Data And Uncertainity
Assignment 8 November 2017

Leonardo Ripoli

Ex1.1)
Let $\Omega = \mathbb{N}$, show that $\boldsymbol{A}$, the set of all cofinite subsets in Ω, forms an algebra.

The proof is done by checking that the 3 properties of an algebra are satisfied: empty set, closeness to complement and closeness to finite unions.

1. $\varnothing \in \boldsymbol{A}$ trivially because it is finite.

2. By definition if $A \in \boldsymbol{A}$ it is cofinite, therefore either it is finite and its complement is infinite, or the other way around: in either case $A \in \boldsymbol{A} \implies A^{\complement} \in \boldsymbol{A}$.

3. Let $A_1, A_2, A_3... \in \boldsymbol{A}$, then if all of the A_i are finite, clearly $\cup_{k=1}^{n} A_i$ is finite and therefore $\cup_{k=1}^{n} A_i \in \boldsymbol{A}$. If at least one of the A_i is infinite, then $\cup_{k=1}^{n} A_i$ is infinite, we have to show that $(\cup_{k=1}^{n} A_i)^{\complement}$ is finite. Let's say at least A_k is infinite (and therefore $A_k^{\complement}$ is finite because A_k is cofinite by hypothesis), we have that $(\cup_{k=1}^{n} A_i)^{\complement} = \cap_{k=1}^{n} A_i^{\complement}$, and since $\cap_{k=1}^{n} A_i^{\complement} \subseteq A_k^{\complement}$ and $A_k^{\complement}$ is finite, it must be that $\cap_{k=1}^{n} A_i^{\complement}$ is finite and therefore we have shown that $\cup_{k=1}^{n} A_i \in \boldsymbol{A}$.

Ex1.2)
Let $\boldsymbol{A}$ be the algebra of cofinite sets and define the set function $\mu(A) = 1$ if A is finite, and 0 otherwise: show that μ is normalised and additive.

1. $\mu(\mathbb{N}) = 1$ since $\mathbb{N}$ is infinite, therefore μ is normalised.

2. For additivity we have to show that $\mu(\cup_{k=1}^{n} A_i) = \sum_{k=1}^{n} \mu(A_i), \forall A_k$ s.t. $A_i \cap A_j = \varnothing$ if $i \neq j$.