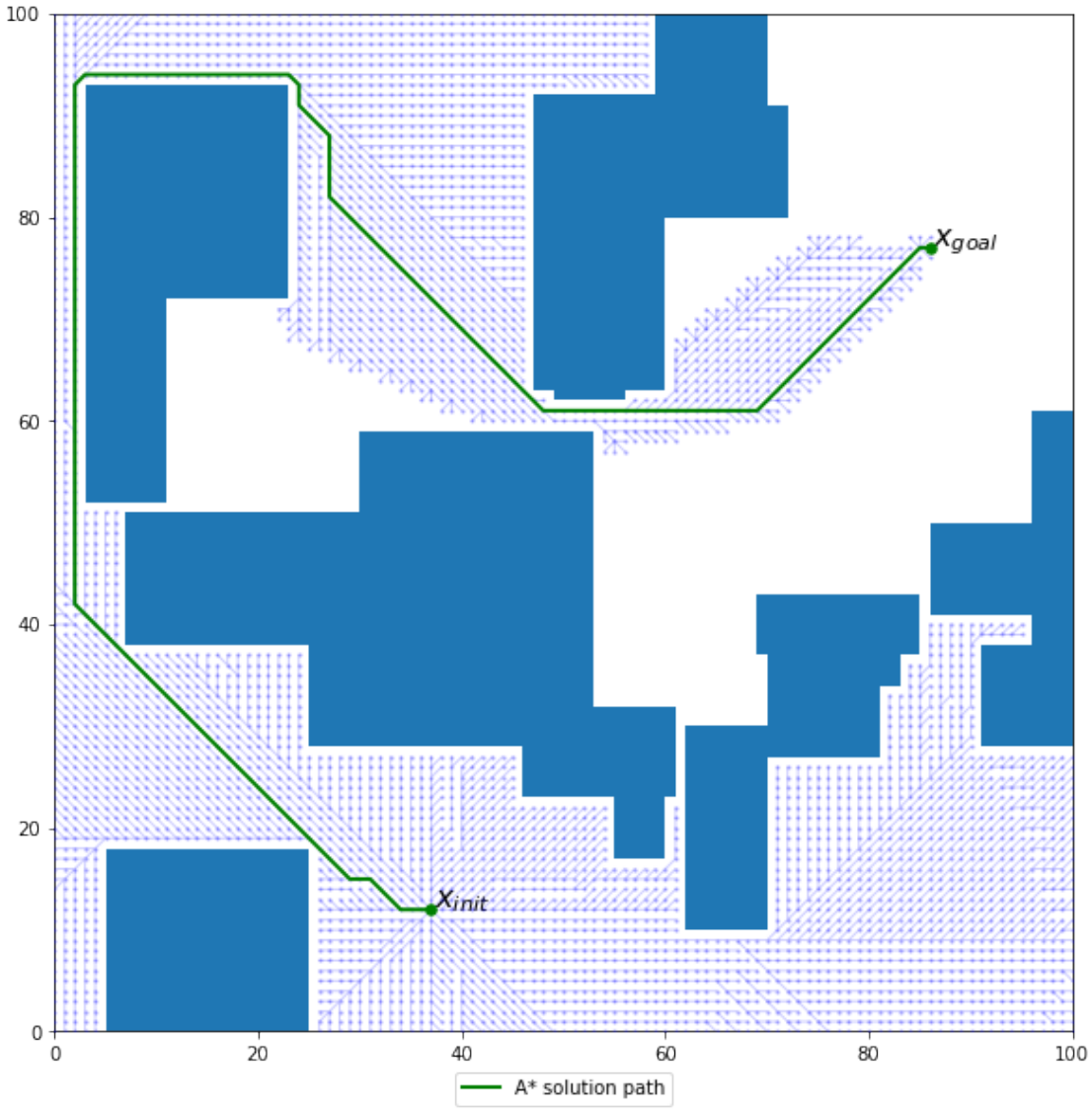
(Try changing these and see what happens)

```
In [134]: width = 100
          height = 100
          num_obs = 25
          min_size = 5
          max_size = 30

          occupancy, x_init, x_goal = generate_planning_problem(width, height, num_obs, min_si
```

1.2.2 Run A* planning

```
In [135]: astar = AStar((0, 0), (width, height), x_init, x_goal, occupancy)
          if not astar.solve():
```

```
        print "No path found"
    else:
        plt.rcParams['figure.figsize'] = [10, 10]
        astar.plot_path()
        astar.plot_tree(point_size=2)
```



In [ ]:

sim_rrt

October 5, 2020

1 RRT Sampling-Based Motion Planning

In [8]:
```
# The autoreload extension will automatically load in new code as you edit files,
# so you don't need to restart the kernel every time
%load_ext autoreload
%autoreload 2

import numpy as np
import matplotlib.pyplot as plt
from P2_rrt import *

plt.rcParams['figure.figsize'] = [8, 8] # Change default figure size
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

1.0.1 Set up workspace

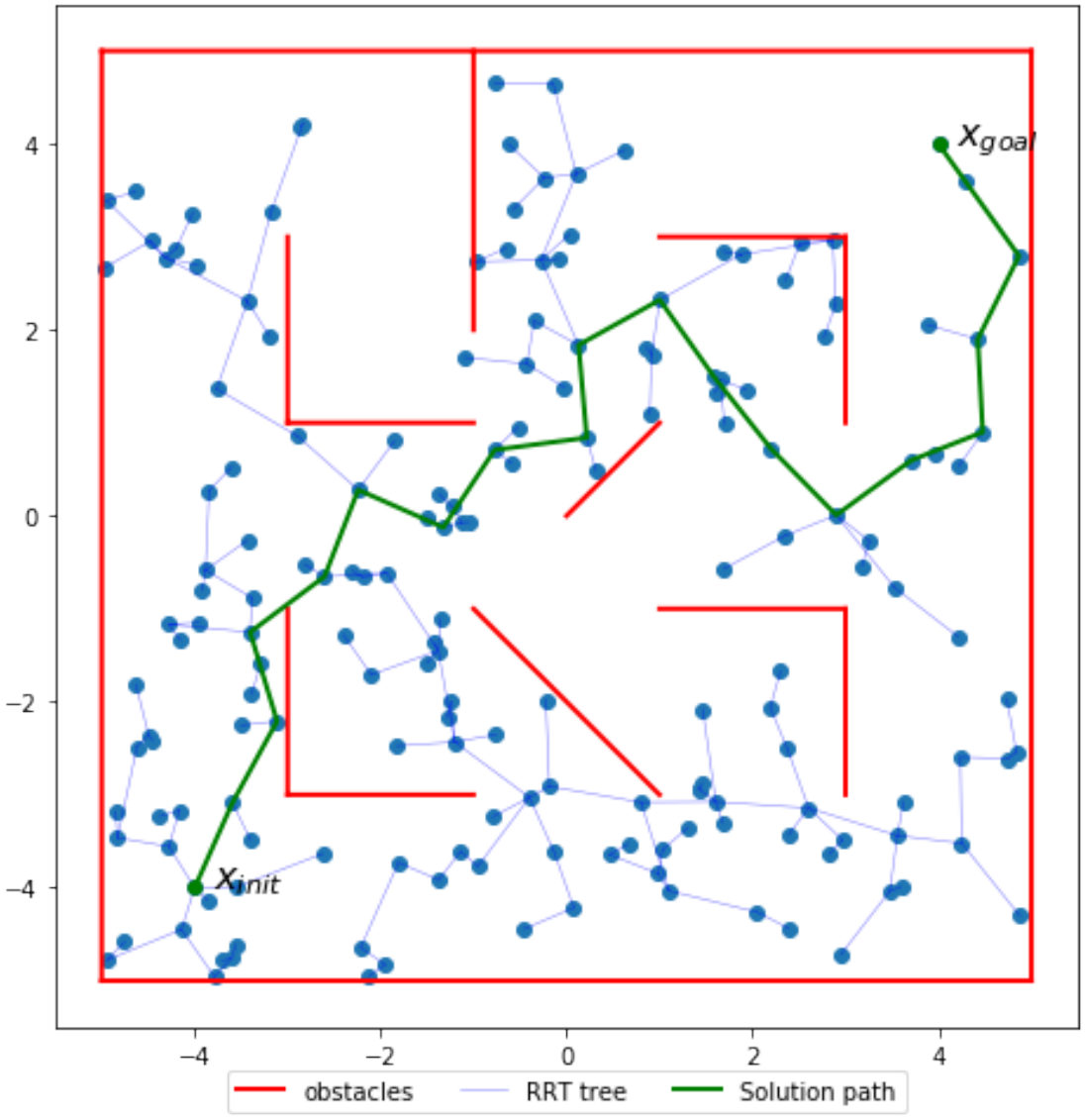
In [9]:
```
MAZE = np.array([
            (( 5, 5), (-5, 5)),
            ((-5, 5), (-5,-5)),
            ((-5,-5), ( 5,-5)),
            (( 5,-5), ( 5, 5)),
            ((-3,-3), (-3,-1)),
            ((-3,-3), (-1,-3)),
            (( 3, 3), ( 3, 1)),
            (( 3, 3), ( 1, 3)),
            (( 1,-1), ( 3,-1)),
            (( 3,-1), ( 3,-3)),
            ((-1, 1), (-3, 1)),
            ((-3, 1), (-3, 3)),
            ((-1,-1), ( 1,-3)),
            ((-1, 5), (-1, 2)),
            (( 0, 0), ( 1, 1))
        ])

