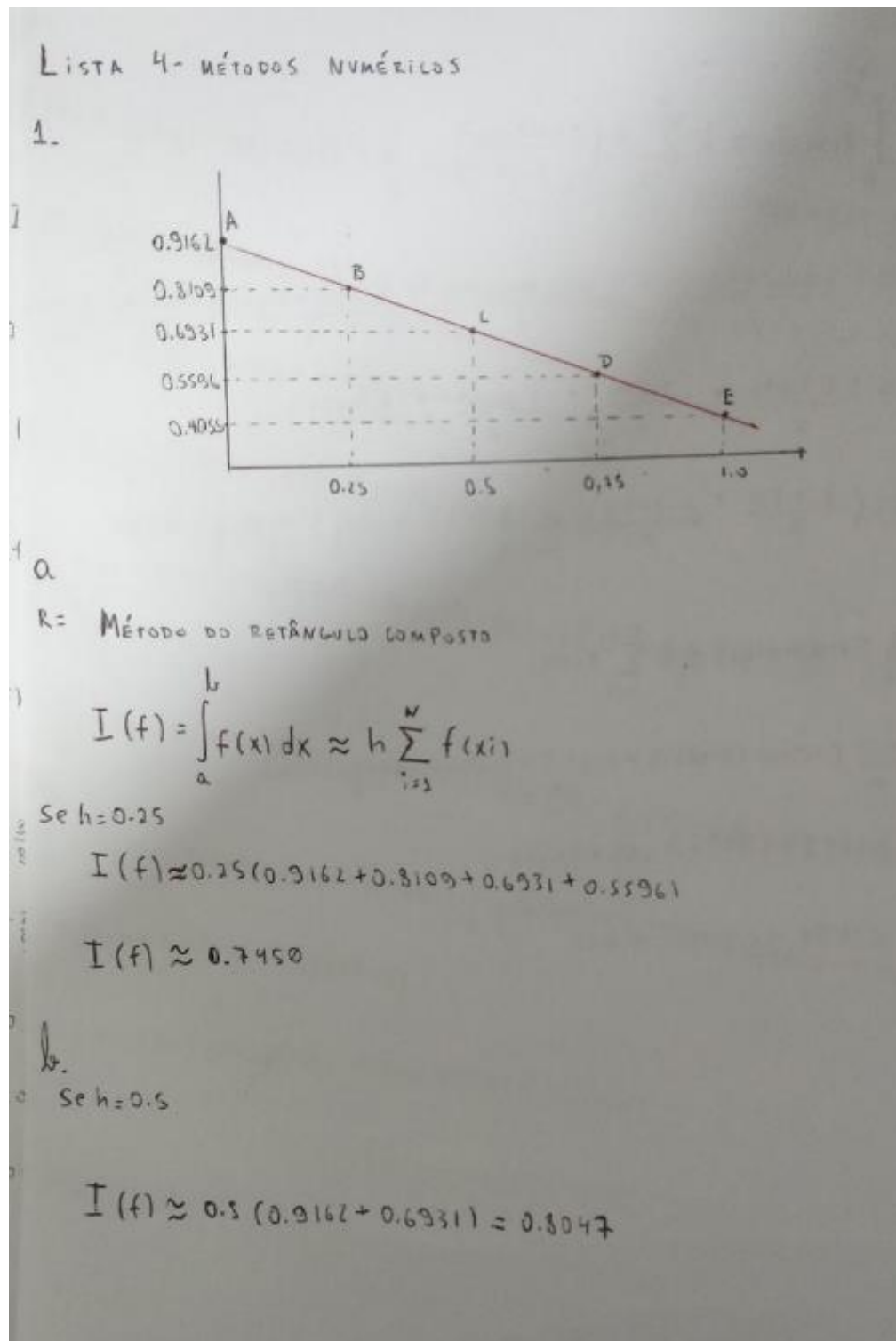


Lista 4 de Métodos numéricos

180145509

Lucas Ramon Alves de Oliveira



b.

