Numerical Computing HW3

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Problem 1

$f(x) = x^2 - 6$ with $x_0 = 1$
$f^{'}(x) = 2x$
$x_1 = 1 - \frac{(x_0)^2-6}{2x_0} = \frac{7}{2}$
$x_2 = x_1 - \frac{f(x_1)}{f^{'}(x_1)} = \frac{7}{2} - \frac{(x_1)^2-6}{2x_1} = 2.60714$

Problem 2

$x_1 = x_0 - \frac{ax_0+b}{a} = \frac{ax_0-ax_0-b}{a} = -\frac{b}{a}$
$x_2 = x_1 - \frac{a\frac{-b}{a}+b}{a} = -\frac{b}{a}$
The value stays at $\frac{-b}{a}$ with more iteration, thus already converged in the first step

Problem 3

$x_0 = 1$ and $x_1 = 2$
$x^3 = 2x + 2$
$f(x) = x^3 - 2x - 2$
$x_2 = x_1 - \frac{f(x_1)(x_1-x_0)}{f(x_1)-f(x_0)} = 2 - \frac{2*(2-1)}{2-(-3)} = 2 - \frac{2}{5} = \frac{8}{5}$
$x_3 = x_2 - \frac{f(x_2)(x_2-x_1)}{f(x_2)-f(x_1)} = \frac{8}{5} - \frac{[(\frac{8}{5})^3-2*\frac{8}{5}-2](\frac{8}{5}-2)}{(\frac{8}{5})^3-2*\frac{8}{5}-2-2} = \frac{169}{97} = 1.74227$

b)

$e^x + x = 7$
$f(x) = e^x + x - 7$
with $x_0 = 1$ and $x_1 = 2$
$x_2 = x_1 - \frac{f(x_1)(x_1-x_0)}{f(x_1)-f(x_0)} = 2 - \frac{e^2+2-7}{e^2+2-7-(e^1+1-7)} = 2 - \frac{e^2-5}{e^2+2-e^1-1} = 1.57871$
$x_3 = x_2 - \frac{f(x_2)(x_2-x_1)}{f(x_2)-f(x_1)} = x_2 - \frac{(e^{x_2}+x_2-7)(x_2-2)}{(e^{x_2}+x_2-7)-(e^{x_1}+x_1-7)} = 1.6602$

Problem 4

```
%Program Newton's method, hw3
% and tolerance tol
%x is x0, initial guess,

function [x, fx]=Newton(f,df,x,maxiter,Tol,del)
fx = f(x);
for n = 1:maxiter
    fp = df(x);
    if abs(fp) < del
        error('small dervative')
    end
    d = fx/fp;
    x = x-d;
    fx = f(x);
    if abs(d)<Tol
        disp('convergence')
        return
    end
end
```

$f = x - cos(x)$ with $df = 1 + sin(x)$ and initial guess $x_0 = \pi/4$
with output $x = 0.739085$

b)

$f = e^x + x - 7$ with $df = 1 + sin(x)$ and initial guess $x_0 = 3/2$
with output $x = 1.6728$

c)

$f = ln(x) + x^2 - 3$ with $df = \frac{1}{x} + 2x$ and initial guess $x_0 = 3/2$
with output $x = 1.59214$

Problem 5

```
%Program Newton's method, hw3
% and tolerance tol
%x is x1, x0 is initial guess,
%windows commands linef=@(x) x^3+x-1;
function [xi, fxi]=Secant(f,x1,x0,maxiter,Tol,del)
xi = x1;
xim1 = x0;
fxi = f(xi)
```

```
fxim1 = f(xim1)
for n = 1:maxiter
    if abs(fxi-fxim1) < del
        error('small dervative')
    end
    n = fxi*(xi-xim1)
    d = fxi-fxim1
    t = n/d
    xim1 = xi
    xi = xi-t
    fxi = f(xi)
    fxim1 = f(xim1);
    if abs(t)<Tol
        disp('convergence')
        return
    end
end
```

$f = x - cos(x)$ with $x_1 = pi/4$ and $x_0 = pi/8$
with output $x = 0.7391$

b)

$f = e^x + x - 7$ with $x_1 = 1.8$ and $x_0 = 1.6$
with output $x = 1.6728$

c)

$f = ln(x) + x^2 - 3$ with $x_1 = 1.8$ and $x_1 = 1.6$
with output $x = 1.5921$