# try changing these!
```

```
x_init = [-4,-4] # reset to [-4,-4] when saving results for submission
x_goal = [4,4] # reset to [4,4] when saving results for submission
```

1.1 Geometric Planning

In [13]:
```
grrt = GeometricRRT([-5,-5], [5,5], x_init, x_goal, MAZE)
grrt.solve(1.0, 2000)
```

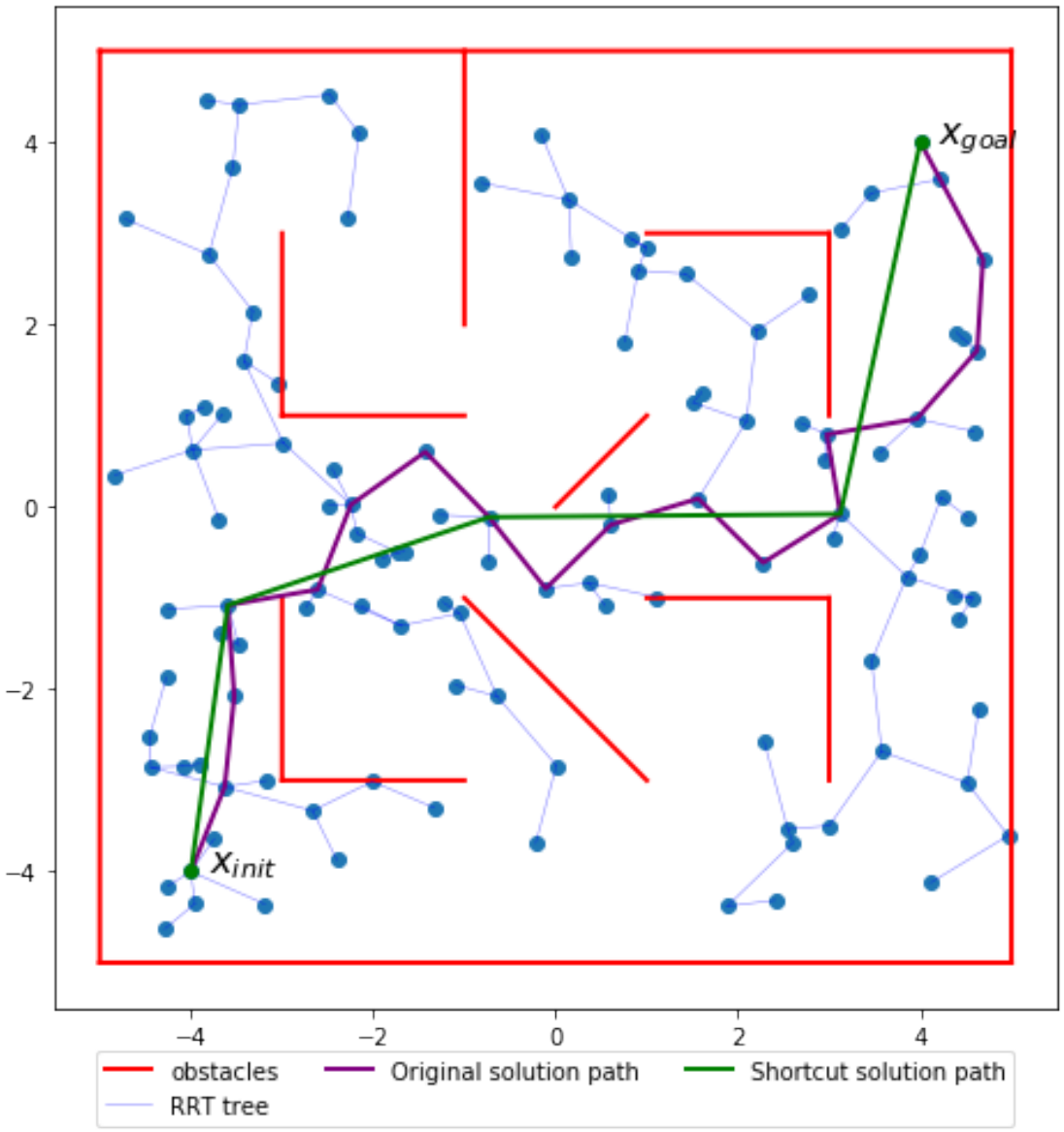
Out[13]: True



1.1.1 Adding shortcutting

In [19]:
```
grrt.solve(1.0, 2000, shortcut=True)
```

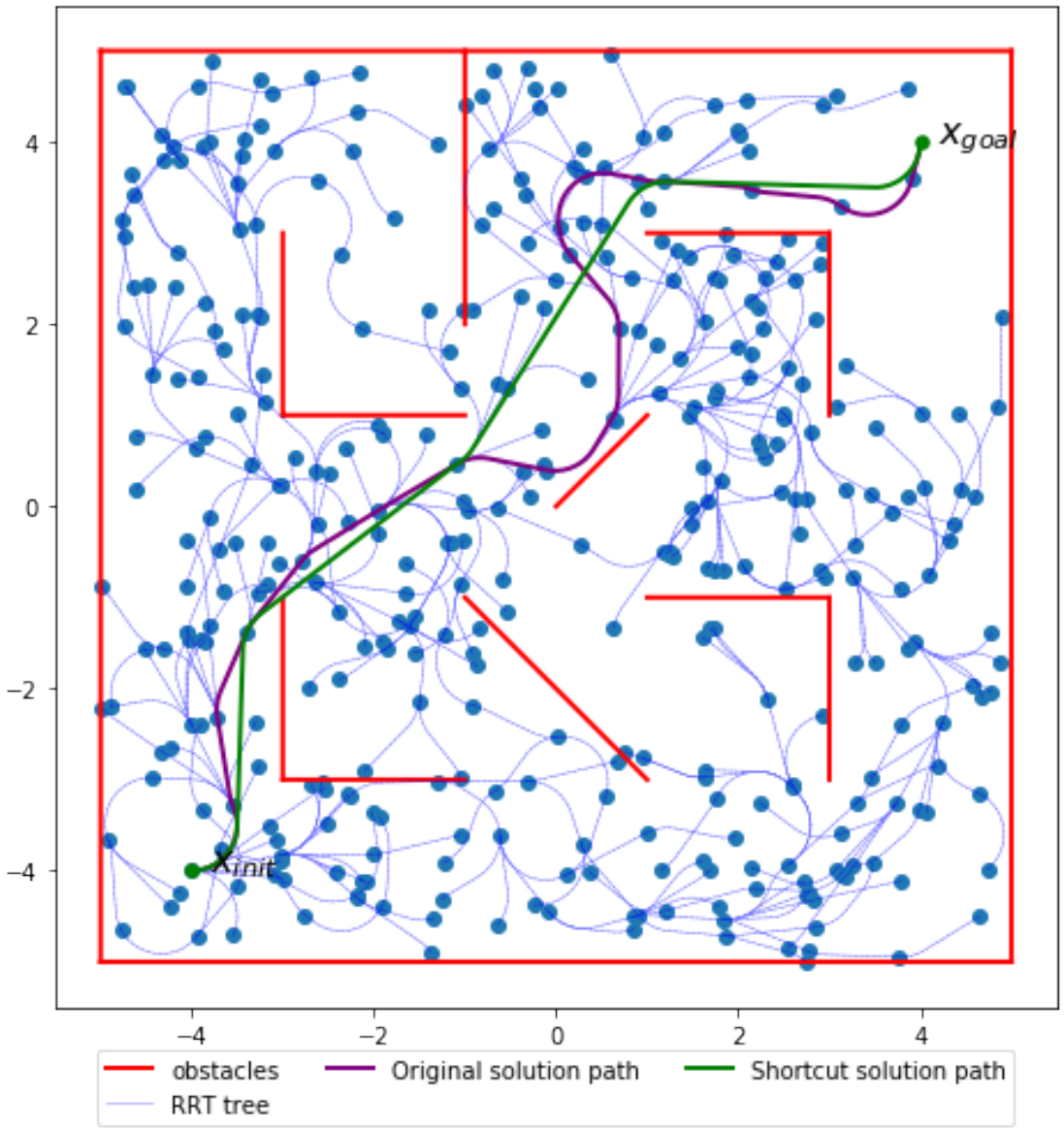
Out[19]: True



1.2 Dubins Car Planning

```
In [39]: x_init = [-4,-4,0]
         x_goal = [4,4,np.pi/2]

         # Occasionally, depending on the generated problem, the solution will fail
         # due to numerical precision invalidating the state equality comparison with the goal
         drrt = DubinsRRT([-5,-5,0], [5,5,2*np.pi], x_init, x_goal, MAZE, .5)
         drrt.solve(1.0, 1000, shortcut=True)
```

Out[39]: True



In [ ]:

sim_traj_planning

October 4, 2020

In [8]:
```
# The autoreload extension will automatically load in new code as you edit files,
# so you don't need to restart the kernel every time
%load_ext autoreload
%autoreload 2

import numpy as np
from P1_astar import DetOccupancyGrid2D, AStar
from P2_rrt import *
from P3_traj_planning import compute_smoothed_traj, modify_traj_with_limits, SwitchingC
import scipy.interpolate
import matplotlib.pyplot as plt
from HW1.P1_differential_flatness import *
from HW1.P2_pose_stabilization import *
from HW1.P3_trajectory_tracking import *
from utils import generate_planning_problem

plt.rcParams['figure.figsize'] = [14, 14] # Change default figure size
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

0.0.1 Generate workspace, start and goal positions

In [29]:
```
width = 100
height = 100
num_obs = 25
min_size = 5
max_size = 30

occupancy, x_init, x_goal = generate_planning_problem(width, height, num_obs, min_size
```

0.0.2 Solve A* planning problem

In [30]:
```
astar = AStar((0, 0), (width, height), x_init, x_goal, occupancy)
if not astar.solve():
    print "No path found"
```

0.1 Smooth Trajectory Generation

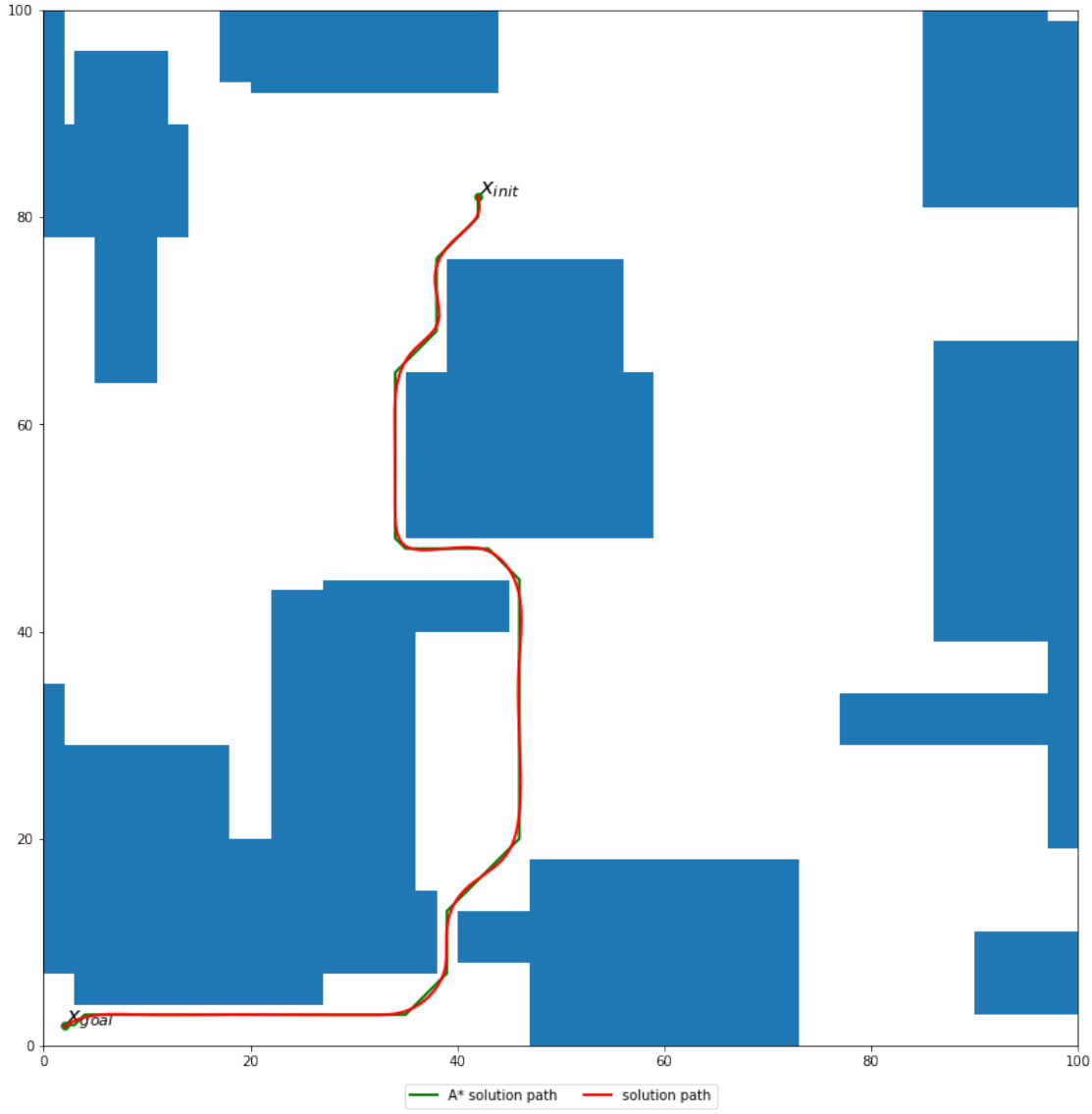
0.1.1 Trajectory parameters

(Try changing these and see what happens)

```
In [31]: V_des = 0.3  # Nominal velocity
         alpha = 3.0   # Smoothness parameter
         dt = 0.05
```

0.1.2 Generate smoothed trajectory

```
In [32]: traj_smoothed, t_smoothed = compute_smoothed_traj(astar.path, V_des, alpha, dt)

         fig = plt.figure()
         astar.plot_path(fig.number)
         def plot_traj_smoothed(traj_smoothed):
             plt.plot(traj_smoothed[:,0], traj_smoothed[:,1], color="red", linewidth=2, label='
         plot_traj_smoothed(traj_smoothed)
         plt.legend(loc='upper center', bbox_to_anchor=(0.5, -0.03), fancybox=True, ncol=3)
         plt.show()
```



0.2 Control-Feasible Trajectory Generation and Tracking

0.2.1 Robot control limits

```
In [33]: V_max = 0.5 # max speed
         om_max = 1 # max rotational speed
```

0.2.2 Tracking control gains

Tune these as needed to improve tracking performance.

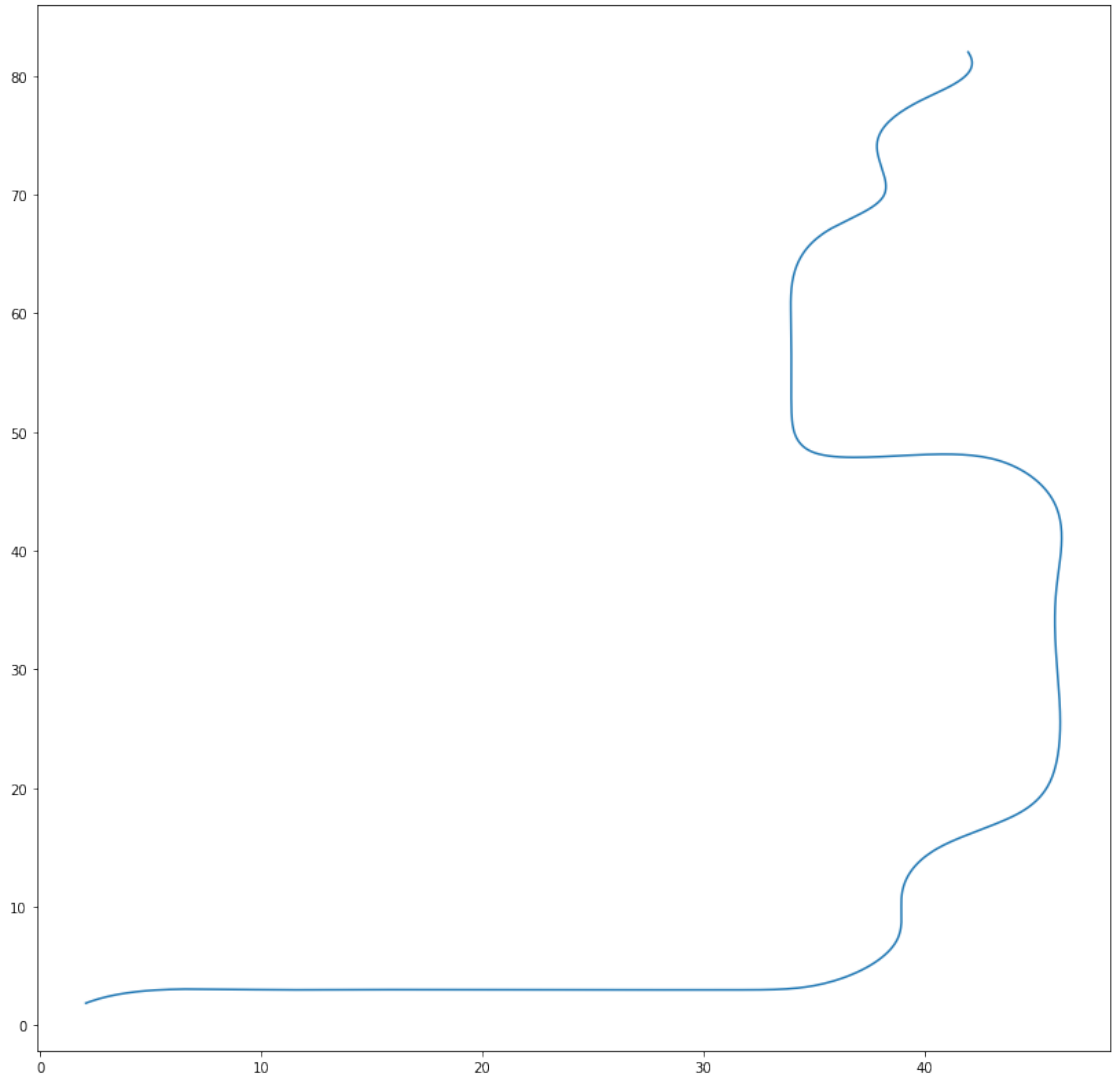
```
In [34]: kpx = 2
         kpy = 2
```

```
kdx = 2
kdy = 2
```

