Kinetic Energy:

$$T=\frac{1}{2}\dot{q}^T M(q)\dot{q}$$ where $M(q)$ is the mass-inertia matrix

from matlab pendubot.m we can get

$$M(q)=\begin{bmatrix} m_2 l_1^2+2m_2 C_2 l_1 l_{2c}+m_1 l_{1c}^2+m_2 l_{2c}^2+I_{C_1zz}+I_{C_2zz} & m_2 l_{2c}^2+l_1 m_2 C_2 l_{2c}+I_{C_2zz} \\ m_2 l_{2c}^2+l_1 m_2 C_2 l_{2c}+I_{C_2zz} & m_2 l_{2c}^2+I_{C_2zz} \end{bmatrix}$$

$I_{C_1zz}=I_{C_2zz}=\frac{l^2}{12}$

$$M(q)=\begin{bmatrix} \frac{5}{3}+C_2 & \frac{1}{3}+\frac{C_2}{2} \\ \frac{1}{3}+\frac{C_2}{2} & \frac{1}{3} \end{bmatrix}$$

kinetic:

$$T=\frac{1}{2}\begin{bmatrix}\dot{q}_1 & \dot{q}_2\end{bmatrix}\begin{bmatrix} \frac{5}{3}+C_2 & \frac{1}{3}+\frac{C_2}{2} \\ \frac{1}{3}+\frac{C_2}{2} & \frac{1}{3} \end{bmatrix}\begin{bmatrix}\dot{q}_1 \\ \dot{q}_2\end{bmatrix}$$

potential energy:

$$P=\frac{m_1 g l_1}{2}S_1+m_2 g(l_1 S_1+\frac{l_2}{2}S_{12})=\frac{g}{2}S_1+g(S_1+\frac{S_{12}}{2})$$

b) equation of motion:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+\frac{\partial G}{\partial q}=\tau-J^T F_T$$

where

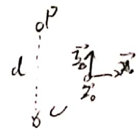
from matlab:

$$C(q,\dot{q})\dot{q}=\begin{bmatrix} -l_1 l_{2c} m_2 \dot{q}_2 S_2(2\dot{q}_1+\dot{q}_2) \\ l_1 l_{2c} m_2 \dot{q}_1^2 S_2 \end{bmatrix}=\begin{bmatrix} -\frac{\dot{q}_2}{2}S_2(2\dot{q}_1+\dot{q}_2) \\ \frac{1}{2}\dot{q}_1^2 S_2 \end{bmatrix}$$

$$\frac{\partial G}{\partial q}=\begin{bmatrix} g m_2(l_{2c}C_{12}+l_1 C_1)+g l_{1c} m_1 C_1 \\ g l_{2c} m_2 C_{12} \end{bmatrix}=\begin{bmatrix} \frac{g}{2}C_{12}+gC_1+\frac{g}{2}C_1 \\ \frac{g}{2}C_{12} \end{bmatrix}$$

$$\begin{bmatrix} (\frac{5}{3}+C_2)\ddot{q}_1+(\frac{1}{3}+\frac{C_2}{2})\ddot{q}_2-\frac{\dot{q}_2}{2}S_2(2\dot{q}_1+\dot{q}_2)+\frac{g}{2}C_{12}+gC_1+\frac{g}{2}C_1 \\ (\frac{1}{3}+\frac{C_2}{2})\ddot{q}_1+\frac{1}{3}\ddot{q}_2+\frac{1}{2}\dot{q}_1^2 S_2+\frac{g}{2}C_{12} \end{bmatrix}=\tau-J^T F_T$$

2.

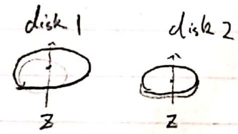


$$I_p = I_c + m r_c^\times r_c^\times \qquad r_c = \begin{bmatrix} 0 \\ -d \\ 0 \end{bmatrix} \begin{matrix} r_x \\ r_y \\ r_z \end{matrix}$$

$$-r_c^\times r_c^\times = \begin{bmatrix} d^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d^2 \end{bmatrix}$$

$$I_p = I_c + \begin{bmatrix} d^2 m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d^2 m \end{bmatrix}$$

3.

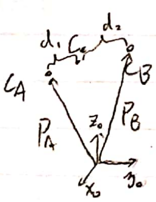


both inertia:

$$I_z = \frac{1}{2} m r^2$$

disk 1 have larger radius thus larger inertia through the center of the disk and perpendicular to the disk

4



mass : $m_A + m_B$

location of the center of mass

$$\vec{P}_{A0} + \frac{m_b \|\vec{P}_{AB}\|}{m_b + m_a} \frac{(\vec{P}_{AB})_0}{\|\vec{P}_{AB}\|}$$

moment of inertia

$$I_A + m_a d_1^2 + I_B + m_b d_2^2$$

where

$$d_1 = \frac{m_b \|\vec{P}_{AB}\|}{m_b + m_a} \qquad d_2 = \frac{m_a \|\vec{P}_{AB}\|}{m_b + m_a}$$