Numerical Computing HW1

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Problem 1

Nested form:
$P(x) = 1 + x + 5x^2 + x^3 + 6x^4$
$= 1 + x(1 + x(5 + x(1 + 6x)))$
$= 1 + \frac{1}{3}\left(1 + \frac{1}{3}\left(5 + \frac{1}{3}(1 + 6 * \frac{1}{3})\right)\right) = 2$
4 times of multiplications
Without Nested form: $P\left(\frac{1}{3}\right) = 1 + \frac{1}{3} + 5 \cdot \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + 6\left(\frac{1}{3}\right)^4 = 2$
8 times of multiplications

Problem 2

$\frac{17}{2} = 8R1$
$\frac{8}{2} = 4R0$
$\frac{4}{2} = 2R0$
$\frac{2}{2} = 1R0$
$\frac{1}{2} = 0R1$
$(17)_{10} = (10001)_2$

b)

$0.875 \times 2 = 1.75 \quad 1$
$0.75 \times 2 = 1.5 \quad 1$
$0.5 \times 2 = 1 \quad 1$
$0 \times 2 = 0 \quad 0$
$\left(\frac{7}{8}\right)_{10} = (0.111)_2$

c)

$(1011.101)_2 = 2^3 + 2^1 + 2^0 + 2^{-1} + 2^{-3} = 11 + \frac{1}{2} + \frac{1}{8} = \frac{93}{8}$

Problem 3

$0.6 \times 2 = 1.2$
$0.2 \times 2 = 0.4$
$0.4 \times 2 = 0.8$
$0.8 \times 2 = 1.6$
$0.6 \times 2 = 1.2$
$(9.6)_{10} = (1001.\overline{1001})_2 = 1.001\overline{1001} \times 2^3 = 1.0011001...10011|00110.. \times 2^3$
applying rounding rules: (since the 53th bit is 0, chopping off)
$\text{fl}(9.6) = 1.0011001\ldots10011 \mid \times 2^3$
to convert to exact decimals:
$0.00\overline{1001} \times 2^{-52} = 0.\overline{1001} \times 2^{-54} = 0.6 \times 2^{-54}$
thus
$\text{fl}(9.6) = 9.6 - 0.6 \times 2^{-54}$

b)

$0.3 \times 2 = 0.6$
$0.6 \times 2 = 1.2$
$0.2 \times 2 = 0.4$
$0.4 \times 2 = 0.8$
$0.8 \times 2 = 1.6$
$0.6 \times 2 = 1.2$
$3.3 = 11.0\overline{1001} = 1.101001...100110|0110... \times 2^1$
With rounding:
$\text{fl}(3.3) = 1.101001...100110 \times 2^1$
error = fl(3.3) - $3.3 = 0.\overline{0110} \times 2^1 \times 2^{-52} = 0.4 \times 2^{-51}$
relative rounding error:
$\frac{|3.3-0.4\times2^{-51}-3.3|}{3.3} = \frac{8}{33} \times 2^{-52} < \frac{1}{2} \times 2^{-52}$

Problem 4

```
function sum=fibevensum(k)
    if k < 2
        disp('invalid input, enter a number large than 1')
        return
    end
    %create fib series
    B(1)=1;
    B(2)=1;
    for i=3:k+1
        B(i)=B(i-1)+B(i-2);
    end
```

```
    B %display fib series, starting from a0
    %find the sum of even elements
    n = floor(k/2); sum = 0;
    for i = 1:n
        sum = sum + B(1+2*i);
    end
end
```

Examples of summation output: