Numerical Methods
Course Assignment Report

by

Leonardo Ripoli

15 November 2017

Teacher: Hilary Weller

Abstract

In this work we present the analysis of the linear advection equation modelled in one dimension, x, without sources or sinks of the advected variable ϕ. The exact expression of the equation is:

$$\phi_t + u\phi_x = 0 \tag{1}$$

We consider the case of constant and uniform wind, u, and with given initial condition $\phi(x, 0) = \phi_0$. It can be shown that the analytic solution of (1) is:

$$\phi(x, t) = \phi_0(x - ut) \tag{2}$$

We have modelled equation (1) using several numerical schemes, currently:

- FTBS
- CTCS

The FTBS scheme has been chosen for its "naivety", being first order accurate in time and space, to show how a "basic" method works. The CTCS method has been chosen because, among the explicit ones, it's the method that guarantees the highest order of accuracy, namely two. A comparison of the two methods has been carried out. This report contains the preliminary results, the full analysis will be presented in the final version of the submission which will contain further developements.

Chapter 1

FTBS

The motion of an incompressible Newtonian fluid is governed by the Navier-Stokes Equations:

$$\rho(\frac{\delta \mathbf{u}}{\delta t} + \mathbf{u}\nabla\mathbf{u}) = -\nabla P + \mu\nabla^2\mathbf{u} + \mathbf{f}, \tag{1.1}$$

$$\nabla \cdot \mathbf{u} = 0, \tag{1.2}$$

where $\mathbf{u}$ is the fluid velocity, ρ and μ are the density and the viscosity of the fluid, and $\mathbf{f}$ a force applied on the fluid.

Contents