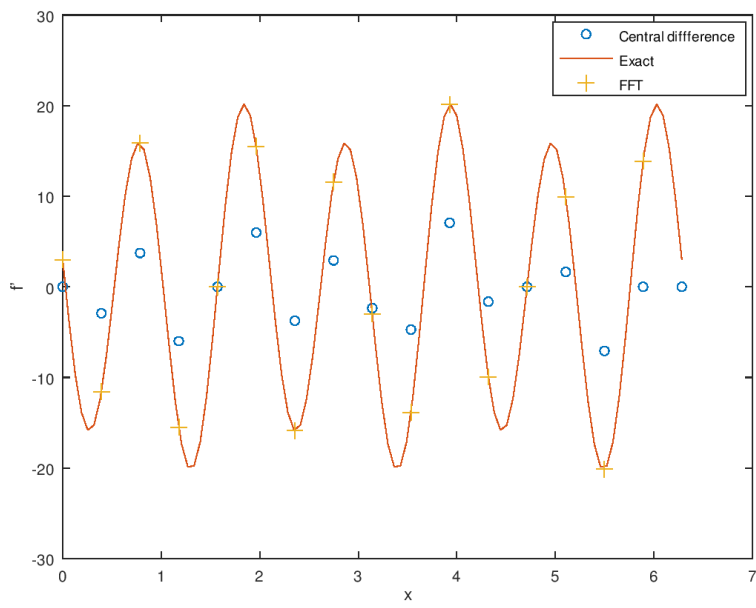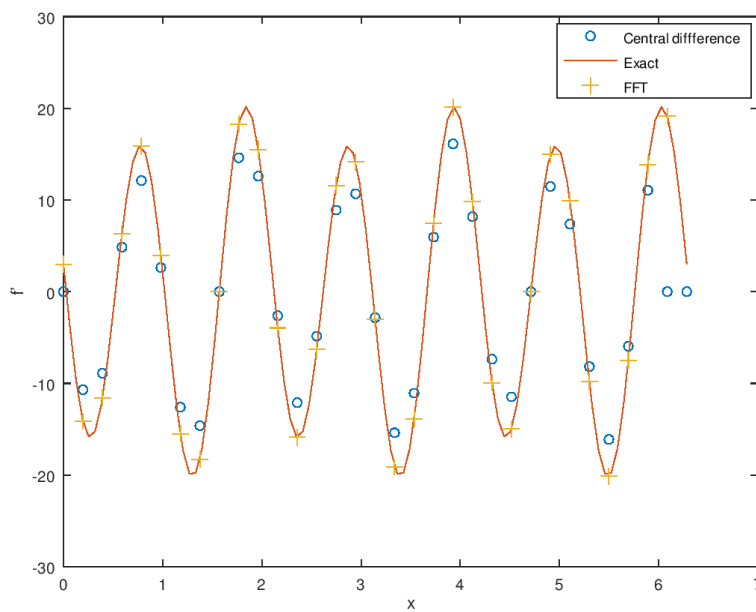