01/21/12 | DUT De-embedding Using a 2x Thru | Lenny Rayzman

For sake of simplicity, we limit this discussion to 2-port analysis.

Assume DUT is embedded between identical (but mirrored) feed-in structures.

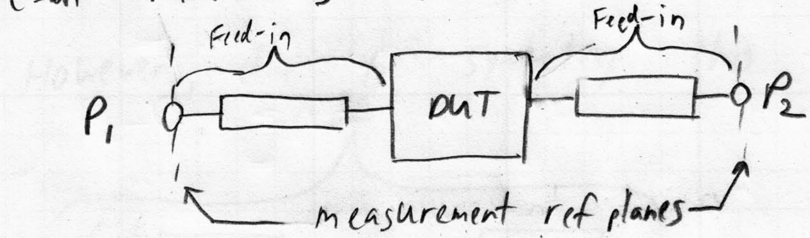


Fig 1

Our goal is to move the reference planes around the DUT. To this end, we design copies of feed-in structure and cascade them.

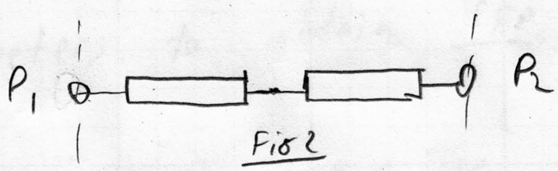


Fig 2

Thus, we have two measurements.

Let $\underline{S_E}$: end-to-end, measured S-param matrix for Fig. 1

$\underline{S_F}$: feed-in structures, measured S-param matrix for Fig. 2

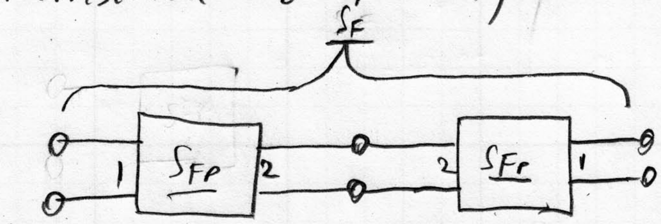
We now show the method to split $\underline{S_F}$ to obtain S-parameters for the individual feed-in structures.

Since we made feed-in structures identical, $\underline{S_F}$ is a cascade of two identical S-parameters.

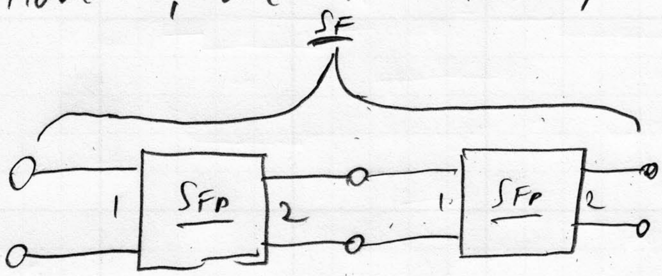
Using the well-known fact that passive linear systems yield symmetrical matrices, we define a matrix $\underline{S_{FP}}$ that represents the individual feed-in structure with the property:

$$\underline{S_{FP}} = \underline{S_{FP}}^T$$

Represented graphically (and observing the mirroring):



However, due to symmetry this is same as



Since this is a cascade, it is easier to use T-parameters to obtain S_{FP}.

Thus,

$$T_F = T_{FP}\, T_{FP} = T_{FP}^2 \implies T_{FP} = \sqrt{T_F}$$

The square root of T_F can be found through the diagonalization procedure (see Appendix II).

Once obtained, T_{FP} can be inverted and cascaded against S_E.

$$S_E \Rightarrow T_E$$

$$T_{OUT} = T_{FP}^{-1}(T_E)\,T_{FP}^{-1} = T_{FP}^{-1}(T_{FP}\, T_{OUT}\, T_{FP})\,T_{FP}^{-1}$$

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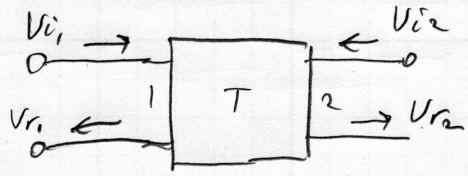
Appendix I

S→T, T→S conversions

The following is ~~are~~ described in terms of voltages rather than power as it is assumed the reference impedance is same for all ports. Thus we describe the parameters in terms of incident & reflected voltages at each port.



where V_{i_x} is incident voltage at port x
V_{r_x} is reflected voltage at port x

We then define (for 2-ports)

$$\begin{bmatrix} V_{i_1} \\ V_{r_1} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_{r_2} \\ V_{i_2} \end{bmatrix} \qquad (1)$$

To see this via example, let

$$\begin{bmatrix} V1_{i_1} \\ V1_{r_1} \end{bmatrix} = \overset{T_1}{\begin{bmatrix} T1_{11} & T1_{12} \\ T1_{21} & T1_{22} \end{bmatrix}} \begin{bmatrix} V1_{i_2} \\ V1_{r_2} \end{bmatrix} \qquad (2a)$$

&

$$\begin{bmatrix} V2_{i_1} \\ V2_{r_1} \end{bmatrix} = \overset{T_2}{\begin{bmatrix} T2_{11} & T2_{12} \\ T2_{21} & T2_{22} \end{bmatrix}} \begin{bmatrix} V2_{i_2} \\ V2_{r_2} \end{bmatrix} \qquad (2b)$$