0.2.3 Generate control-feasible trajectory

In [36]:
```
t_new, V_smooth_scaled, om_smooth_scaled, traj_smooth_scaled = modify_traj_with_limits

#TODO REMOVE BELOW
traj_t = traj_smooth_scaled.T
x = traj_t[0]
y = traj_t[1]

V_t = V_smooth_scaled.T

fig = plt.figure()
plt.plot(x, y)
```

Out[36]:
```
[<matplotlib.lines.Line2D at 0x263e73c8>]
```



0.2.4 Create trajectory controller and load trajectory

```
In [45]: traj_controller = TrajectoryTracker(kpx=kpx, kpy=kpy, kdx=kdx, kdy=kdy, V_max=V_max, c
         traj_controller.load_traj(t_new, traj_smooth_scaled)
```

0.2.5 Set simulation input noise

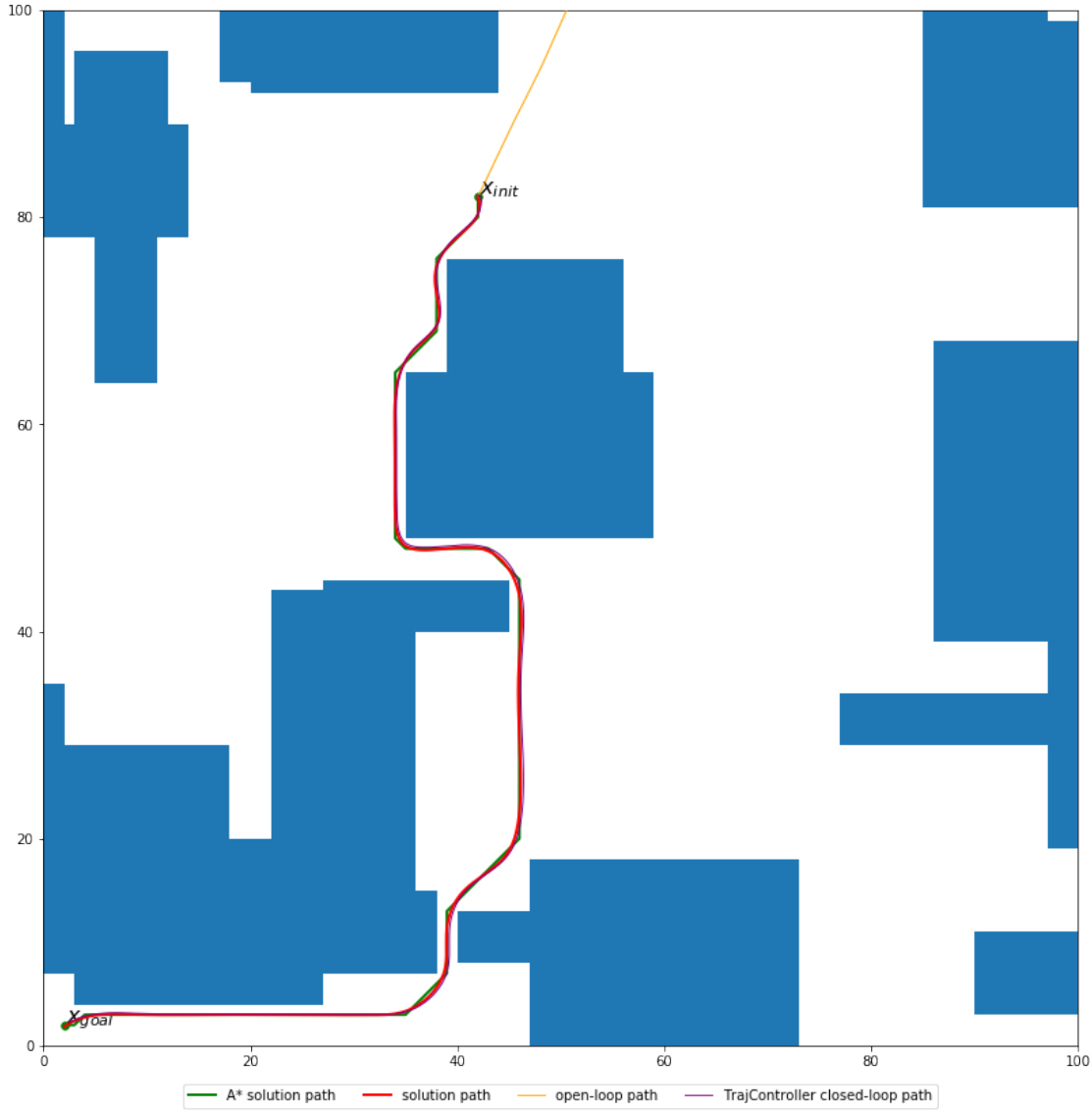
(Try changing this and see what happens)

```
In [46]: noise_scale = 0.05
```

0.2.6 Simulate closed-loop tracking of smoothed trajectory, compare to open-loop

```
In [47]: tf_actual = t_new[-1]
         times_cl = np.arange(0, tf_actual, dt)
```

