Robotics HW9

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Problem 1

Using cam_image_round.m and make the target point increase to 9 points. It turns out when the number of distinct views equal to 5, there are possibility that the maximum error of $(u_0, v_0, \lambda_x, \lambda_y)$ being less than 2 percent. An example shows:

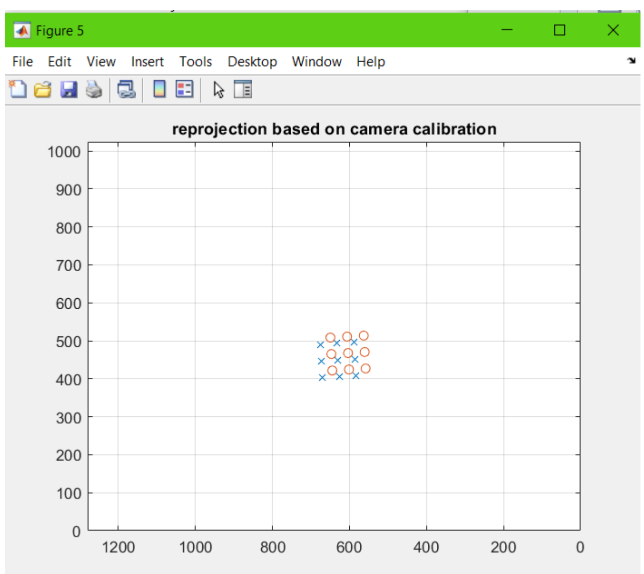


Figure 1: 5 distinct view with maximum of 1.76 percentage error

with 9 target points fixed, as the number of distinct view keeps increase, the number of times when maximum of error percentage less than 2 does not significantly increases.

Use cam_image_round.m again, now the distinct view is 20. When the number of targets equal to 45, there are significant amounts of times when error is less than 2 percent, a good example shows:

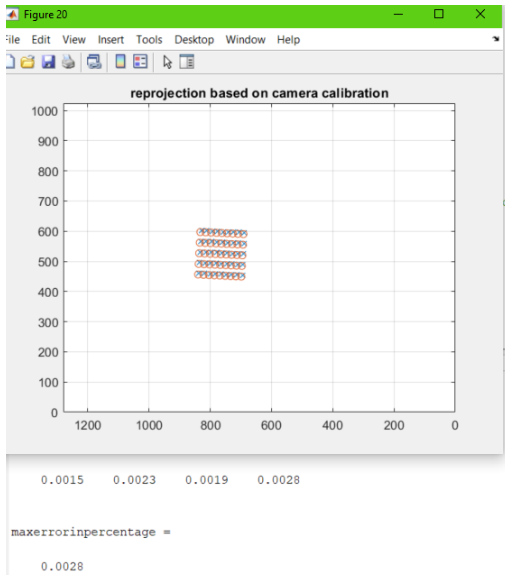


Figure 2: 20 distinct view with maximum of 0.28 percentage error

b)

Now use the cam_image.m to generate camera images, and use 1 pixel of noise to find a good combination of target points and the number of distinct views to make error goes less than 2 percent. It turns out when the number of distinct views is about 30, and the target points are 81. If the program runs successfully, it is likely that the error will goes below 2 percent with 1 pixel error. An example shows:

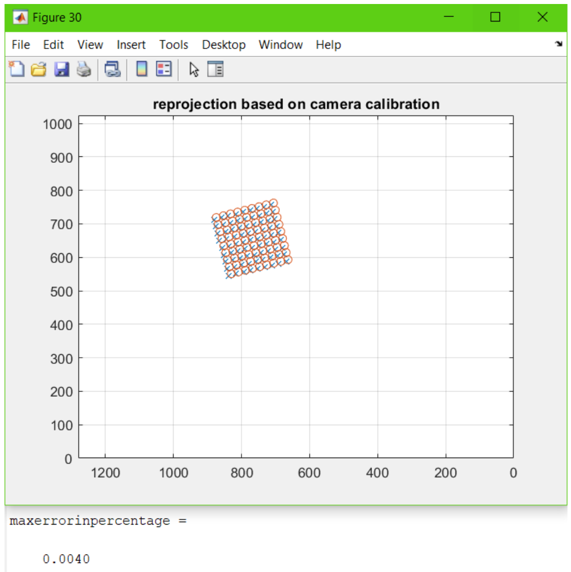


Figure 3: 1 pixel of noise, 30 images-81 points, 0.4 percentage error

As the number of target points increases, the error percentage does not get better significantly, since there are noise that will be amplified with the increasing amount of the points.

Problem 2

P2

$$wu = a_1^T P_{0L} + \alpha\, a_1^T e_L + a_{14}$$

$$wV = a_2^T P_{0L} + \alpha\, a_2^T e_L + a_{24}$$

$$w = a_3^T P_{0L} + \alpha\, a_3^T e_L + a_{34}$$

$$(a_3^T P_{0L} + \alpha\, a_3^T e_L + a_{34})\, u = a_1^T P_{0L} + \alpha\, a_1^T e_L + a_{14}$$

$$\alpha\, a_3^T e_L u - \alpha (a_1^T e_L) = a_1^T P_{0L} + a_{14} - a_3^T P_{0L} u - a_{34} u$$

$$\alpha = \frac{a_1^T P_{0L} + a_{14} - a_3^T P_{0L} u - a_{34} u}{a_3^T e_L u - a_1^T e_L}$$

$$(a_3^T P_{0L} + \alpha\, a_3^T e_L + a_{34}) V = a_2^T P_{0L} + \alpha\, a_2^T e_L + a_{24}$$

$$\left(a_3^T P_{0L} + \frac{a_1^T P_{0L} + a_{14} - (a_3^T P_{0L} + a_{34}) u}{a_3^T e_L u - a_1^T e_L}\, a_3^T e_L + a_{34}\right) V = a_2^T P_{0L} + \frac{a_1^T P_{0L} + a_{14} - (a_3^T P_{0L} + a_{34}) u}{a_3^T e_L u - a_1^T e_L}\, a_2^T e_L + a_{24}$$

$$a_3^T P_{0L}\, a_3^T e_L u - a_3^T P_{0L}\, a_1^T e_L + \left(a_1^T P_{0L} + a_{14} - (a_3^T P_{0L} + a_{34}) u\right) a_3^T e_L + (a_3^T e_L u - a_1^T e_L)\, a_{34}$$

$$\cancel{a_3^T P_{0L}\, a_3^T e_L u} - a_3^T P_{0L}\, a_1^T e_L + a_1^T P_{0L}\, a_3^T e_L + a_{14}\, a_3^T e_L - \cancel{a_3^T P_{0L} u\, a_3^T e_L} - \cancel{a_{34} u\, a_3^T e_L} + \cancel{a_3^T e_L u\, a_{34}} - a_1^T e_L\, a_{34}$$

$$(a_{14}\, a_3^T e_L - a_1^T e_L\, a_{34} + a_1^T P_{0L} \cdot a_3^T e_L - a_3^T P_{0L} \cdot a_1^T e_L) V$$

$$= a_2^T P_{0L} \cdot (a_3^T e_L u - a_1^T e_L) + \left[a_1^T P_{0L} + a_{14} - (a_3^T P_{0L} + a_{34}) u\right] a_2^T e_L + a_{24} (a_3^T e_L u - a_1^T e_L)$$

$$= a_2^T P_{0L}\, a_3^T e_L u - a_2^T P_{0L}\, a_1^T e_L + a_1^T P_{0L}\, a_2^T e_L + a_{14}\, a_2^T e_L - a_3^T P_{0L} u\, a_2^T e_L - a_{34} u\, a_2^T e_L + a_{24}\, a_3^T e_L u - a_{24}\, a_1^T e_L$$

$$= (a_2^T P_{0L} \cdot a_3^T e_L - a_3^T P_{0L}\, a_2^T e_L) u + (a_{24}\, a_3^T e_L - a_{34}\, a_2^T e_L) u + a_1^T P_{0L}\, a_2^T e_L - a_2^T P_{0L}\, a_1^T e_L + a_{14}\, a_2^T e_L - a_{24}\, a_1^T e_L$$

$$V = \frac{(a_2^T P_{0L}\, a_3^T e_L - a_3^T P_{0L}\, a_2^T e_L + a_{24}\, a_3^T e_L - a_{34}\, a_2^T e_L) u}{a_{14}\, a_3^T e_L - a_1^T e_L\, a_{34} + a_1^T P_{0L}\, a_3^T e_L - a_3^T P_{0L}\, a_1^T e_L} + \frac{a_1^T P_{0L}\, a_2^T e_L - a_2^T P_{0L}\, a_1^T e_L + a_{14}\, a_2^T e_L - a_{24}\, a_1^T e_L}{a_{14}\, a_3^T e_L - a_1^T e_L\, a_{34} + a_1^T P_{0L}\, a_3^T e_L - a_3^T P_{0L}\, a_1^T e_L}$$

Problem 3

From previous question: we know the equation for a line in image plane.

for three parallel lines: denote $\beta_1 P_{0L} + \alpha e_L$; $\beta_2 P_{0L} + \alpha e_L$; $\beta_3 P_{0L} + \alpha e_L$

Thus the equation of line becomes.

$$V_1 = \frac{(a_2^T \beta_1 P_{0L} a_3^T e_L - a_3^T \beta_1 P_{0L} a_2^T e_L + a_{24} a_3^T e_L - a_{34} a_2^T e_L) u_1}{a_{14} a_3^T e_L - a_1^T e_L a_{34} + a_1^T \beta_1 P_{0L} a_3^T e_L - a_3^T \beta_1 P_{0L} a_1^T e_L} + \frac{a_1^T \beta_1 P_{0L} a_2^T e_L - a_2^T \beta_1 P_{0L} a_1^T e_L + a_{14} a_2^T e_L - a_{24} a_1^T e_L}{a_{14} a_3^T e_L - a_1^T e_L a_{34} + a_1^T \beta_1 P_{0L} a_3^T e_L - a_3^T \beta_1 P_{0L} a_1^T e_L}$$

$$V_2 = \frac{(a_2^T \beta_2 P_{0L} a_3^T e_L - a_3^T \beta_2 P_{0L} a_2^T e_L + a_{24} a_3^T e_L - a_{34} a_2^T e_L) u_2}{a_{14} a_3^T e_L - a_2^T e_L a_{34} + a_1^T \beta_2 P_{0L} a_3^T e_L - a_3^T \beta_2 P_{0L} a_1^T e_L} + \frac{a_1^T \beta_2 P_{0L} a_2^T e_L - a_2^T \beta_2 P_{0L} a_1^T e_L + a_{14} a_2^T e_L - a_{24} a_1^T e_L}{a_{14} a_3^T e_L - a_1^T e_L a_{34} + a_1^T \beta_2 P_{0L} a_3^T e_L - a_3^T \beta_2 P_{0L} a_1^T e_L}$$

$$V_3 = \frac{(a_2^T \beta_3 P_{0L} a_3^T e_L - a_3^T \beta_3 P_{0L} a_2^T e_L + a_{24} a_3^T e_L - a_{34} a_2^T e_L) u_3}{a_{14} a_3^T e_L - a_1^T e_L a_{34} + a_1^T \beta_3 P_{0L} a_3^T e_L - a_3^T \beta_3 P_{0L} a_1^T e_L} + \frac{a_1^T \beta_3 P_{0L} a_2^T e_L - a_2^T \beta_3 P_{0L} a_1^T e_L + a_{14} a_2^T e_L - a_{24} a_1^T e_L}{a_{14} a_3^T e_L - a_1^T e_L a_{34} + a_1^T \beta_3 P_{0L} a_3^T e_L - a_3^T \beta_2 P_{0L} a_1^T e_L}$$

