Numerical Computing HW5

Yunfan Gao

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Problem 1

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
$\|A\|_\infty = |3| + |4| = 7$
b)
$A = \begin{bmatrix} 1 & 5 & 1 \\ -1 & 2 & -3 \\ 1 & -7 & 0 \end{bmatrix}$
$\|A\|_\infty = |1| + |-7| = 8$

Problem 2

$D = \begin{bmatrix} d_1 \cdots & \\ & d_n \end{bmatrix}$ $D^{-1} = \begin{bmatrix} \frac{1}{d_1} & \\ & \frac{1}{d_n} \end{bmatrix}$
$cond(D) = \|D\|_\infty \left\|D^{-1}\right\|_\infty$
$= \max\{di\} \cdot \frac{1}{\min\{d_i\}}$
$= \frac{max\{di\}}{min\{d_i\}}$ for $i = 1, 2, \cdots n$

Problem 3

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & -1 \\ (\frac{1}{2}) & \frac{1}{2} & \frac{1}{2} \\ (-\frac{1}{2}) & \frac{3}{2} & -\frac{3}{2} \end{bmatrix} \to \begin{bmatrix} 2 & 1 & -1 \\ (-\frac{1}{2}) & \frac{3}{2} & -\frac{3}{2} \\ (\frac{1}{2}) & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}}_{p} \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix}}_{A} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-1}{2} & 1 & 0 \\ \frac{1}{2} & \frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 0 & \frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 4

$$\begin{pmatrix} 10 & 5 & 0 & 0 \\ 5 & 10 & -4 & 0 \\ 0 & -4 & 8 & 1 \\ 0 & 0 & -1 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 6 \\ 25 \\ -11 \\ -11 \end{pmatrix}$$

$$\begin{aligned} 10x_1 + 5x_2 &= 6 \\ 5x_1 + 10x_2 - 4x_3 &= 25 \\ -4x_2 + 8x_3 + x_4 &= -11 \\ -x_3 + 5x_4 &= -11 \end{aligned}$$

a)Jacobi

$x_1 = \frac{6-5x_2}{10}$
$x_2 = \frac{25-5x_1+4x_3}{10}$
$x_3 = \frac{-11+4x_2-x_4}{8}$
$x_4 = \frac{-11+x_3}{5}$
$x^0 = 0$
$x^1 = \left[\frac{6}{10} \quad \frac{25}{10} \quad \frac{-11}{8} \quad -\frac{11}{5} \right]$
$x^2 = \left[\frac{-13}{20} \, \frac{33}{20} \, \frac{3}{20} \, \frac{-99}{40} \right]$

b)Gauss Siedel $\quad \begin{aligned} x_1^{k+1} &= \frac{6-5x_2^k}{10} \\ x_2^{k+1} &= \frac{25-5x_1^{k+1}+4x_3}{10} \\ x_3^{k+1} &= \frac{-11+4x_2^{k+1}-x_4}{8} \\ x_4^{k+1} &= \frac{-11+x_3^{k+1}}{5} \end{aligned}$

$X^0 = 0$
$X^1 = \left[\frac{6}{10} \quad \frac{11}{5} \quad -\frac{11}{40} \quad -\frac{451}{200} \right]$
$X^2 = \left[-\frac{1}{2} \, \frac{66}{25} \, \frac{363}{1600} \, -\frac{17237}{8000} \right]$

Problem 5

$2x_1 - x_2 + x_3 = -1$
$x_1 + x_2 + x_3 = 2$
$-x_1 - x_2 + 2x_3 = -5$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & & \\ & 1 & \\ & & 2 \end{bmatrix} \quad L = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -1 & -1 & 0 \end{bmatrix} \quad u = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T_J = -D^{-1}(L+u) = -\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ -1 & 0 & -1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

$$\begin{vmatrix} \lambda & -\frac{1}{2} & \frac{1}{2} \\ 1 & \lambda & 1 \\ -\frac{1}{2} & -\frac{1}{2} & \lambda \end{vmatrix}$$

$\lambda^3 + \frac{5}{4}\lambda = 0$

$\lambda\left(\lambda^2 + \frac{5}{4}\right) = 0$
 $\lambda_1 = 0$
 $\lambda_2 = \frac{\sqrt{5}}{2} i$
$P\left(T_J\right) = \max\left\{\left|\lambda_1\right|, \left|\lambda_2\right| = \frac{\sqrt{5}}{2}\right.$

$$-(L+D)^{-1}U = -\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 0 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

$\frac{\lambda(2\lambda+1)^2}{4} = 0$
$\lambda = 0,\ \lambda = -\frac{1}{2}$
$P\left(T_G\right) = max\left\{0, \frac{1}{2}\right\} = \frac{1}{2}$

Problem 6

```
function [x,error,count]=Jacobi(A,b,x,eps,maxcount)
    error = 1e8;
    count = 0;

    while (error > eps) && (count < maxcount)
        D = diag(diag(A));
        r = A-D;
        x = -inv(D)*r*x+inv(D)*b;
        error = norm(A*x-b);
        count = count + 1;
    end

end
```

```
>> n = 50;
>> dx = 1/(n+1);
>> A = -(2+10*dx^2)*diag(ones(n,1))+diag(ones(n-1,1),-1)+diag(ones(n-1,1),1);
>> b = zeros(n,1);
>> b(n)  = -1;
>> esp = 10^(-5);
>> maxcount = 10000;
>> Jacobi(A,b,x,eps,maxcount)
```

```
      -
      0
      0
 0.0001
 0.0001
 0.0010
 0.0018
 0.0072
 0.0127
 0.0344
 0.0562
 0.1161
 0.1765
 0.2970
 0.4186
 0.6014
 0.7864
```

```
function [x,error,count]= GaussSeidel(A,b,x,eps,maxcount)
    error = 1e8;
    count = 0;

    while (error > eps) && (count < maxcount)
        D = diag(diag(A));
        L = tril(A)-D;
        U = triu(A)-D;
        x = -inv(L+D)*U*x+inv(L+D)*b;
        error = norm(A*x-b);
        count = count + 1;
    end

end
```

```
>> n = 50;
dx = 1/(n+1);
A = -(2+10*dx^2)*diag(ones(n,1))+diag(ones(n-1,1),-1)+diag(ones(n-1,1),1);
b = zeros(n,1);
b(n) = -1;
esp = 10^(-5);
maxcount = 10000;
GaussSeidel(A,b,x,eps,maxcount)
```

0

0

0.0154

0.0695

0.1777

0.3403

0.5444

0.7707