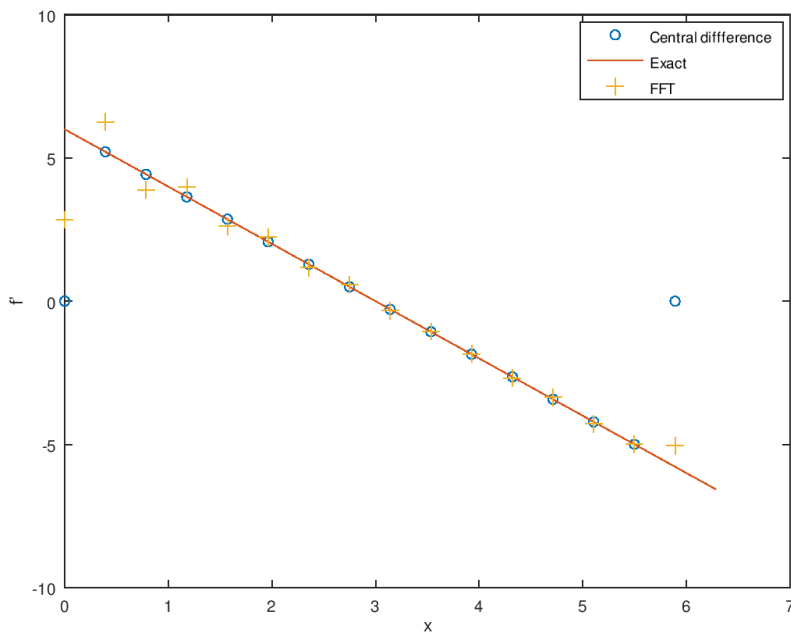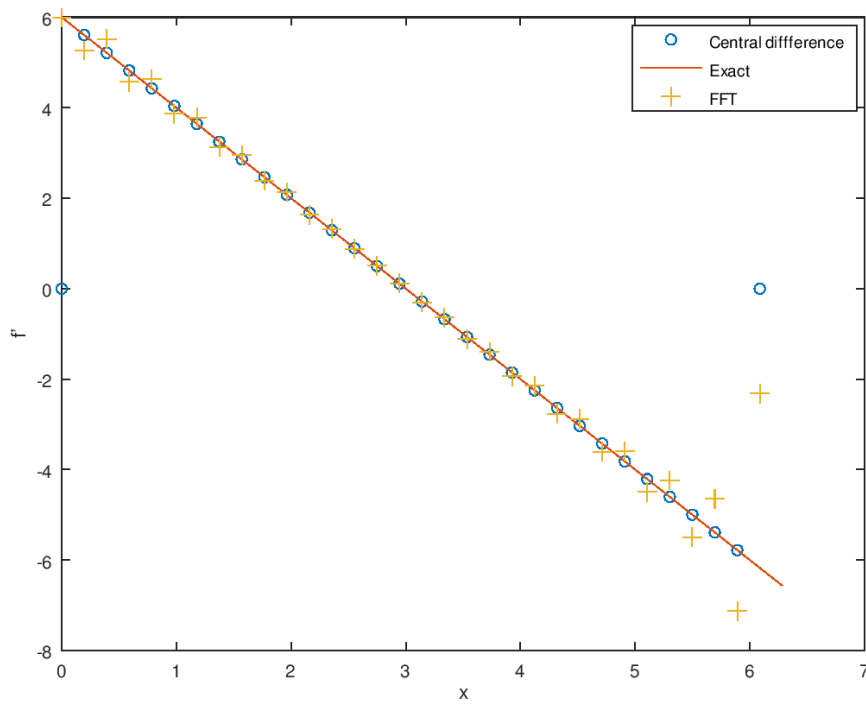B.1