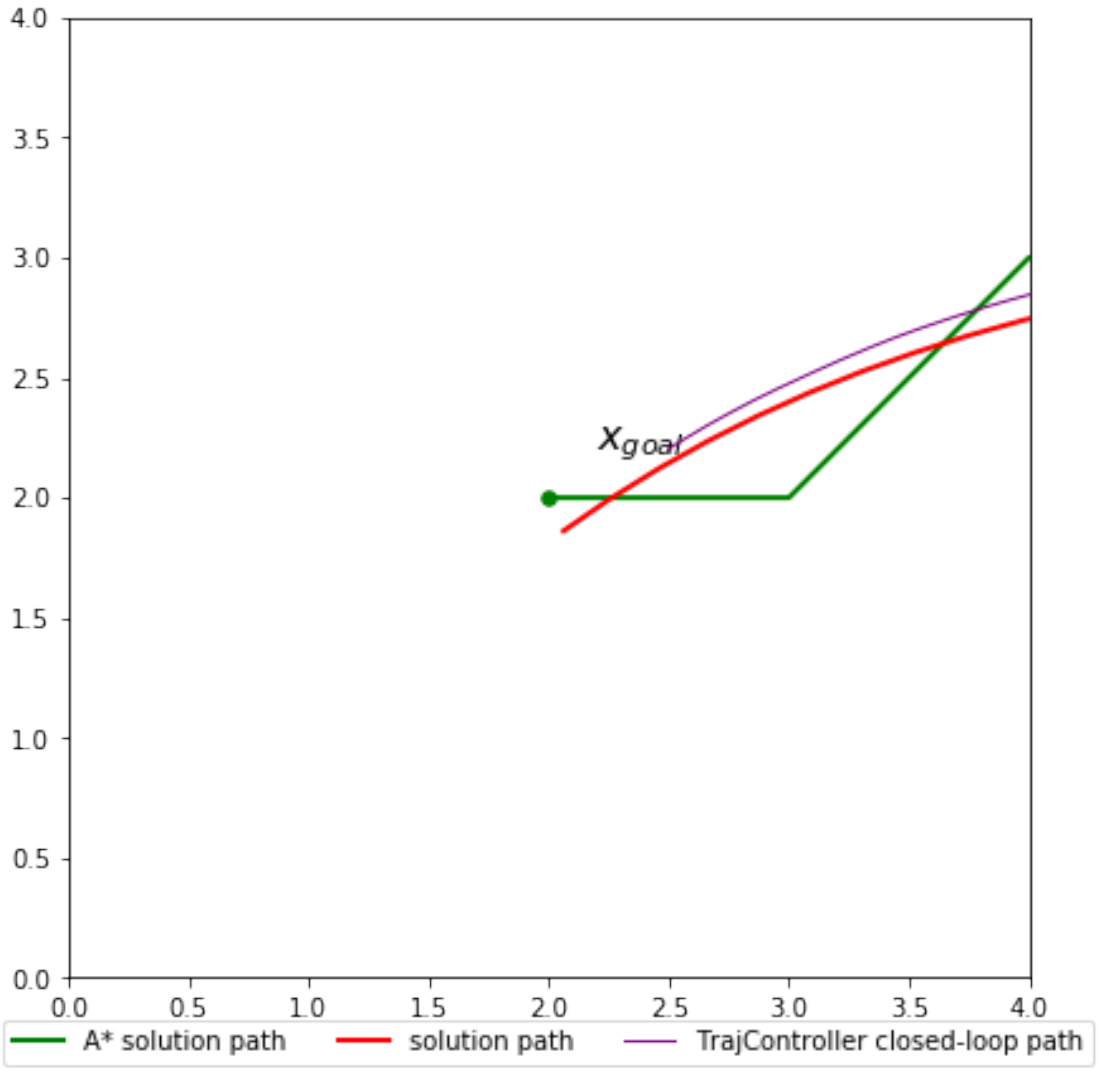
```
s_0 = State(x=x_init[0], y=x_init[1], V=V_max, th=traj_smooth_scaled[0,2])
s_f = State(x=x_goal[0], y=x_goal[1], V=V_max, th=traj_smooth_scaled[-1,2])

actions_ol = np.stack([V_smooth_scaled, om_smooth_scaled], axis=-1)
states_ol, ctrl_ol = simulate_car_dyn(s_0.x, s_0.y, s_0.th, times_cl, actions=actions
states_cl, ctrl_cl = simulate_car_dyn(s_0.x, s_0.y, s_0.th, times_cl, controller=traj

fig = plt.figure()
astar.plot_path(fig.number)
plot_traj_smoothed(traj_smoothed)
def plot_traj_ol(states_ol):
    plt.plot(states_ol[:,0],states_ol[:,1], color="orange", linewidth=1, label="open-
def plot_traj_cl(states_cl):
    plt.plot(states_cl[:,0], states_cl[:,1], color="purple", linewidth=1, label="Traj
plot_traj_ol(states_ol)
plot_traj_cl(states_cl)
plt.legend(loc='upper center', bbox_to_anchor=(0.5, -0.03), fancybox=True, ncol=4)
plt.show()
```



0.3 Switching from Trajectory Tracking to Pose Stabilization Control

0.3.1 Zoom in on final pose error

In [48]:
```
l_window = 4.

fig = plt.figure(figsize=[7,7])
astar.plot_path(fig.number)
plot_traj_smoothed(traj_smoothed)
plot_traj_cl(states_cl)
plt.legend(loc='upper center', bbox_to_anchor=(0.5, -0.03), fancybox=True, ncol=3)
plt.axis([x_goal[0]-l_window/2, x_goal[0]+l_window/2, x_goal[1]-l_window/2, x_goal[1]+
plt.show()
```



0.3.2 Pose stabilization control gains

Tune these as needed to improve final pose stabilization.

```
In [49]: k1 = 1.
         k2 = 1.
         k3 = 1.
```

0.3.3 Create pose controller and load goal pose

Note we use the last value of the smoothed trajectory as the goal heading θ

```
In [50]: pose_controller = PoseController(k1, k2, k3, V_max, om_max)
         pose_controller.load_goal(x_goal[0], x_goal[1], traj_smooth_scaled[-1,2])
```

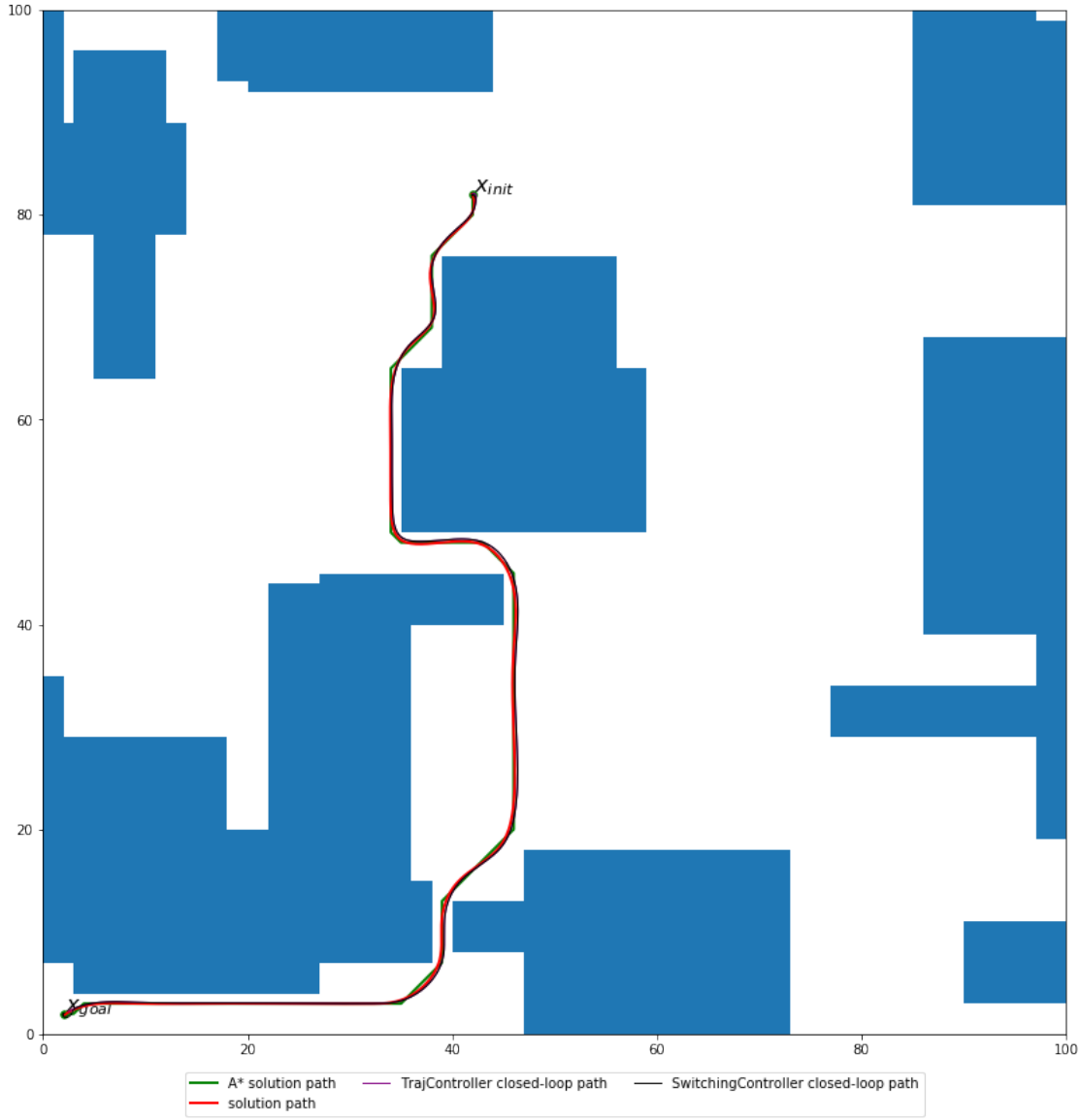
0.3.4 Time before trajectory-tracking completion to switch to pose stabilization