$$R: I(f) = \int_a^b f(x) dx \approx h \sum_{i=1}^N f\left(\frac{x_i + x_{i+1}}{2}\right)$$

$$I(f) = 0.25 \left(\frac{0.9162 + 0.8109}{2} + \frac{0.8109 + 0.6931}{2} + \frac{0.6931 + 0.5596}{2} + \frac{0.5596 + 0.4055}{2} \right)$$

$$I(f) = 0.25 \left(\frac{1.7271}{2} + \frac{1.504}{2} + \frac{1.2527}{2} + \frac{0.9651}{2} \right)$$

$$I(f) = 0.25 \left(\frac{3.2311}{2} + \frac{2.2178}{2} \right) \therefore I(f) = 0.25 \left(\frac{5.4489}{2} \right) = 0.6811$$

c.

$$I(f) \approx \frac{h}{2} [f(a) + f(b)] + h \sum_{i=2}^{N-1} f(x_i)$$

$$I(f) \approx \frac{0.25}{2} [0.9162 + 0.4055] + 0.25 [0.8109 + 0.6931 + 0.5596]$$

$$I(f) \approx 0.125 [1.3217] + 0.25 [2.0636]$$

$$I(f) \approx 0.1652 + 0.5154 = 0.6811$$

2.

$$I(f) = \frac{h}{3} (y_1 + 4y_2 + y_3) + \frac{h}{3} (y_3 + 4y_4 + y_5) + \frac{h}{3} (y_5 + 4y_6 + y_7) + \frac{h}{3} (y_7 + 4y_8 + y_9)$$

$$I(f) = \frac{0.25}{3} (4(0.6614) + 0.8660) + \frac{0.25}{3} (0.8660 + 4(0.9682) + 1) + \frac{0.25}{3} (1 + 4(0.9682) +$$

$$0.8660) + \frac{0.25}{3} (0.8660 + 4(0.6614) + 0)$$

$$I(f) = \frac{0.25}{3} (3.5116) + \frac{0.25}{3} (5.7388) + \frac{0.25}{3} (3.5116)$$

$$I(f) = \frac{0.25}{3} (3.5116 + 5.7388 + 5.7388 + 3.5116) \therefore I(f) = \frac{0.25}{3} (18.5008) = 4.5417$$

b.

$$I(f) = \frac{3h}{8} (y_1 + 3y_2 + 3y_3 + y_4) + \frac{3h}{8} (y_4 + 3y_5 + 3y_6 + y_7) + \frac{3h}{8} (y_7 + 3y_8 + 3y_9 + y_{10})$$

$$I = \frac{3}{8} \cdot \frac{2}{9} (0 + 3(0.6285) + 3(0.8315) + 0.9423) + \frac{3}{8} \cdot \frac{2}{9} (0.9423 + 3 \cdot 0.9938 + 3 \cdot 0.9938 + 0.9423)$$

$$+ \frac{3}{8} \cdot \frac{2}{9} (0.9423 + 3 \cdot 0.8315 + 3 \cdot 0.6285 + 0)$$

$$I = 0.0833 (5.3228) + 0.0833 (7.8484) + 0.0833 (5.3228)$$

$$I = 0.0833 (5.3228 + 7.8484 + 5.3228)$$

$$I = 0.0833 (18.494) \approx 1.5406$$

$$2. \int_{-1}^1 f(x) dx \approx C_1 f(x_1) + C_2 f(x_2)$$

$$C_1 = 1, x_1 = -0.57735027, C_2 = 1, x_2 = 0.57735027$$

$$I = \int_{-1}^1 f(x) dx \approx C_1 f(x_1) + C_2 f(x_2) = \sqrt{1 - (-0.57735027)^2} + \sqrt{1 - (0.57735027)^2}$$

$$I_1 = 0.8365 + 0.8365 = 1.6730$$

3.

$$a. x_{i+1} = x_i + h = x_i + 0.6$$

$$y_{i+1} = y_i + f(x_i, y_i)h = y_i + (y_i x_i - x_i^3) 0.6$$

$$E_{\text{RK2}}: E_1 = x_1^2 - e^{\frac{1}{2}x_1^2} + 2 - y_1$$

1º passo:

$$i=1, x_1=0 \leftarrow y_1=1$$

$$x_2 = 0 + 0.6 = 0.6$$

$$y_2 = y_1 + (y_1 x_1 - x_1^3) 0.6 = 1 + (1 \cdot 0 - 0^3) 0.6 = 1$$

$$E_2 = 0.6^2 - e^{\frac{1}{2}0.6^2} + 2 - 1 = 0.1628$$

$$x_3 = 0.6 + 0.6 = 1.2$$

$$y_3 = y_2 + (y_2 x_2 - x_2^3) 0.6 = 1 + (1 \cdot 0.6 - 0.6^3) 0.6 = 1.1304$$

$$E_3 = 1.2^2 - e^{\frac{1}{2}1.2^2} + 2 - 1.1304 = 0.1551$$

$$x_4 = 1.2 + 0.6 = 1.8$$

$$y_4 = y_3 + (y_3 x_3 - x_3^3) 0.6 = 1.1304 + (1.1304 \cdot 1.2 - 1.2^3) 0.6 = 1.0795$$

$$E_4 = 1.8^2 - e^{\frac{1}{2}1.8^2} + 2 - 1.0795 = -0.8926$$

b.

$$x_{i+1} = x_i + h = x_i + 0.6$$

$$y_i^{EV} + 1 = y_i + f(x_i, y_i)h = y_i + (y_i x_i - x_i^3) \cdot 0.6$$

$$y_{i+1} = y_i + 0.5 [f(x_i, y_i) + f(x_{i+1}, y_i^{EV} + 1)]h = y_i + 0.5 [y_i x_i - x_i^3 + y_{i+1}^{EV} x_{i+1} - x_{i+1}^3]h$$

Ex: $f(x,y) = x^2 - e^{\frac{1}{2}xy} + 2 - y$ para o primeiro passo $i=1, x_1=0$ e $y_1=1$

$$x_2 = 0 + 0.6 = 0.6$$

$$y_2^{EV} = y_1 + (y_1 x_1 + x_1^3) \cdot 0.6 = 1 + (1 \cdot 0 - 0^3) \cdot 0.6 = 1$$

$$y_2 = y_1 + 0.5 [y_1 x_1 + x_1^3 + y_2^{EV} x_2 + x_2^3] \cdot 0.6 = 1 + 0.5 [1 \cdot 0 - 0^3 + 1 \cdot 0.6 - 0.6^3] \cdot 0.6 = 1.1152$$

$$E_2 = 0.6^2 - e^{\frac{1}{2} \cdot 0.6^2} + 2 - 1.1152 = 0.0476$$

Segundo passo

$$x_3 = 0.6 + 0.6 = 1.2$$

$$y_3^{EV} = y_2 + (y_2 x_2 + x_2^3) \cdot 0.6 = 1.1152 + (1.1152 \cdot 0.6 - 0.6^3) \cdot 0.6 = 1.3871$$

$$y_3 = y_2 + 0.5 [y_2 x_2 + x_2^3 + y_3^{EV} x_3 + x_3^3] \cdot 0.6 = 1.1152 + 0.5 [1.1152 \cdot 0.6^3 + 1.3871 \cdot 1.2 - 1.2^3] \cdot 0.6 = 1.2321$$

$$E_3 = 1.2^2 - e^{\frac{1}{2} \cdot 1.2^2} + 2 - 1.2321 = 0.1555$$

$$x_4 = 1.2 + 0.6 = 1.8$$

$$y_4^{EV} = y_3 + (y_3 x_3 + x_3^3) \cdot 0.6 = 1.2321 + (1.2321 \cdot 1.2 - 1.2^3) \cdot 0.6 = 1.0824$$

$$Y_4 = Y_3 + 0.5 [Y_3 X_3 + X_3^3 + Y_4^{EV} X_4 + X_4^3] \cdot 0.6 = 1.2321 + 0.5(1.2321 \cdot 1.2$$

$$(-1.2^3 + 1.0524 \cdot 1.3 - 1.9^3) \cdot 0.6 = -0.00785$$

$$E_4 = 1.3^2 - e^{\frac{1}{2} \cdot 1.3^2} + 2(-0.00785) = 0.1948$$

2-

$$X_{i+1} = x_i + h = x_i + 0.6$$

$$f(x_i, y_i) = y_i x_i - x_i^3$$

$$K_1 = f(x_i, y_i)$$

$$K_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1h)$$

$$K_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_2h)$$

$$K_4 = f(x_i + h, y_i + K_3h)$$

$$y_{i+2} = y_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)h$$

$$E_{\text{approx}}: E_i = x_i^2 - e^{\frac{1}{2}x_i^2}$$

$$x_1 = 0 + 0.6 = 0.6$$

$$K_1 = y_1 x_1 - y_1^3 = 1 \cdot 0 - 0^3 = 0$$

$$x_2 + \frac{1}{2}h = 0 + \frac{1}{2} \cdot 0.6 = 0.3 \quad y_2 + \frac{1}{2}K_1h = 1 + \frac{1}{2} \cdot 0 \cdot 0.6 = 1$$

$$K_2 = 1 \cdot 0.3 - 0.3^3 = 0.273$$

$$y_2 + \frac{1}{2}K_2h = 1 + \frac{1}{2} \cdot 0.273 \cdot 0.6 = 1.0819$$

$$k_3 = 1.0319 \cdot 0.3 - 0.3^3 = 0.2976$$

$$y_2 + k_3 h = 1 + 0.2976 \cdot 0.6 = 1.1786$$

$$k_4 = 1.1786 \cdot 0.6 - 0.6^3 = 0.4912$$

$$y_2 = y_1 + \frac{1}{6} (k_1 + 2k_2 + k_4) h = 1 + \frac{1}{6} (0 + 2 \cdot 0.273 + 2 \cdot 0.2976 +$$

$$0.4912) \cdot 0.6 = 1.1632$$

$$E_2 = 0.6^2 - 0.1 \cdot \frac{0.6^2}{2} + 2 \cdot 1.1632 = -0.000417$$

$$x_3 = 0.6 + 0.6 = 1.2$$

$$k_L = y_2 x_2 - x_2^3 = 1.1632 \cdot 0.6 - 0.6^3 = 0.49192$$

$$x_2 + \frac{1}{2} h = 0.6 + \frac{1}{2} \cdot 0.6 = 0.9 \quad y_2 + \frac{1}{2} k_2 h = 1.1632 + \frac{1}{2} \cdot 0.49192 \cdot 0.6 = 1.3108$$

$$k_2 = 1.3108 \cdot 0.9 - 0.9^3 = 0.4507$$

$$y_2 + \frac{1}{2} k_2 h = 1.1632 + \frac{1}{2} \cdot 0.4507 \cdot 0.6 = 1.298$$

$$k_3 = 1.2108 \cdot 0.9 - 0.9^3 = 0.4392$$

$$y_2 + k_3 h = 1.1632 + 0.4392 \cdot 0.6 = 1.4267$$

$$k_4 = 1.4267 \cdot 1.2 - 1.2^3 = -0.01596$$

$$y_3 = y_2 + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) h = 1.1632 + \frac{1}{6} (0.49192 + 2 \cdot 0.4507 + 2 \cdot 0.4392 +$$

$$-0.01596) \cdot 0.6 = 1.389$$

$$E_3 = 1.2^2 - e^{\frac{1}{2} \cdot 1.2^2} + 2 - 1.389 = -0.00343$$

$$X_4 = 1.2 + 0.6 = 1.8$$

$$K_1 = Y_3 \cdot X_3 - X_3^3 = 1.389 \cdot 1.2 - 1.2^3 = -0.0612$$

$$X_3 + \frac{1}{2}h = 1.2 + \frac{1}{2} \cdot 0.6 = 1.5 \quad Y_3 + \frac{1}{2}K_1h = 1.389 + \frac{1}{2}(-0.0612) \cdot 0.6 = 1.3706$$

$$K_2 = 0.9933 \cdot 1.5 - 1.5^3 = -1.8851$$

$$Y_3 + K_2h = 1.389 + (-1.8851) \cdot 0.6 = 0.2579$$

$$K_4 = 0.2579 \cdot 1.8 - 1.8^3 = -5.368$$

$$Y_4 = Y_3 + \frac{1}{6}(K_1 + 2K_2 + K_3 + K_4)h = 1.389 + \frac{1}{6}((-0.0612) + 2 \cdot (-1.7191) +$$

$$2(-1.8851) - 5.368) \cdot 0.6 = 0.2052$$

$$E_4 = 1.8^2 - e^{\frac{1}{2} \cdot 1.8^2} + 2 - 0.02$$

4-

```
function dhdt=ODEHW8_19(t,h)
```

```
At=3.13; Ap=0.06; K1=300; K2=200; rho=1000;
```

```
C=pi/12; g=9.81;
```

```
dhdt=(K1+K2*cos(C*t)-rho*Ap*sqrt(2*g*h))/(At*rho);
```

