Numerical Methods
Course Assignment Report

by

Leonardo Ripoli

15 November 2017

Teacher: Hilary Weller

Abstract

In this work we present the analysis of the linear advection equation modelled in one dimension, x, without sources or sinks of the advected variable ϕ. The exact expression of the equation is:

$$\phi_t + u\phi_x = 0 \tag{1}$$

We consider the case of constant and uniform wind, u, and with given initial condition $\phi(x, 0) = \phi_0$. It can be shown that the analytic solution of (1) is:

$$\phi(x, t) = \phi_0(x - ut) \tag{2}$$

We have modelled equation (1) using several numerical schemes, currently:

- FTBS
- CTCS

The FTBS scheme has been chosen for its "naivety", being first order accurate in time and space, to show how a "basic" method works. The CTCS method has been chosen because, among the explicit ones, it's the method that guarantees the highest order of accuracy, namely two. A comparison of the two methods has been carried out. This report contains the preliminary results, the full analysis will be presented in the final version of the submission which will contain further developements.

Contents

Chapter 1

FTBS

The first method to be explored is FTBS. Characteristics of the method can be found in [1]. FTBS has been chosen as ”naive” method to be compared to a more ”sophisticathed” method and show the quantitative and qualitative differences.

1.1 General characteristics

We will explore the characteristics of the FTBS method. Unless otherwise stated, the results used in this chapter will be taken without proof from [1] and [2].

1.1.1 Conservation of mass

With reference to equation (2), the mass of ϕ is conserved under the linear advection in the exact solution and also in the FTBS numerical implementation, whereas the variance of ϕ is conserved only in the exact solution but not in the FTBS implementation. The variance decreases in FTBS, and this is consistent with what we will find in the subsection 1.1.2, that the method is damping.

1.1.2 Stability

1.1.3 Accuracy

The FTBS method is first order accurate

1.1.4 Monotonicity

1.1.5 Dispersion errors

1.1.6 Diffusion errors

1.1.7 Computational modes

What are these??? I read they refer to nothing specifically, but in general to modes that can be either physical or coming from numerical implementations. Not sure I understood how to compute these.

1.1.8 Computational Cost

1.1.9 Variable resolution

Chapter 2

CTCS

The second method to be explored is CTCS. Characteristics of the method can be found in [1].

2.1 General characteristics

We will explore the characteristics of the CTCSS method.

2.1.1 Conservation of mass

2.1.2 Stability

2.1.3 Accuracy

2.1.4 Monotonicity

2.1.5 Dispersion errors

2.1.6 Diffusion errors

2.1.7 Computational modes

What are these??? I read they refer to nothing specifically, but in general to modes that can be either physical or coming from numerical implementations. Not sure I understood how to compute these.

2.1.8 Computational Cost

2.1.9 Variable resolution

Chapter 3

Python Implementation

3.1 General characteristics

In this section we will explain the python implementation of the numerical schemes. Currently the implementation consists of 4 files, each implementing one of the following:

- Main file used to run the project
- Initial condition file, stores different possible choices of initial conditions that can be used to test different properties
- Advection schemes file
- Diagnostics functions file

3.2 Code Repository

The code repository is:
https://github.com/lr411/Numerics.git
The hash of the commit the report refers to for the code part, is:
43f4315e753289bfaab8d4c84fa88d7124c732dc

3.3 Advection Schemes Implementation

The function used for the method is in the file advectionSchemes.py, and the function in the code is the following, with comments included on the meaning of inputs:

```
def FTCS(phiOld, c, nt):
"Linear advection scheme using FTCS, with Courant number c and"
"                    nt time-steps"
"inputs are:"
"phiOld: initial condition on phi (to save space the array will then"\
"             be used to store values from the previous time step)"
"c: Courant number"
"nt: nr of time steps"
```

Bibliography

[1] J. Bröcker, B. Calderhead, D. Cheraghi, C. Cotter, D. Holm, T. Kuna, B. Pelloni, T. Shepherd, H. Weller. Mαthematics of Planet Earth *World Scientific*, 2017

[2] H. Weller. Lecture notes, teacher version , 2017

[3] H. Weller. Videos for the Numerical Methods course *http://mpecdt.bitbucket.io*, 2017