Try changing this!

```
In [58]: t_before_switch = 3.0
```

0.3.5 Create switching controller and compare performance

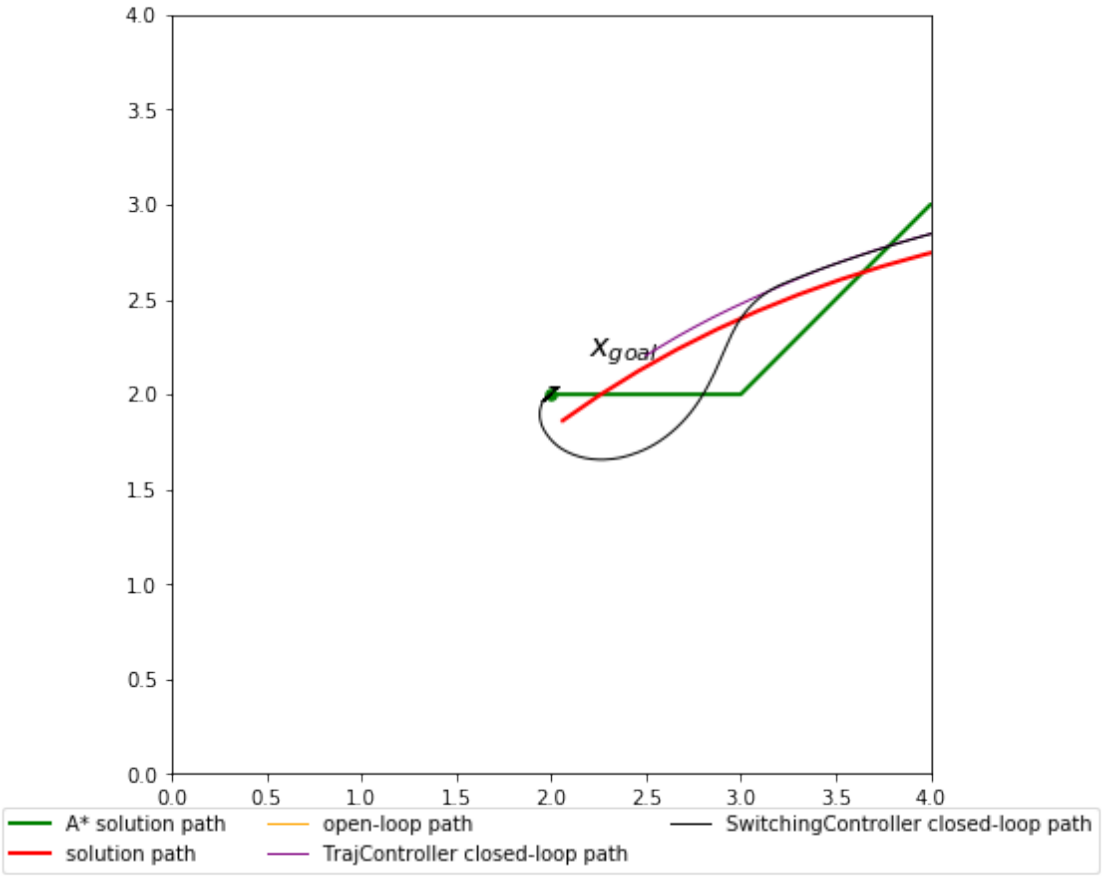
```
In [59]: switching_controller = SwitchingController(traj_controller, pose_controller, t_before

         t_extend = 60.0 # Extra time to simulate after the end of the nominal trajectory
         times_cl_extended = np.arange(0, tf_actual+t_extend, dt)
         states_cl_sw, ctrl_cl_sw = simulate_car_dyn(s_0.x, s_0.y, s_0.th, times_cl_extended,

         fig = plt.figure()
         astar.plot_path(fig.number)
         plot_traj_smoothed(traj_smoothed)
         plot_traj_cl(states_cl)
         def plot_traj_cl_sw(states_cl_sw):
             plt.plot(states_cl_sw[:,0], states_cl_sw[:,1], color="black", linewidth=1, label="
         plot_traj_cl_sw(states_cl_sw)
         plt.legend(loc='upper center', bbox_to_anchor=(0.5, -0.03), fancybox=True, ncol=3)
         plt.show()
```



0.3.6 Zoom in on final pose

In [60]:
```
l_window = 4.

fig = plt.figure(figsize=[7,7])
astar.plot_path(fig.number)
plot_traj_smoothed(traj_smoothed)
plot_traj_ol(states_ol)
plot_traj_cl(states_cl)
plot_traj_cl_sw(states_cl_sw)
plt.legend(loc='upper center', bbox_to_anchor=(0.5, -0.03), fancybox=True, ncol=3)
```

```
plt.axis([x_goal[0]-l_window/2, x_goal[0]+l_window/2, x_goal[1]-l_window/2, x_goal[1]+
plt.show()
```



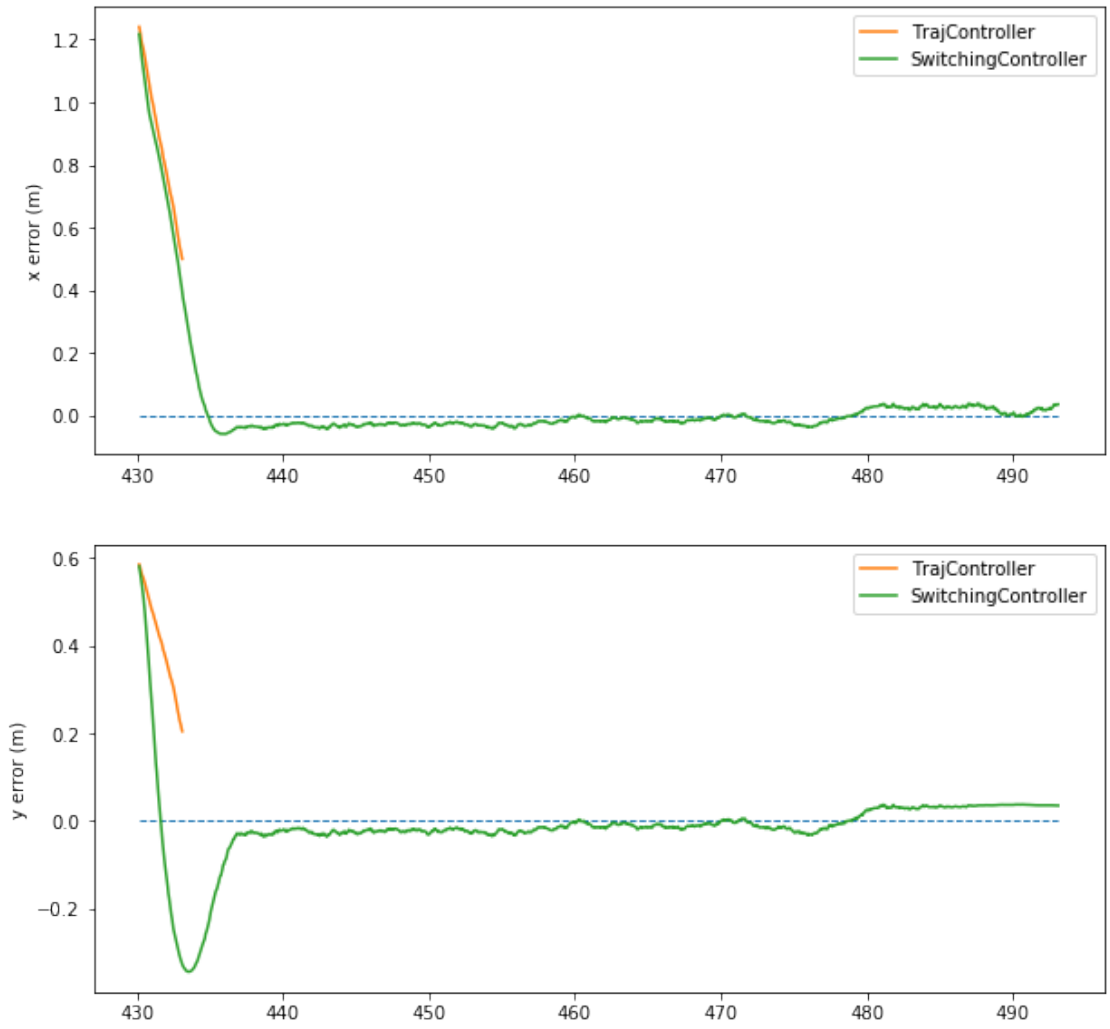
0.3.7 Plot final sequence of states

To see just how well we're able to arrive at the target point (and to assist in choosing values for the pose stabilization controller gains k_1, k_2, k_3), we plot the error in x and y for both the tracking controller and the switching controller at the end of the trajectory.

```
In [61]: T = len(times_cl) - int(t_before_switch/dt)
         fig = plt.figure(figsize=[10,10])
         plt.subplot(2,1,1)
         plt.plot([times_cl_extended[T], times_cl_extended[-1]], [0,0], linestyle='--', linewi
         plt.plot(times_cl[T:], states_cl[T:,0] - x_goal[0], label='TrajController')
         plt.plot(times_cl_extended[T:], states_cl_sw[T:,0] - x_goal[0], label='SwitchingContr
         plt.legend()
         plt.ylabel("x error (m)")
         plt.subplot(2,1,2)
         plt.plot([times_cl_extended[T], times_cl_extended[-1]], [0,0], linestyle='--', linewi
         plt.plot(times_cl[T:], states_cl[T:,1] - x_goal[1], label='TrajController')
```

```
        plt.plot(times_cl_extended[T:], states_cl_sw[T:,1] - x_goal[1], label='SwitchingContro
        plt.legend()
        plt.ylabel("y error (m)")
```