Since $\alpha = \frac{a_1^T \beta P_{0L} + a_{14} - a_3^T \beta P_{0L} u - a_{34} u}{a_3^T e_L u - a_1^T e_L}$ need to be infinite to be a line $\Rightarrow a_3^T e_L u = a_1^T e_L$

Thus $$V = \frac{(-a_3^T \beta P_{0L} a_2^T e_L - a_{34} a_2^T e_L) u + a_1^T \beta P_{0L} a_2^T e_L + a_{14} a_2^T e_L}{a_{14} a_3^T e_L - a_1^T e_L a_{34} + a_1^T \beta P_{0L} a_3^T e_L - a_3^T \beta P_{0L} a_1^T e_L}$$

For simplicity denote $V = \frac{Au + B}{M}$ so $V_1 = \frac{A_1 u + B_1}{M_1}$ $V_2 = \frac{A_2 u + B_2}{M_2}$ $V_3 = \frac{A_3 u + B_3}{M_3}$

for intersection.

$$\frac{A_1 u + B_1}{M_1} = \frac{A_2 u + B_2}{M_2} \Rightarrow u = \frac{B_2 M_1 - B_1 M_2}{A_1 M_2 - A_2 M_1} \quad v = \frac{A_1 B_2 M_1 - A_1 B_1 M_2}{M_1 (A_1 M_2 - A_2 M_1)} + \frac{B_1}{M_1}$$

$$\frac{A_1 u + B_1}{M_1} = \frac{A_3 u + B_3}{M_3} \Rightarrow u = \frac{B_3 M_1 - B_1 M_3}{A_1 M_3 - A_3 M_1} \quad V = \frac{A_1 B_3 M_1 - A_1 B_1 M_3}{M_1 (A_1 M_3 - A_3 M_1)} + \frac{B_1}{M_1}$$

$$\frac{A_2 u + B_2}{M_2} = \frac{A_3 u + B_3}{M_3} \Rightarrow u = \frac{B_3 M_2 - B_2 M_3}{A_2 M_3 - A_3 M_2} \quad v = \frac{A_2 B_3 M_2 - A_2 B_2 M_3}{M_2 (A_2 M_3 - A_3 M_2)} + \frac{B_2}{M_2}$$

where

$A_1 = -a_3^T \beta_1 P_{0L} a_2^T e_L - a_{34} a_2^T e_L$ $A_2 = -a_3^T \beta_2 P_{0L} a_2^T e_L - a_{34} a_2^T e_L$ $A_3 = -a_3^T \beta_3 P_{0L} a_2^T e_L - a_{34} a_2^T e_L$

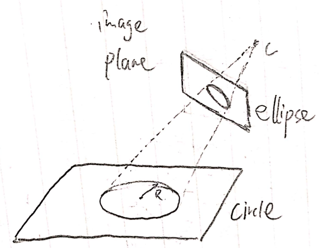
$B_1 = a_1^T \beta_1 P_{0L} a_2^T e_L + a_{14} a_2^T e_L$ $B_2 = a_1^T \beta_2 P_{0L} a_2^T e_L + a_{14} a_2^T e_L$ $B_3 = a_1^T \beta_3 P_{0L} a_2^T e_L + a_{14} a_2^T e_L$

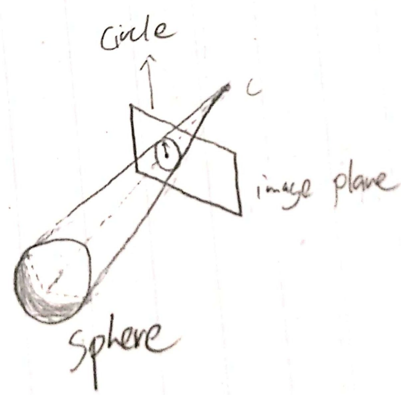
$M_1 = a_{14} a_3^T e_L - a_1^T e_L a_{34} + a_1^T \beta_1 P_{0L} a_3^T e_L - a_3^T \beta_1 P_{0L} a_1^T e_L$

