Numerical Methods
Course Assignment Report

by

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Abstract

In this work we present the analysis of the linear advection equation modelled in one dimension, x, without sources or sinks of the advected variable ϕ. The exact expression of the equation is:

$$\phi_t + u\phi_x = 0 \tag{1}$$

We consider the case of constant and uniform wind, u, and with given initial condition $\phi(x, 0) = \phi_0$. It can be shown that the analytic solution of (1) is:

$$\phi(x, t) = \phi_0(x - ut) \tag{2}$$

We have modelled equation (1) using several numerical schemes, currently:

- FTBS
- CTCS

The FTBS scheme has been chosen for its "naivety", being first order accurate in time and space, to show how a "basic" method works. The CTCS method has been chosen because, among the explicit ones, it's the method that guarantees the highest order of accuracy, namely two. A comparison of the two methods has been carried out. This report contains the preliminary results, the full analysis will be presented in the final version of the submission which will contain further developements.

Contents

Chapter 1

FTBS

The first method to be explored is FTBS. Characteristics of the method can be found in the book

The function used for the method is in the file advectionSchemes.py, and the function in the code is the following, with comments included on the meaning of inputs:

```
def FTCS(phiOld, c, nt):
"Linear advection scheme using FTCS, with Courant number c and"
"                          nt time-steps"
"inputs are:"
"phiOld: initial condition on phi (to save space the array will then"\
"                 be used to store values from the previous time step)"
"c: Courant number"
"nt: nr of time steps"
```

1.1 General characteristics

We will explore the characteristics of the FTBS method, in terms of

Bibliography

[1] J.P. Hernandez-Ortiz, C.G. Stoltz, M.D. Graham. Transport and collective dynamics in suspensions of confined self-propelled particles, *Phys. Rev. Lett.* 95:204501, 2005.

[2] E. Lauga, T. R. Powers, The hydrodynamics of swimming microorganisms *Rep. Prog. Phys.* 72 096601, 2009.

[3] H.C. Berg. E. coli in Motion *New York: Springer*, 2004

[4] S. S. Suarez, A. A. Pacey. Sperm transport in the Female Reproductive Tract *Human Reproduction Update* 12: 23-37, 2006

[5] S. Childress. Mechanics of Swimming and Flying *Cambridge University Press*, 1981

[6] C. P. Ellington. The Aerodynamics of Hovering Insect *Flight (London: The Royal Society)*, 1984

[7] S. Vogel. Life in Moving Fluids *Princeton University Press*, 1996