Out[61]: Text(0,0.5,'y error (m)')



In [ ]:

sim_bidirectional_rrt

October 5, 2020

1 Bidirectional Sampling-Based Motion Planning

```
In [ ]: # The autoreload extension will automatically load in new code as you edit files,
        # so you don't need to restart the kernel every time
        %load_ext autoreload
        %autoreload 2

        import numpy as np
        import matplotlib.pyplot as plt
        from P2_rrt import *
        from P4_bidirectional_rrt import *

        plt.rcParams['figure.figsize'] = [7, 7] # Change default figure size
```

1.0.1 Set up workspace

```
In [27]: MAZE = np.array([
             (( 5, 5), (-5, 5)),
             ((-5, 5), (-5,-5)),
             ((-5,-5), ( 5,-5)),
             (( 5,-5), ( 5, 5)),
             ((-5, 2), (-1, 2)),
             ((-1, 2), (-1,-1)),
             (( 0, 2), ( 0,-1)),
             (( 0, 2), ( 5, 2))
         ])
```

1.1 Normal RRT

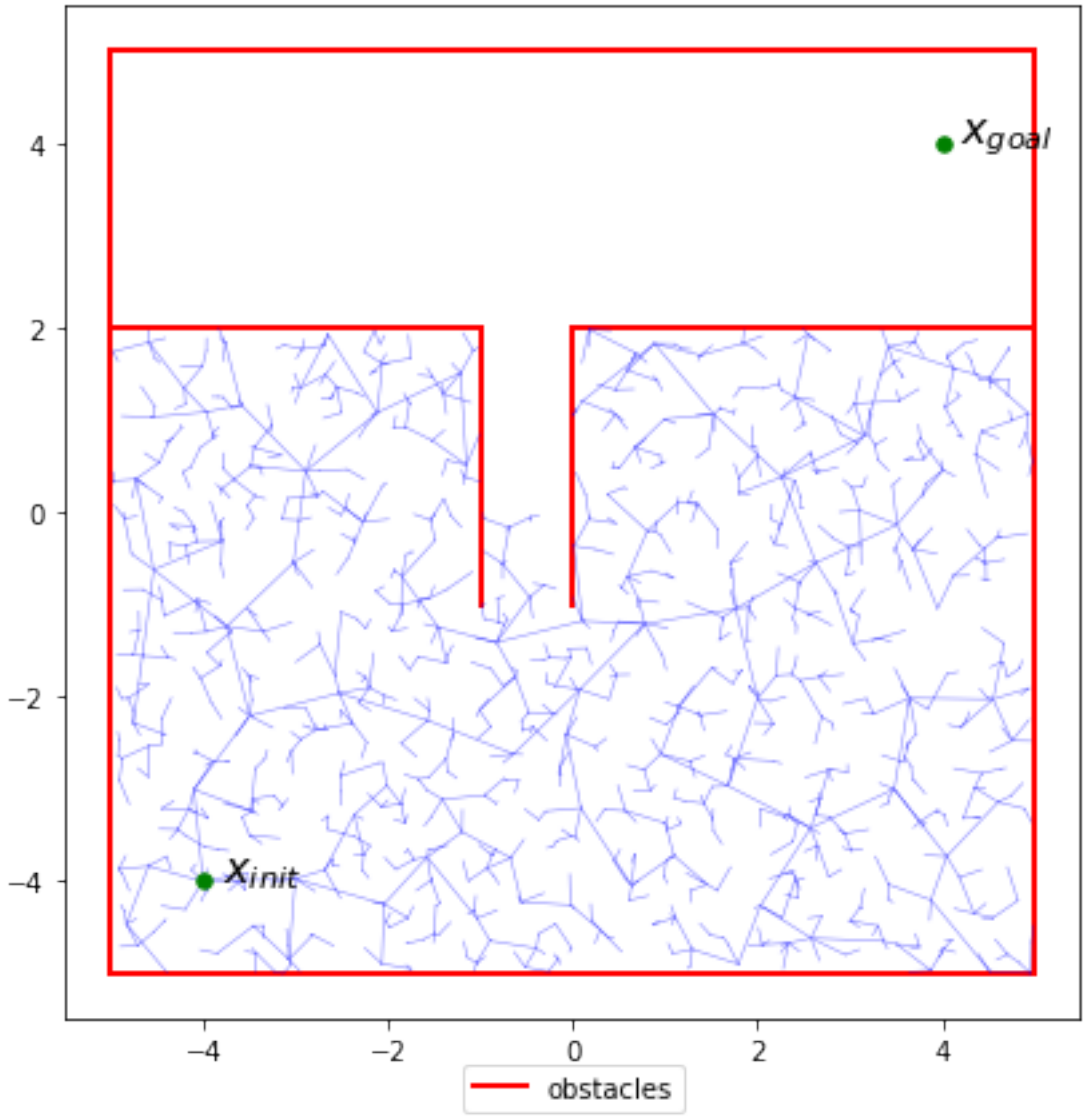
On this "bugtrap" problem, normal RRT often will fail to find a find a path.

1.1.1 Geometric planning

```
In [30]: grrt = GeometricRRT([-5,-5], [5,5], [-4,-4], [4,4], MAZE)
         grrt.solve(1.0, 2000)
```

```
Solution not found!
```

Out[30]: False

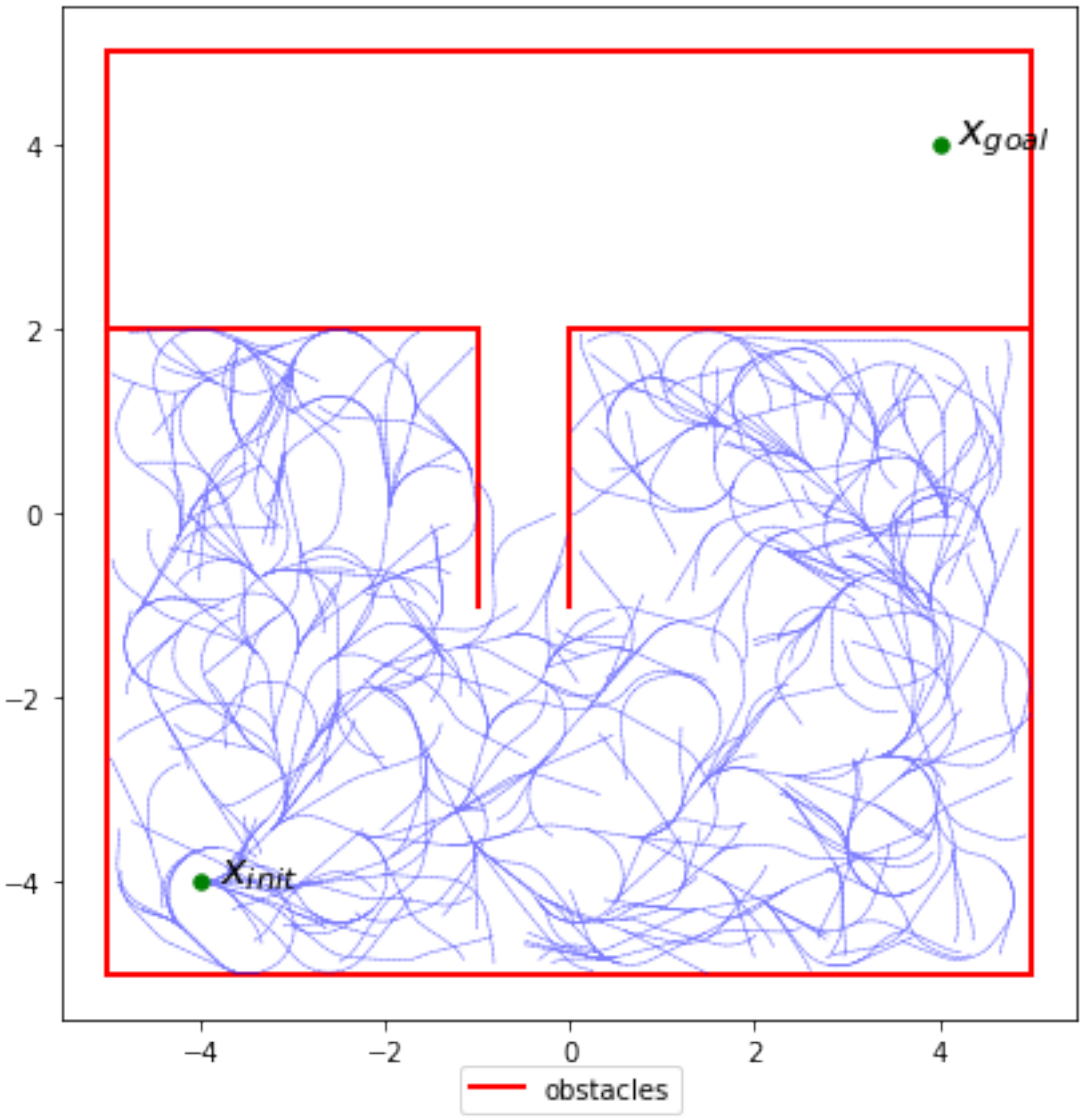


1.1.2 Dubins car planning

```
In [29]: drrt = DubinsRRT([-5,-5,0], [5,5,2*np.pi], [-4,-4,0], [4,4,np.pi/2], MAZE, .5)
         drrt.solve(1.0, 1000)
```

```
Solution not found!
```