$M_2 = a_{14} a_3^T e_L - a_1^T e_L a_{34} + a_1^T \beta_2 P_{0L} a_3^T e_L - a_1^T \beta_2 P_{0L} a_1^T e_L$

$M_3 = a_{14} a_3^T e_L - a_1^T e_L a_{34} + a_1^T \beta_3 P_{0L} a_3^T e_L - a_3^T \beta_3 P_{0L} a_1^T e_L$

Problem 4





Problem 5

P4

Camera 1:

$$W_1 \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \underbrace{K_1 T_1}_{\substack{\text{known} \\ 3\times 4}} \begin{bmatrix} P_{OA} \\ 1 \end{bmatrix} \qquad K_1 T_1 = \begin{bmatrix} a_{11}^T & a_{114} \\ a_{12}^T & a_{124} \\ a_{13}^T & a_{134} \end{bmatrix}$$

Camera 2:

$$W_2 \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = K_2 T_2 \begin{bmatrix} P_{OA} \\ 1 \end{bmatrix} \qquad K_2 T_2 = \begin{bmatrix} a_{21}^T & a_{214} \\ a_{22}^T & a_{224} \\ a_{23}^T & a_{234} \end{bmatrix}$$

Thus

$$W_1 u_1 = a_{11}^T P_{OA} + a_{114} \qquad W_2 u_2 = a_{21}^T P_{OA} + a_{214}$$
$$W_1 v_1 = a_{12}^T P_{OA} + a_{124} \qquad W_2 v_2 = a_{22}^T P_{OA} + a_{224}$$
$$W_1 = a_{13}^T P_{OA} + a_{134} \qquad W_2 = a_{23}^T P_{OA} + a_{234}$$

$\times u_1$: $W_1 u_1 = u_1 a_{13}^T P_{OA} + u_1 a_{134}$ $\qquad$ $\times u_2$: $W_2 u_2 = u_2 a_{13}^T P_{OA} + u_2 a_{234}$

$\times v_1$: $W_1 v_1 = v_1 a_{13}^T P_{OA} + v_1 a_{134}$ $\qquad$ $\times v_2$: $W_2 v_2 = v_2 a_{13}^T P_{OA} + v_2 a_{234}$

$$W_1 v_1 - W_1 v_1 = a_{12}^T P_{OA} - v_1 a_{13}^T P_{OA} + a_{124} - v_1 a_{134} \qquad W_2 v_2 - W_2 v_2 = a_{22}^T P_{OA} + a_{224} - v_2 a_{13}^T P_{OA} - v_2 a_{234}$$
$$W_1 u_1 - W_1 u_1 = a_{11}^T P_{OA} - u_1 a_{13}^T P_{OA} + a_{114} - u_1 a_{134} \qquad W_2 u_2 - W_2 u_2 = a_{21}^T P_{OA} + a_{214} - u_2 a_{11}^T P_{OA} - u_2 a_{234}$$

$$\begin{aligned}
u_1 a_{134} - a_{114} &= (a_{11}^T - u_1 a_{13}^T) P_{OA} \\
v_1 a_{134} - a_{124} &= (a_{12}^T - v_1 a_{13}^T) P_{OA} \\
u_2 a_{234} - a_{214} &= (a_{21}^T - u_2 a_{13}^T) P_{OA} \\
\underbrace{v_2 a_{234} - a_{224}}_{B:\ 4\times 1} &= \underbrace{(a_{22}^T - v_2 a_{13}^T)}_{A:\ 4\times 3} P_{OA}
\end{aligned}$$

$$P_{OA} = (A^T A)^{-1} A^T B$$