Numerical Computing HW8

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Problem 1

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} = [\begin{smallmatrix} 5 & 4 \\ 4 & 6 \end{smallmatrix}]$$

$$A'b = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 5 & 4 & 5 \\ 0 & 6 - \frac{16}{5} & 4 \end{array} \right]$$

$\frac{14}{5} x_2 = 4- > x_2 = \frac{10}{7}$

$5x_1 + \frac{40}{7} = 5- > x_1 = -\frac{1}{7}$

$$r = b - A\bar{x} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} \frac{-1}{7} \\ \frac{10}{7} \end{bmatrix} = \begin{bmatrix} 0.2857 \\ -0.4286 \\ -0.1429 \end{bmatrix}$$

RMSE: $\frac{\sqrt{r_1^2 + r_1^2 + r_2^2}}{2} = 0.2494$

b)

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 & 2 \\ 3 & 6 & 3 \\ 2 & 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 10 \\ 9 \end{bmatrix}$$

$x = \begin{bmatrix} 2 \\ -\frac{1}{3} \\ 2 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -\frac{1}{3} \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ -\frac{1}{3} \\ 0 \end{bmatrix}$

RMSE: $\sqrt{\frac{(\frac{1}{3})^2+(\frac{1}{3})^2+(-\frac{1}{3})^2}{4}} = \sqrt{\frac{1}{12}} = \frac{1}{2}\sqrt{\frac{1}{3}}$

Problem 2

$(-3,3)(1,-1)$
$(-1,2)(3,-4)$
$(0,1)$
$y = C_1 + C_2 t$
$3 = C_1 - 3C_2$
$-1 = C_1 + C_2$
$2 = C_1 - C_2$
$-4 = C_1 + 3C_2$
$1 = C_1 + C_2(0)$

$\begin{bmatrix} 1 & -3 \\ 1 & 1 \\ 1 & -1 \\ 1 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \\ -4 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -3 & 1 & -1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 1 & 1 \\ 1 & -1 \\ 1 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -3 & 1 & -1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 2 \\ -4 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 5 & 0 \\ 0 & 20 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -24 \end{bmatrix}$

$C_1 = \frac{1}{5}$
$C_2 = -\frac{6}{5}$
$y = 5 - \frac{6}{5}t$

$b - Ax = \begin{bmatrix} 3 \\ -1 \\ 2 \\ -4 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & -3 \\ 1 & 1 \\ 1 & -1 \\ 1 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{5} \\ -\frac{6}{5} \end{bmatrix} = \begin{bmatrix} -0.8 \\ 0 \\ 0.6 \\ -0.6 \\ 0.8 \end{bmatrix}$

RMSE: $\sqrt{\frac{(-0.8)^2+0+0.6^2+(-0.6)^2+0.8^2}{5}} = 0.6325$

Problem 3

$(0,1),(1/4,3),(1/2,2),(3/4,0)$

$y=C_1+C_2\cos 2\pi t+C_3\sin 2\pi t$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}=\begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}=\begin{bmatrix} 6 \\ -1 \\ 3 \end{bmatrix}$$

$C_3=\frac{3}{2}\quad C_1=\frac{3}{2}\ C_2=-\frac{1}{2}$

$$\begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix}-\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} \frac{3}{2} \\ \frac{-1}{2} \\ \frac{3}{2} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\|e\|_2=0\quad RMSE=0$

Problem 4

$$\begin{pmatrix} 3 & -1 & 2 \\ 4 & 1 & 0 \\ -3 & 2 & 1 \\ 1 & 1 & 5 \\ -2 & 0 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 10 \\ 10 \\ -5 \\ 15 \\ 0 \end{pmatrix}$$

Input and output:

```
>> A = [3 -1 2;4 1 0;-3 2 1; 1 1 5; -2 0 3];
>> b = [10; 10; -5; 15; 0];
>> AA = A'*A;bb = A'*[10;10;-5;15;0];
>> esp = 1^(-10);maxcount = 10000;
>> x = GaussSeidel(AA,bb,x,eps,maxcount)

x =

    2.5246
    0.6616
    2.0934

>> norm(A*x-b)

ans =

    2.4135

>> x = Jacobi(AA,bb,x,eps,maxcount)

x =

    2.5246
    0.6616
    2.0934

>> norm(A*x-b)

ans =

    2.4135
```

b)

$$\begin{pmatrix} 4 & 2 & 3 & 0 \\ -2 & 3 & -1 & 1 \\ 1 & 3 & -4 & 2 \\ 1 & 0 & 1 & -1 \\ 3 & 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ 2 \\ 0 \\ 5 \end{pmatrix}$$

Input and output:

```
>> A = [4 2 3 0;-2 3 -1 1;1 3 -4 2;1 0 1 -1;3 1 3 2];
>> b = [10; 0; 2; 0; 5];
>> AA = A'*A;bb = A'*b;
>> esp = 1^(-10);maxcount = 10000;
>> x0 = [0;0;0;0];
>> x = GaussSeidel(AA,bb,x0,eps,maxcount)

x =

    1.2823
    1.2067
    0.4744
   -0.3788

>> norm(A*x-b)

ans =

    2.4998

>> x = Jacobi(AA,bb,x0,eps,maxcount)

x =

    1.2823
    1.2067
    0.4744
   -0.3788

>> norm(A*x-b)

ans =

    2.4998
```

```
function [x,error,count]= GaussSeidel(A,b,x,eps,maxcount)
    error = 1e8;
    count = 0;

    while (error > eps) && (count < maxcount)
        D = diag(diag(A));
        L = tril(A)-D;
        U = triu(A)-D;
        x = -inv(L+D)*U*x+inv(L+D)*b;
        error = norm(A*x-b);
        count = count + 1;
    end

end
```

```
function [x,error,count]=Jacobi(A,b,x,eps,maxcount)
    error = 1e8;
    count = 0;

    while (error > eps) && (count < maxcount)
        D = diag(diag(A));
        r = A-D;
        x = -inv(D)*r*x+inv(D)*b;
```

```
        error = norm(A*x-b);
        count = count + 1;
    end

end
```