作业：

1. 推导四足机器人单刚体模型的 mpc 的 QP 形式。[轨迹跟踪任务]。

reference: $x_{ref} = (x_{1,ref}, x_{2,ref}, \cdots, x_{N,ref})^T$. $x_{0,ref} = x_0$.

cost: $J = \frac{1}{2}\sum_{i=0}^{N-1}\left[(x_i - x_{i,ref})^T Q_i (x_i - x_{i,ref}) + u_i^T R_i u_i\right] + \frac{1}{2}(x_N - x_{N,ref})^T Q_N (x_N - x_{N,ref})$.

dynamics: $x_{i+1} = Ax_i + Bu_i$. $i = 0, 1, 2, \cdots, N-1$.

2. 参考 LQR 的推导过程，推导 iLQR 的迭代公式。

cost: $\min_{u,x} \sum_{i=0}^{N-1} f_i(x_i, u_i) + f_N(x_N)$.

dynamics: $x_{i+1} = F(x_i, u_i)$.

对 cost 二阶近似，对 dynamics 一阶近似：

$\Rightarrow \min_{\delta u, \delta x} q_N + \delta x_N^T \hat{q}_N + \frac{1}{2}\delta x_N^T Q_N \delta x_N + \sum_{i=0}^{N-1} q_i + \delta x_i^T \hat{q}_i + \delta u_i^T r_i + \frac{1}{2}\delta x_i^T Q_i \delta x_i + \frac{1}{2}\delta u_i^T R_i \delta u_i + \delta u_i^T P_i \delta x_i$.

s.t. $\delta x_{i+1} = A_i \delta x_i + B_i \delta u_i$.

define:
$$\begin{cases} C_i(\delta x_i, \delta u_i) = q_i + \delta x_i^T \hat{q}_i + \delta u_i^T r_i + \frac{1}{2}\delta x_i^T Q_i \delta x_i + \frac{1}{2}\delta u_i^T R_i \delta u_i + \delta u_i^T P_i \delta x_i \\ Q_i(\delta x_i, \delta u_i) = c_i(\delta x_i, \delta u_i) + V_{i+1}(\delta x_{i+1}) \\ V_i(\delta x_i) = \min_{\delta u_i} Q_i(\delta x_i, \delta u_i) \\ V_N(\delta x_N) = \frac{1}{2}\delta x_N^T Q_N \delta x_N + \delta x_N^T \hat{q}_N + q_N \end{cases}$$

3 编译运行附件 Cartpole_mujoco，尝试粗略理解 iLQR 的实现。自行了解自动微分技术。(CppAD、CasADi 等)。