a-

```
function [x, y] = odeRK3(ODE,a,b,h,y1)
```



```

x(1) = a; y(1) = y1;
n = (b-a)/h;
for i = 1:n
    x(i+1) = x(i) + h;
    K1 = feval(ODE,x(i),y(i));
    xhalf = x(i) + h/2;
    yK1 = y(i) + K1*h/2;
    K2 = feval(ODE,xhalf,yK1);
    yK2 = y(i) - K1*h + 2*K2*h;
    K3 = feval(ODE,xhalf,yK2);
    y(i+1) = y(i) + (K1 + 4*K2 + K3)*h/6;
end

```

```

% Solution of Problem 8.19 (script file)
clear all
[t, h] = odeRK3('ODEHW8_19',0,150,0.1,3);
plot(t,h)
xlabel('Time (s)'); ylabel('Water Height (m)')

```

b-

```

clear all
[t,h]=ode45('ODEHW8_19',[0:1:150],3);
plot(t,h)
xlabel('Time (s)'); ylabel('Water Height (m)')

```

5-

a-

```
function [t, x, y] = Sys2ODEsRK4(ODE1,ODE2,a,b,h,x1,y1)
```

```
t(1)=a; x(1) = x1; y(1) = y1;
n = (b-a)/h;
for i = 1:n
    t(i+1) = t(i) + h;
    tm = t(i) + h/2;
    Kx1 = feval(ODE1,t(i),x(i),y(i));
    Ky1 = feval(ODE2,t(i),x(i),y(i));
    Kx2 = feval(ODE1,tm,x(i)+Kx1*h/2,y(i)+Ky1*h/2);
    Ky2 = feval(ODE2,tm,x(i)+Kx1*h/2,y(i)+Ky1*h/2);
    Kx3 = feval(ODE1,tm,x(i)+Kx2*h/2,y(i)+Ky2*h/2);
    Ky3 = feval(ODE2,tm,x(i)+Kx2*h/2,y(i)+Ky2*h/2);
    Kx4 = feval(ODE1,t(i+1),x(i)+Kx3*h,y(i)+Ky3*h);
    Ky4 = feval(ODE2,t(i+1),x(i)+Kx3*h,y(i)+Ky3*h);
    x(i+1) = x(i) + (Kx1 + 2*Kx2 + 2*Kx3 + Kx4)*h/6;
    y(i+1) = y(i) + (Ky1 + 2*Ky2 + 2*Ky3 + Ky4)*h/6;
end
```

at

```
function dydt = ODEdydtHW8_21a(t,y,u)
dydt = u;
```

```

clear all
[t, y, u] = Sys2ODEsRK4('ODEdydtHW8_21a','ODEdudtHW8_21a',0,3,0.1,0,0)
subplot(3,1,1)
plot(t,y)
xlabel('Time (s)')
ylabel('Position (ft)')
subplot(3,1,2)
plot(t,u)
xlabel('Time (s)')
ylabel('Velocity (ft/s)')
Accel= ODEdudtHW8_21a(t,y,u)
subplot(3,1,3)
plot(t,Accel)
xlabel('Time (s)')
ylabel('Acceleration (ft/s^2)')

```

b-

```

function dydt = ODEHW8_21b(t, y)
T=7000; g=32.2;
w=3000-80*t;
m=w/g;
D=0.008*g*y(2)^2;
dydt(1,1) = y(2);
dydt(2,1) = (T-w-D)/m;

```

```

clear all
tspan = [0:0.1:3];
yini = [0 0];
[Time y] = ode45(@ODEHW8_21b,tspan,yini);
subplot(3,1,1)
plot(Time,y(:,1))
xlabel('Time (s)')
ylabel('Position (ft)')
subplot(3,1,2)
plot(Time,y(:,2))
xlabel('Time (s)')
ylabel('Velocity (ft/s)')
w=3000-80*Time;
Accel= (7000-w-(0.008*32.2*y(:,2).^2))*32.2./w;
subplot(3,1,3)
plot(Time,Accel)
xlabel('Time (s)')

ylabel('Acceleration (ft/s^2)')
Answer=[Time, y(:,1), y(:,2), Accel];
disp('    Time (s)  Pos(ft)  vel(ft/s)  Acc (ft/s^2)')
disp(Answer)

```