N=16

N=32

B.2

N=16

N=32

B.1 的 f 微分後為週期性函數,所以用 FFT 會比較準確。
B.2 的 f 微分後是線性,所以 central difference 會沒有誤差。

C.1

```
>> f
f =

 Columns 1 through 12:

    0.00000    0.40219    0.66884    0.97944    0.92929    1.00703    0.61472    0.48076   -0.10000   -0.28460   -0.79949   -0.84073

 Columns 13 through 24:

   -1.07071   -0.86832   -0.74538   -0.36317    0.00000    0.36317    0.74538    0.86832    1.07071    0.84073    0.79949    0.28460

 Columns 25 through 32:

    0.10000   -0.48076   -0.61472   -1.00703   -0.92929   -0.97944   -0.66884   -0.40219

>> g
g =

 Columns 1 through 9:

    1.0000e-01    2.8460e-01    7.9949e-01    8.4073e-01    1.0707e+00    8.6832e-01    7.4538e-01    3.6317e-01   -1.4702e-16

 Columns 10 through 18:

   -3.6317e-01   -7.4538e-01   -8.6832e-01   -1.0707e+00   -8.4073e-01   -7.9949e-01   -2.8460e-01   -1.0000e-01    4.8076e-01

 Columns 19 through 27:

    6.1472e-01    1.0070e+00    9.2929e-01    9.7944e-01    6.6884e-01    4.0219e-01    1.8864e-15   -4.0219e-01   -6.6884e-01

 Columns 28 through 32:

   -9.7944e-01   -9.2929e-01   -1.0070e+00   -6.1472e-01   -4.8076e-01

>> H
H =

 Columns 1 through 12:

    0.00000    0.11447    0.53473    0.82344    0.99500    0.87442    0.45820    0.17460    0.00000    0.10336    0.59592    0.73003

 Columns 13 through 24:

    1.14642    0.73003    0.59592    0.10336   -0.00000    0.17460    0.45820    0.87442    0.99500    0.82344    0.53473    0.11447

 Columns 25 through 32:

    0.00000    0.19336    0.41115    0.98632    0.86358    0.98632    0.41115    0.19336

>> Hhat
Hhat =

 Columns 1 through 5:

   16.00000 +  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.08000i    0.00000 -  0.00000i   -8.00000 +  0.00000i

 Columns 6 through 10:

   -0.00000 +  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 -  0.00000i

 Columns 11 through 15:

    0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 -  0.00000i    0.80000 +  0.80000i   -0.00000 +  0.00000i

 Columns 16 through 20:

   -0.80000 +  0.80000i    0.00000 +  0.00000i   -0.80000 -  0.80000i   -0.00000 -  0.00000i    0.80000 -  0.80000i

 Columns 21 through 25:

   -0.00000 +  0.00000i   -0.00000 -  0.00000i    0.00000 -  0.00000i   -0.00000 +  0.00000i   -0.00000 -  0.00000i

 Columns 26 through 30:

   -0.00000 -  0.00000i    0.00000 -  0.00000i   -0.00000 -  0.00000i   -8.00000 -  0.00000i    0.00000 +  0.00000i

 Columns 31 and 32:

   -0.00000 -  0.08000i    0.00000 -  0.00000i

>>
```

Real function:

C.2

```
>> fhat
fhat =

 Columns 1 through 5:

    0.00000 +  0.00000i    0.00000 +  0.00000i   -0.00000 - 16.00000i    0.00000 -  0.00000i   -0.00000 +  0.00000i

 Columns 6 through 10:

   -0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 +  0.00000i    0.00000 -  0.00000i    0.00000 -  0.00000i

 Columns 11 through 15:

    0.00000 -  0.00000i    0.00000 +  0.00000i    0.00000 +  0.00000i    0.00000 +  0.00000i    0.00000 +  0.00000i

 Columns 16 through 20:

   -0.00000 -  1.60000i   -0.00000 +  0.00000i   -0.00000 +  1.60000i    0.00000 -  0.00000i    0.00000 -  0.00000i

 Columns 21 through 25:

    0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 +  0.00000i    0.00000 +  0.00000i    0.00000 +  0.00000i

 Columns 26 through 30:

    0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 -  0.00000i    0.00000 +  0.00000i

 Columns 31 and 32:

   -0.00000 + 16.00000i    0.00000 -  0.00000i


>> ghat
ghat =

 Columns 1 through 5:

   -0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 - 16.00000i    0.00000 -  0.00000i    0.00000 -  0.00000i

 Columns 6 through 10:

   -0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 +  0.00000i

 Columns 11 through 15:

   -0.00000 +  0.00000i   -0.00000 +  0.00000i    0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 -  0.00000i

 Columns 16 through 20:

    1.60000 -  0.00000i    0.00000 +  0.00000i    1.60000 +  0.00000i   -0.00000 +  0.00000i    0.00000 -  0.00000i

 Columns 21 through 25:

    0.00000 +  0.00000i   -0.00000 -  0.00000i   -0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 +  0.00000i

 Columns 26 through 30:

    0.00000 +  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i    0.00000 +  0.00000i    0.00000 +  0.00000i

 Columns 31 and 32:

   -0.00000 + 16.00000i   -0.00000 -  0.00000i

>> hhat
hhat =

 Columns 1 through 5:

    0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 -  0.00000i    0.80000 -  0.80000i   -0.00000 +  0.00000i

 Columns 6 through 10:

   -0.00000 +  0.00000i    0.00000 -  0.00000i   -0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 -  0.00000i

 Columns 11 through 15:

    0.00000 -  0.00000i   -0.00000 -  0.00000i   -8.00000 -  0.00000i    0.00000 +  0.00000i    0.00000 -  0.00000i

 Columns 16 through 20:

    0.00000 -  0.00000i   16.00000 +  0.00000i    0.00000 +  0.00000i    0.00000 +  0.00000i    0.00000 -  0.00000i

 Columns 21 through 25:

   -8.00000 +  0.00000i   -0.00000 +  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 -  0.00000i

 Columns 26 through 30:

   -0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 -  0.00000i   -0.00000 -  0.00000i    0.80000 +  0.80000i

 Columns 31 and 32:

   -0.00000 -  0.00000i    0.00000 -  0.00000i
```