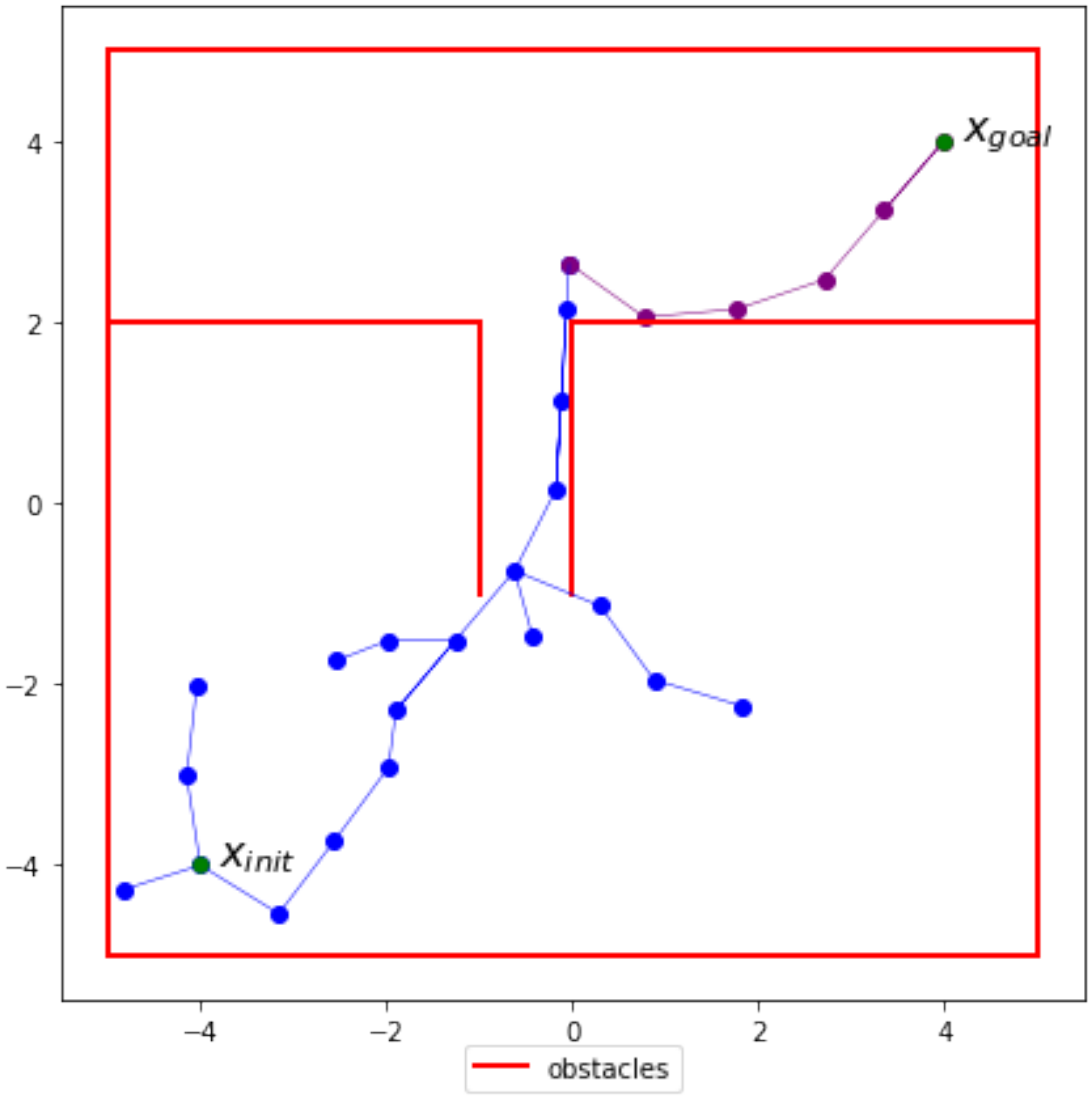
Out[29]: False



1.2 RRTConnect

1.2.1 Geometric planning

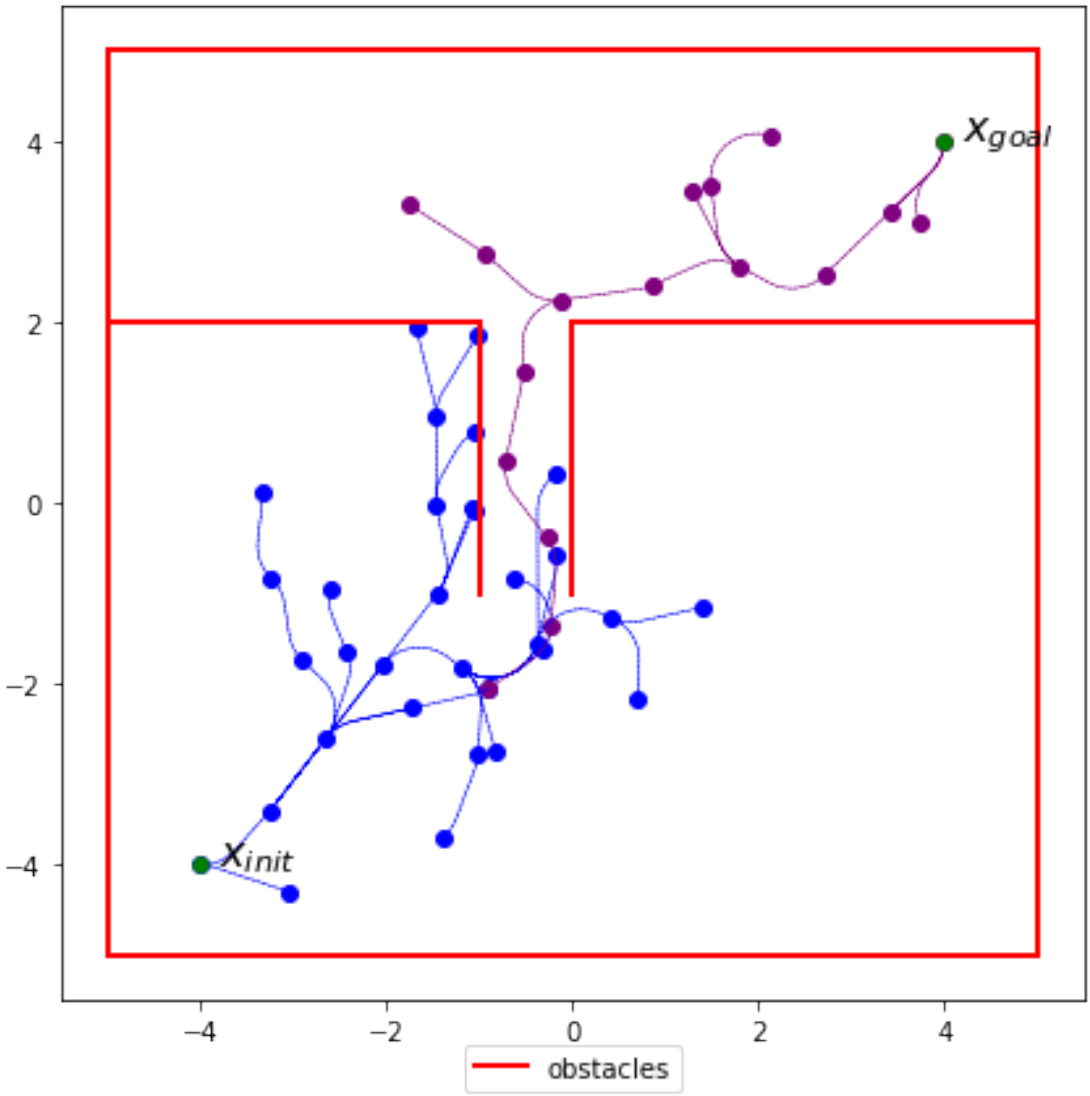
```
In [32]: grrt = GeometricRRTConnect([-5,-5], [5,5], [-4,-4], [4,4], MAZE)
         grrt.solve(1.0, 2000)

found: True
outofmem: False
```



1.2.2 Dubins car planning

```
In [33]: drrt = DubinsRRTConnect([-5,-5,0], [5,5,2*np.pi], [-4,-4,0], [4,4,np.pi/2], MAZE, .5)
         drrt.solve(1.0, 1000)
```

```
found: True
outofmem: False
```



In [ ]: