复旦大学计算机科学技术学院

2021～2022 学年第二学期期末考试试卷

☐A 卷　　☐B 卷

课程名称：_算法设计与分析_　　课程代码：_COMP130011.03______

开课院系：计算机科学技术学院______试形式：开卷/闭卷/课程论文/

姓 名：____________　学 号：____________　专 业：____________

声明：我已知悉学校对于考试纪律的严肃规定，将秉持诚实守信宗旨，严守考试纪律，不作弊，不剽窃；若有违反学校考试纪律的行为，自愿接受学校严肃处理。

学生（签名）：____________

年 月 日

题 号	1	2	3	4	5	6	总 分
得 分							

Design and Analysis of Algorithms
Final Examination

June 15, 2022

Notice: This exam is closed book, no books, papers or recording devices permitted. You may use theorems from class, or the book provided you can recall them correctly. Add some annotation to your algorithm and pseudo code when essential.

Problem 1. Fundamentals (3%*7=21%)

1. Give the tight Θ-notation bound of recurrences $\mathrm{T(n)} = aT\left(\frac{n}{8}\right) + \sqrt[3]{n}$:________________________.
2. Consider a modification to QUICKSORT, such that each time PARTITION is called, the median of the partitioned array is found (using the SELECT algorithm) and used as a pivot. The worst-case running time of this algorithm is _______.
3. The tight upper bound and lower bound of log(n!) are ________ and ___________.
4. Any comparison-based algorithm for constructing a binary search tree from an arbitrary list of ***n*** elements takes _________ time in the worst case.
5. How many comparisons occurred in MergeSort? ______________. How many comparisons in QuickSort? _____________. (Assume the size of problem is n, express the answer in order of growth)

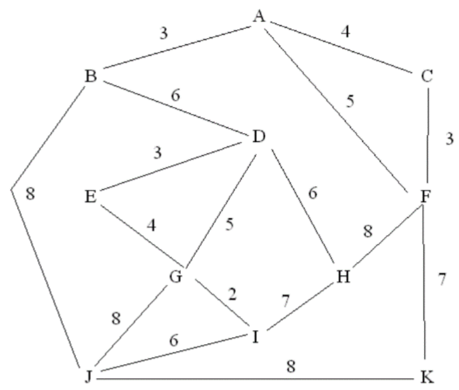
6. For a binary search tree, if both ***preorder*** tree walk and ***post-order*** tree walk are known, then the binary search tree is determined. _________ True or false?
7. What is the **black height** of the RB tree that result after successively inserting the keys 39; 32; 27, 19; 8 into an initially empty red-black tree. ______________

Problem 2. Computation and Justification (44%)

1. Let G=(V, E) be a connected graph with ***n*** vertices, ***m*** edges, and positive edge costs that you may assume are all distinct. Let T= (V, E') be a spanning tree of G; we define the bottleneck edge of T to be the edge of T with the ***greatest*** cost. A spanning tree T of G is a ***minimum bottleneck spanning tree*** if there is no spanning tree T' of G with a cheaper bottleneck edge. Prove or give a counterexample for the following questions.
   a) Is every minimum-bottleneck spanning tree of G a minimum spanning tree of G? (4%)
   b) Is every minimum spanning tree of G a minimum bottleneck spanning tree of G？(4%)

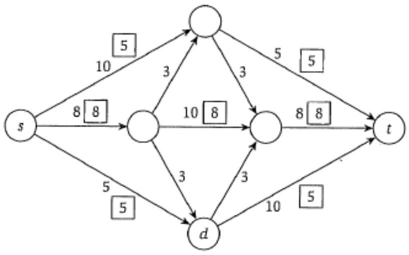
2. A directed graph is **strongly connected** if and only if a DFS started from ***any*** vertex will visit every vertex in the graph without needing to be restarted. True or False? Justify your answer (6%)

3. Consider the following graph.



   1）Give the minimum spanning tree of this graph. (4%)
   2）Using Dijkstra's algorithm, determine the shortest path from node A to I. (4%)

4. (8%) The figure shows a flow network on which a flow is defined. The capacity of each edge appears as a label next to the edge and the numbers in boxes give the flow.
   a) What is the value of this flow? Is this a maximum flow? If not, find the maximum flow. (4%)
   b) Find a minimum cut in the flow network and say what its capacity is. (4%)



5. We have a connected graph G=(*V*, *E*), and a specific vertex ***u*** in *V*. Suppose we compute a ***depth-first*** search tree rooted at ***u***, and obtain a tree *T* that includes all nodes of G. Suppose we then compute a ***breadth-first*** search tree rooted at ***u*** and obtain the same tree *T*. Prove that G=T. (8%)

6. A sequence of $\boldsymbol{n}$ operations is performed on a data structure. The $\boldsymbol{i}$th operation costs $\sqrt{i}$ if $\boldsymbol{i}$ is a perfect square, and 1 otherwise. Using amortized analysis (e.g., aggregate analysis, accounting method, or potential method) to determine the amortized cost per operation（6%）

Problem 3: Highways between cities are usually modeled as a directed graph G= (V, E) in which vertices represent cities and edges represent roads between cities. A truck company is planning new routes for shipments from city A (vertex s) to city B (vertex t). It is very costly when a shipment is delayed. For a given road e∈E，probability p(e) ∈[0,1] is calculated to show that e will close without warning. Give an efficient algorithm for finding a route with the minimum probability of encountering a closed road. You should assume that all road closings are independent.

1) Present your algorithm in pseudo-code. **(6%)**
2) Prove and analyze your algorithm. **(6%)**

Problem 4. (12%) Your friends have studied stock patterns and defined something called **a rising trend**. A rising trend in a sequence of stock prices is a subsequence of prices $p[i_1], p[i_2], \ldots p[i_k]$ $for\ days\ i_1 < i_2 < \cdots < i_k, so\ that\ i_1 = 1, and\ p[i_j] < p[i_{j+1}], for\ j = 1, 2, \ldots k-1$. The problem is to find the longest rising trend in a given sequence of prices. E.g. n=7 and the sequence of prices is 10, 1, 2, 11, 3, 4, 12, then the longest rising trend is prices given on days 1, 4, 7. Give an efficient algorithm that takes a sequence of prices and returns the longest rising trend and its length. Your algorithm should be presented in pseudo-code.

Problem 5. (11%) Suppose in activity-selection problem, instead of always selecting the first activity to finish, we select the last activity to start that is compatible with all previously selected activities.

1）Show this strategy a greedy property and prove that it yields an optimal solution. (3%)
2）Present your algorithm with the selection strategy in pseudo-code. (5%)
3）Analyze the efficiency of your algorithm. (3%)