

Numerical Computing HW1

Yunfan Gao

February 9, 2021

Problem 1

Nested form:

$$\begin{aligned}P(x) &= 1 + x + 5x^2 + x^3 + 6x^4 \\&= 1 + x(1 + x(5 + x(1 + 6x))) \\&= 1 + \frac{1}{3} \left(1 + \frac{1}{3} \left(5 + \frac{1}{3} (1 + 6 * \frac{1}{3}) \right) \right) = 2\end{aligned}$$

4 times of multiplications

$$\text{Without Nested form: } P\left(\frac{1}{3}\right) = 1 + \frac{1}{3} + 5 \cdot \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + 6 \left(\frac{1}{3}\right)^4 = 2$$

8 times of multiplications

Problem 2

$$\begin{aligned}\frac{17}{2} &= 8R1 \\ \frac{8}{2} &= 4R0 \\ \frac{4}{2} &= 2R0 \\ \frac{2}{2} &= 1R0 \\ \frac{1}{2} &= 0R1 \\ (17)_{10} &= (10001)_2\end{aligned}$$

b)

$$\begin{aligned}0.875 \times 2 &= 1.75 & 1 \\ 0.75 \times 2 &= 1.5 & 1 \\ 0.5 \times 2 &= 1 & 1 \\ 0 \times 2 &= 0 & 0 \\ \left(\frac{7}{8}\right)_{10} &= (0.111)_2\end{aligned}$$

c)

$$(1011.101)_2 = 2^3 + 2^1 + 2^0 + 2^{-1} + 2^{-3} = 11 + \frac{1}{2} + \frac{1}{8} = \frac{93}{8}$$

Problem 3

$$0.6 \times 2 = 1.2$$

$$0.2 \times 2 = 0.4$$

$$0.4 \times 2 = 0.8$$

$$0.8 \times 2 = 1.6$$

$$0.6 \times 2 = 1.2$$

$$(9.6)_{10} = (1001.\overline{1001})_2 = 1.001\overline{1001} \times 2^3 = 1.0011001\dots 10011|00110\dots \times 2^3$$

applying rounding rules: (since the 53th bit is 0, chopping off)

$$\text{fl}(9.6) = 1.0011001\dots 10011 \mid \times 2^3$$

to convert to exact decimals:

$$0.00\overline{1001} \times 2^{-52} = 0.\overline{1001} \times 2^{-54} = 0.6 \times 2^{-54}$$

thus

$$\text{fl}(9.6) = 9.6 - 0.6 \times 2^{-54}$$

b)

$$0.3 \times 2 = 0.6$$

$$0.6 \times 2 = 1.2$$

$$0.2 \times 2 = 0.4$$

$$0.4 \times 2 = 0.8$$

$$0.8 \times 2 = 1.6$$

$$0.6 \times 2 = 1.2$$

$$3.3 = 11.0\overline{1001} = 1.101001\dots 100110|0110\dots \times 2^1$$

With rounding:

$$\text{fl}(3.3) = 1.101001\dots 100110 \times 2^1$$

$$\text{error} = \text{fl}(3.3) - 3.3 = 0.\overline{0110} \times 2^1 \times 2^{-52} = 0.4 \times 2^{-51}$$

relative rounding error:

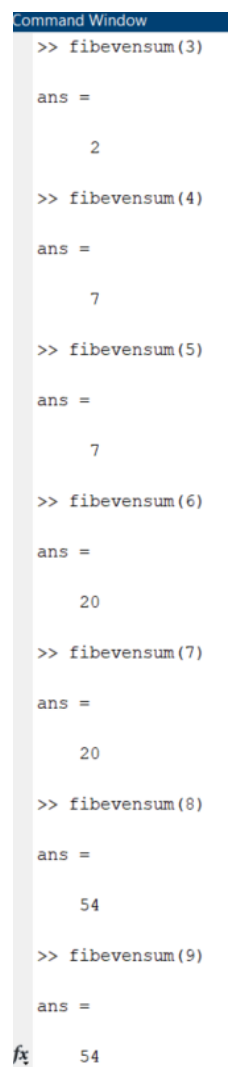
$$\frac{|3.3 - 0.4 \times 2^{-51} - 3.3|}{3.3} = \frac{8}{33} \times 2^{-52} < \frac{1}{2} \times 2^{-52}$$

Problem 4

```
function sum=fibevensum(k)
    if k < 2
        disp('invalid input, enter a number large than 1')
        return
    end
    %create fib series
    B(1)=1;
    B(2)=1;
    for i=3:k+1
        B(i)=B(i-1)+B(i-2);
    end
```

```
B %display fib series, starting from a0
%find the sum of even elements
n = floor(k/2); sum = 0;
for i = 1:n
    sum = sum + B(1+2*i);
end
end
```

Examples of summation output:



The image shows a screenshot of a MATLAB Command Window. The title bar is labeled "Command Window". The window contains the following text:

```
>> fibevensum(3)

ans =

     2

>> fibevensum(4)

ans =

     7

>> fibevensum(5)

ans =

     7

>> fibevensum(6)

ans =

    20

>> fibevensum(7)

ans =

    20

>> fibevensum(8)

ans =

    54

>> fibevensum(9)

ans =

    54
```

At the bottom left of the window, there is a small icon of a notepad and a cursor.