Real function:

C.3

```
>> E
E =

 Columns 1 through 12:

   0.00000   0.11447   0.53473   0.82344   0.99500   0.87442   0.45820   0.17460   0.00000   0.10336   0.59592   0.73003

 Columns 13 through 24:

   1.14642   0.73003   0.59592   0.10336  -0.00000   0.17460   0.45820   0.87442   0.99500   0.82344   0.53473   0.11447

 Columns 25 through 32:

  -0.00000   0.19336   0.41115   0.98632   0.86358   0.98632   0.41115   0.19336

Ehat =

 Columns 1 through 5:

   16.00000 +  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.08000i    0.00000 +  0.00000i   -8.00000 +  0.00000i

 Columns 6 through 10:

   -0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 -  0.00000i    0.00000 -  0.00000i

 Columns 11 through 15:

    0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 -  0.00000i    0.80000 +  0.80000i   -0.00000 -  0.00000i

 Columns 16 through 20:

   -0.80000 +  0.80000i    0.00000 +  0.00000i   -0.80000 -  0.80000i   -0.00000 +  0.00000i    0.80000 -  0.80000i

 Columns 21 through 25:

   -0.00000 +  0.00000i    0.00000 -  0.00000i    0.00000 +  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i

 Columns 26 through 30:

   -0.00000 -  0.00000i    0.00000 -  0.00000i   -0.00000 +  0.00000i   -8.00000 -  0.00000i    0.00000 -  0.00000i

 Columns 31 and 32:

   -0.00000 -  0.08000i    0.00000 -  0.00000i

``
```
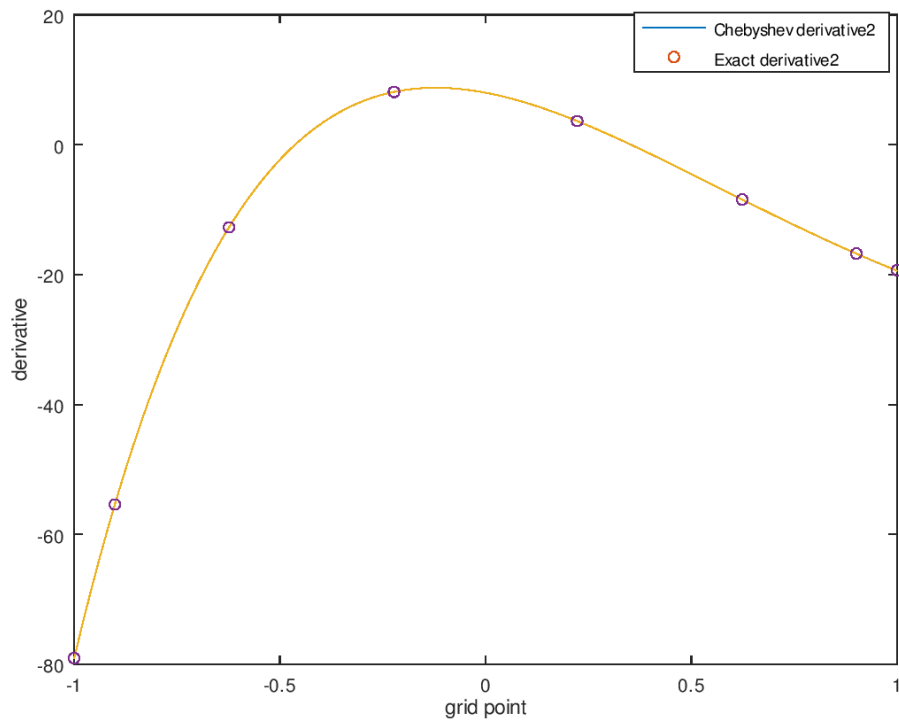
D.1


```
x =

   1.00000
   0.90097
   0.62349
   0.22252
  -0.22252
  -0.62349
  -0.90097
  -1.00000

u =

   0.00000
   0.38957
   0.69591
   0.16843
   0.21041
   1.29818
   0.95911
   0.00000


D =

   16.50000  -20.19567    5.31194   -2.57242    1.63596   -1.23191    1.05210   -0.50000
    5.04892   -2.39295   -3.60388    1.47395   -0.89008    0.65597   -0.55496    0.26302
   -1.32799    3.60388   -0.51000   -2.49396    1.18202   -0.80194    0.65597   -0.30798
    0.64310   -1.47395    2.49396   -0.11706   -2.24698    1.18202   -0.89008    0.40899
   -0.40899    0.89008   -1.18202    2.24698    0.11706   -2.49396    1.47395   -0.64310
    0.30798   -0.65597    0.80194   -1.18202    2.49396    0.51000   -3.60388    1.32799
   -0.26302    0.55496   -0.65597    0.89008   -1.47395    3.60388    2.39295   -5.04892
    0.50000   -1.05210    1.23191   -1.63596    2.57242   -5.31194   20.19567  -16.50000

uderv =

   -4.85015
   -3.05993
    0.46577
    1.34965
   -1.89666
   -2.16625
    6.57305
   13.18722
```

