# Data And Uncertainity
# Assignment 8 November 2017

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**Ex1.1)**
Let $\Omega = \mathbb{N}$, show that $\boldsymbol{A}$, the set of all cofinite subsets in $\Omega$, forms an algebra.

The proof is done by checking that the 3 properties of an algebra are satisfied: empty set, closeness to complement and closeness to finite unions.

1. $\varnothing \in \boldsymbol{A}$ trivially because it is finite.

2. By definition if $A \in \boldsymbol{A}$ it is cofinite, therefore either it is finite and its complement is infinite, or the other way around: in either case $A \in \boldsymbol{A} \implies A^{\complement} \in \boldsymbol{A}$.

3. Let $A_1, A_2, A_3... \in \boldsymbol{A}$, then if all of the $A_i$ are finite, clearly $\cup_{k=1}^{n} A_i$ is finite and therefore $\cup_{k=1}^{n} A_i \in \boldsymbol{A}$. If at least one of the $A_i$ is infinite, then $\cup_{k=1}^{n} A_i$ is infinite, we have to show that $(\cup_{k=1}^{n} A_i)^{\complement}$ is finite. Let's say at least $A_k$ is infinite (and therefore $A_k^{\complement}$ is finite because $A_k$ is cofinite by hypothesis), we have that $(\cup_{k=1}^{n} A_i)^{\complement} = \cap_{k=1}^{n} A_i^{\complement}$, and since $\cap_{k=1}^{n} A_i^{\complement} \subseteq A_k^{\complement}$ and $A_k^{\complement}$ is finite, it must be that $\cap_{k=1}^{n} A_i^{\complement}$ is finite and therefore we have shown that $\cup_{k=1}^{n} A_i \in \boldsymbol{A}$.

**Ex1.2)**
Let $\boldsymbol{A}$ be the algebra of cofinite sets and define the set function $\mu(A) = 1$ if A is finite, and 0 otherwise: show that $\mu$ is normalised and additive.

1. $\mu(\mathbb{N}) = 1$ since $\mathbb{N}$ is infinite, therefore $\mu$ is normalised.