If cascaded, then

$$\begin{bmatrix} V1_{i_2} \\ V1_{r_2} \end{bmatrix} = \begin{bmatrix} V2_{r_1} \\ V2_{i_1} \end{bmatrix} \qquad (3)$$

So that, using (2a), (2b), (3)

$$\begin{bmatrix} V1_{i_1} \\ V1_{r_1} \end{bmatrix} = \begin{bmatrix} T_1 \end{bmatrix} \begin{bmatrix} T_2 \end{bmatrix} \begin{bmatrix} V2_{i_2} \\ V2_{r_2} \end{bmatrix} \qquad (4)$$

$\boxed{S \to T}$

Expressing (1) as

$$Vi_1 = T_{11} Vr_2 + T_{12} Vi_2 \qquad (5a)$$
$$Vr_1 = T_{21} Vr_2 + T_{22} Vi_2$$

For S-params

$$Vr_1 = S_{11} Vi_1 + S_{12} Vi_2$$
$$Vr_2 = S_{21} Vi_1 + S_{22} Vi_2 \qquad (5b)$$

$$Vi_1 = \frac{1}{S_{21}} Vr_2 - \frac{S_{22}}{S_{21}} Vi_2$$

$$Vr_1 = S_{11}\left[\frac{1}{S_{21}} Vr_2 - \frac{S_{22}}{S_{21}} Vi_2\right] + S_{12} Vi_2$$

$$= \frac{S_{11}}{S_{21}} Vr_2 + \left(S_{12} - \frac{S_{22}}{S_{21}}\right) Vi_2$$

$$\Downarrow$$

$$\underline{T} = \begin{bmatrix} \frac{1}{S_{21}} & -\frac{S_{22}}{S_{21}} \\ \frac{S_{11}}{S_{21}} & \frac{S_{21}S_{12} - S_{11}S_{22}}{S_{21}} \end{bmatrix}$$

$\boxed{T \to S}$

Similarly comparing (5a) & (5b)

$$Vr_2 = \frac{Vi_1 - T_{12} Vi_2}{T_{11}}$$

$$Vr_1 = T_{21}\left[\frac{Vi_1 - T_{12} Vi_2}{T_{11}}\right] + T_{22} Vi_2 = \frac{T_{21}}{T_{11}} Vi + \left[\frac{T_{11}T_{22} - T_{21}T_{12}}{T_{11}}\right] Vi_2$$

$$\Downarrow$$

$$\underline{S} = \begin{bmatrix} \frac{T_{21}}{T_{11}} & \frac{T_{11}T_{22} - T_{21}T_{12}}{T_{11}} \\ \frac{1}{T_{11}} & -\frac{T_{12}}{T_{11}} \end{bmatrix}$$

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Appendix II

matrix square root through diagonalization

Recall eigenvector decomposition:

$$\underline{A}\,\vec{c_i} = \lambda_i \vec{c_i} \qquad (1)$$

where

$\underline{A}$ is $n \times n$ matrix

λ_i is i^{th} eigenvalue of $\underline{A}$

$\vec{c_i}$ is i^{th} eigenvector of $\underline{A}$ of length n.

We can apply a diagonalization procedure as follows:

Let $\underline{C} = [\vec{c_1}\ \vec{c_2} \ldots \vec{c_i} \ldots \vec{c_n}]$ (2)

& $\underline{\Lambda} = \text{diag}([\lambda_1, \lambda_2 \ldots \lambda_i \ldots \lambda_n]) = \begin{bmatrix} \lambda_1 & & & \phi \\ & \lambda_2 & & \\ & & \lambda_i & \\ \phi & & & \lambda_n \end{bmatrix}$ (3)

Then we can rewrite the decomposition in (1) as

$$\underline{A}\,\vec{c_i} = \lambda_i \vec{c_i} \Rightarrow \underline{A}\,\underline{C} = \underline{\Lambda}\,\underline{C} \qquad (4)$$

If $\text{rank}(\underline{C}) = n$ (i.e. eigenvectors are linearly independent)

then $\underline{C}$ is invertible:

$$\underline{C}\,\underline{C}^{-1} = \underline{I} \qquad (5)$$

where $\underline{I}$ is identity matrix

Then, $\underline{A} = \underline{C}\,\underline{\Lambda}\,\underline{C}^{-1}$ (6)

Next, recognize that power of $\underline{\Lambda}$ can be computed simply by taking the power of diagonal elements λ_i.

In the particular case of square root,

$$\text{Let} \quad \underline{\Lambda} = \underline{\Lambda}'\underline{\Lambda}' \qquad (7)$$

Then, using (4) & (5)

$$\underline{A} = \underline{C}\,\underline{\Lambda}'\underline{\Lambda}'\,\underline{C}^{-1} \qquad (8)$$

$$= \underline{C}\,\underline{\Lambda}'\,\underline{C}^{-1}\,\underline{C}\,\underline{\Lambda}'\,\underline{C}^{-1}$$

$$= \underline{A}'\underline{A}'$$

Therefore, $\underline{A}' = \sqrt{\underline{A}}$ (9)