D.2


```
x =

   1.00000
   0.90097
   0.62349
   0.22252
  -0.22252
  -0.62349
  -0.90097
  -1.00000

u =

   0.00000
   0.38957
   0.69591
   0.16843
   0.21041
   1.29818
   0.95911
   0.00000
>> D*D
ans =

   160.00000  -258.59203   147.07734   -78.27242    51.31043   -39.13555    33.61223   -16.00000
    77.80268  -113.20778    43.22360   -11.39925     5.84434    -4.00000     3.27193    -1.53553
    -5.69963    22.29986   -28.85180    14.98352    -4.00000     2.10419    -1.52969     0.69354
     1.50377    -4.00000    11.85580   -17.94043    10.62388    -3.07106     1.79288    -0.76484
    -0.76484     1.79288    -3.07106    10.62388   -17.94043    11.85580    -4.00000     1.50377
     0.69354    -1.52969     2.10419    -4.00000    14.98352   -28.85180    22.29986    -5.69963
    -1.53553     3.27193    -4.00000     5.84434   -11.39925    43.22360  -113.20778    77.80268
   -16.00000    33.61223   -39.13555    51.31043   -78.27242   147.07734  -258.59203   160.00000

>> uderv2
uderv2 =

  -19.3404
  -16.7671
   -8.4445
    3.6388
    8.1303
  -12.7193
  -55.3902
  -79.0538
```