

Data And Uncertainty
Assignment 8 November 2017

Leonardo Ripoli

Ex1.1)

Let $\Omega = \mathbb{N}$, show that \mathbf{A} , the set of all cofinite subsets in Ω , forms an algebra.

The proof is done by checking that the 3 properties of an algebra are satisfied: empty set, closeness to complement and closeness to finite unions.

1. $\emptyset \in \mathbf{A}$ trivially because it is finite.
2. By definition if $A \in \mathbf{A}$ it is cofinite, therefore either it is finite and its complement is infinite, or the other way around: in either case $A \in \mathbf{A} \implies A^c \in \mathbf{A}$.
3. Let $A_1, A_2, A_3 \dots \in \mathbf{A}$, then if all of the A_i are finite, clearly $\cup_{k=1}^n A_i$ is finite and therefore $\cup_{k=1}^n A_i \in \mathbf{A}$. If at least one of the A_i is infinite, then $\cup_{k=1}^n A_i$ is infinite, we have to show that $(\cup_{k=1}^n A_i)^c$ is finite. Let's say at least A_k is infinite (and therefore A_k^c is finite because A_k is cofinite by hypothesis), we have that $(\cup_{k=1}^n A_i)^c = \cap_{k=1}^n A_i^c$, and since $\cap_{k=1}^n A_i^c \subseteq A_k^c$ and A_k^c is finite, it must be that $\cap_{k=1}^n A_i^c$ is finite and therefore we have shown that $\cup_{k=1}^n A_i \in \mathbf{A}$.

Ex1.2)

Let \mathbf{A} be the algebra of cofinite sets and define the set function $\mu(A) = 1$ if A is finite, and 0 otherwise: show that μ is normalised and additive.