B.1

clear

clc

%%N=16

x=0:pi/8:2\*pi

cendiff=zeros(1,17)

for i=2:15

cendiff(i)=(sin(3\*x(i+1))+3\*cos(6\*x(i+1))-sin(3\*x(i-1))-3\*cos(6\*x(i-1)))/(pi/4);

end

plot(x,cendiff,"o")

hold on

t=linspace(0,2\*pi)

actdiff=3\*cos(3\*t)-18\*sin(6\*t)

plot(t,actdiff)

hold on

cutx=0:pi/8:15\*pi/8

actf=sin(3\*cutx)+3\*cos(6\*cutx)

fhat=fft(actf)

ikfhat=zeros(1,16)

k=-8:7

for s=1:8

ikfhat(s)=fhat(s).\*1i.\*k(s+8)

end

for s=1:8

ikfhat(s+8)=fhat(s+8).\*1i.\*k(s)

end

difffhat=ifft(ikfhat)

plot(cutx,difffhat,"+","markersize",10)

legend("Central diffference","Exact","FFT")

hold on

xlabel("x")

ylabel("f'")

print -dpng output.png

clear

clc

%%N=32

x=0:pi/16:2\*pi

cendiff=zeros(1,33)

for i=2:31

cendiff(i)=(sin(3\*x(i+1))+3\*cos(6\*x(i+1))-sin(3\*x(i-1))-3\*cos(6\*x(i-1)))/(pi/8);

end

plot(x,cendiff,"o")

hold on

t=linspace(0,2\*pi)

actdiff=3\*cos(3\*t)-18\*sin(6\*t)

plot(t,actdiff)

hold on

cutx=0:pi/16:31\*pi/16

actf=sin(3\*cutx)+3\*cos(6\*cutx)

fhat=fft(actf)

ikfhat=zeros(1,32)

k=-16:15

for s=1:16

ikfhat(s)=fhat(s).\*1i.\*k(s+16)

end

for s=1:16

ikfhat(s+16)=fhat(s+16).\*1i.\*k(s)

end

difffhat=ifft(ikfhat)

plot(cutx,difffhat,"+","markersize",10)

legend("Central diffference","Exact","FFT")

hold on

xlabel("x")

ylabel("f'")

print -dpng output.png

B.2

clear

clc

%%N=16

x=0:pi/8:2\*pi

cendiff=zeros(1,17)

for i=2:15

cendiff(i)=(6\*x(i+1)-x(i+1).\*x(i+1)-6\*x(i-1)+x(i-1).\*x(i-1))/(pi/4);

end

plot(x,cendiff,"o")

hold on

t=linspace(0,2\*pi)

actdiff=6-2\*t

plot(t,actdiff)

hold on

cutx=0:pi/8:15\*pi/8

actf=6\*cutx-cutx.\*cutx

fhat=fft(actf)

ikfhat=zeros(1,16)

k=-8:7

for s=1:8

ikfhat(s)=fhat(s).\*1i.\*k(s+8)

end

for s=1:8

ikfhat(s+8)=fhat(s+8).\*1i.\*k(s)

end

difffhat=ifft(ikfhat)

plot(cutx,difffhat,"+","markersize",10)

legend("Central diffference","Exact","FFT")

hold on

xlabel("x")

ylabel("f'")

print -dpng output.png

clear

clc

%%N=32

x=0:pi/16:2\*pi

cendiff=zeros(1,33)

for i=2:31

cendiff(i)=(6\*x(i+1)-x(i+1).\*x(i+1)-6\*x(i-1)+x(i-1).\*x(i-1))/(pi/8);

end

plot(x,cendiff,"o")

hold on

t=linspace(0,2\*pi)

actdiff=6-2\*t

plot(t,actdiff)

hold on

cutx=0:pi/16:31\*pi/16

actf=6\*cutx-cutx.\*cutx

fhat=fft(actf)

ikfhat=zeros(1,32)

k=-16:15

for s=1:16

ikfhat(s)=fhat(s).\*1i.\*k(s+16)

end

for s=1:16

ikfhat(s+16)=fhat(s+16).\*1i.\*k(s)

end

difffhat=ifft(ikfhat)

plot(cutx,difffhat,"+","markersize",10)

legend("Central diffference","Exact","FFT")

hold on

xlabel("x")

ylabel("f'")

print -dpng output.png

C.1

clear

clc

x=0:pi/16:2\*pi/32\*31;

f=sin(2\*x)+0.1\*sin(15\*x);

g=sin(2\*x)+0.1\*cos(15\*x);

H=f.\*g;

Hhat=fft(H);

C.2

clear

clc

N=32;

x=0:2\*pi/N:2\*pi/N\*(N-1);

f=sin(2\*x)+0.1\*sin(15\*x);

g=sin(2\*x)+0.1\*cos(15\*x);

fhat=fft(f);

ghat=fft(g);

%%change position

fhat2=[];

ghat2=[];

for i=1:(N/2)

fhat2(i)=fhat(i+N/2);

ghat2(i)=ghat(i+N/2);

fhat2(i+N/2)=fhat(i);

ghat2(i+N/2)=ghat(i);

end

%%put 0 in gkm

gkm=[];

for i=1:(N/2-1)

gkm(i)=0;

end

for i=(3/2\*N):(2\*N-1)

gkm(i)=0;

end

for i=(N/2):(3/2\*N-1)

gkm(i)=ghat2(i-N/2+1);

end

%%multiply

hhat=zeros(1,N);

for k=1:N

h=0;

for m=1:N

h+=fhat2(m).\*gkm(k-m+N);

hhat(k)=h/32;

endfor

endfor

C.3

clear

clc

N=32;

x=0:2\*pi/N:2\*pi/N\*(N-1);

E=sin(2\*x).\*sin(2\*x)+0.05\*(cos(13\*x)-cos(17\*x)+sin(17\*x)-sin(13\*x))+0.005\*(sin(30\*x));

Ehat=fft(E)

D.1 D.2

clear

clc

N=7;

for i=1:8

x(i)=cos(pi\*(i-1)/7);

end

x=x'

u=4.\*(x.^2-x.^4).\*exp(-x./2)

That=zeros(8,8);

T=[];

for i=1:8

for j=1:8

T(i,j)=cos((i-1)\*(j-1)/N\*pi);

That(i,j)=T(i,j)\*2/N;

if (i==1)||(i==8)

That(i,j)=That(i,j)/2;

end

if (j==8)||(j==1)

That(i,j)=That(i,j)/2;

end

end

end

G=zeros(8,8);

for p=1:8

for n=1:8

if (p==1)||(p==8)

cp=2;

else

cp=1;

end

if (p>=n)||(rem((p+n),2)==0)

G(p,n)=0;

else

G(p,n)=2\*(n-1)/cp;

end

end

end

D=T\*G\*That

uderv=D\*u

s=linspace(-1,1);

uactderv=(2\*s.^4-16\*s.^3-2\*s.^2+8\*s).\*exp(-s/2);

figure(1);

plot(s,uactderv);

hold on;

plot(x,uderv,"o");

legend("Chebyshev derivative","Exact derivative");

xlabel("grid point");

ylabel("derivative");

hold off;

figure(2);

uactderv2=(-1\*s.^4+16\*s.^3-47\*s.^2-8\*s+8).\*exp(-s/2);

plot(s,uactderv2);

hold on;

uderv2=D\*uderv;

plot(x,uderv2,"o");

legend("Chebyshev derivative2","Exact derivative2");

xlabel("grid point");

ylabel("derivative");