Numerical Computing HW6

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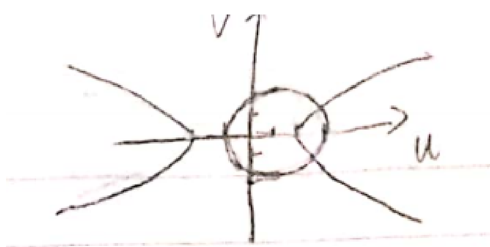
Problem 1

$u^2 - 4v^2 = 4$
$(u-1)^2 + v^2 = 4$
yields: $\pm\sqrt{4-(u-1)^2} = v$ and $\pm\sqrt{\frac{u^2-4}{4}} = v$



solve for the intersections: $4(u-1)^2 + u^2 - 4 = 16$
$u = -1.15459$
$u = 2.75459(\checkmark)$
$v = \pm\sqrt{\frac{u^2-4}{4}} = \pm\frac{\sqrt{8\cdot\sqrt{6}+3}}{5} = \pm 0.950703$

$f_1(u,v) = u^2 - 4v^2 - 4$
$f_2(u,v) = (u-1)^2 + v^2 - 4$

Jacobian Matrix $\begin{bmatrix} 2u & -8v \\ 2(u-1) & 2v \end{bmatrix}$

$\begin{bmatrix} 2 & -8 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = -\begin{bmatrix} -7 \\ -3 \end{bmatrix}$

$x_1 = x_0 + s = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{19}{2} \\ \frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{21}{2} \\ \frac{5}{2} \end{bmatrix}$

$\begin{bmatrix} 21 & -20 \\ 19 & 5 \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = -\begin{bmatrix} \frac{325}{4} \\ \frac{185}{2} \end{bmatrix}$

$S = \begin{bmatrix} -4.6526 \\ -0.822165 \end{bmatrix}$

$x_2 = x_1 + s$
$x_2 = \begin{bmatrix} \frac{21}{2} \\ \frac{5}{2} \end{bmatrix} + \begin{bmatrix} -4.65206 \\ -0.92246 \end{bmatrix} = \begin{bmatrix} 5.84794 \\ 1.67184 \end{bmatrix}$

Problem 2

$\sum_{k=0}^{n} \prod_{i=1,i\neq k}^{i=n} \frac{x-x_i}{x_k-x_i} = \frac{(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_2)(x_1-x_3)\cdots-(x_1-x_n)} + \cdots \frac{(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)} +$
$\cdots \frac{(x-x_1)(x-x_2)\cdots\cdots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\cdots-(x_n-x_{n-1})}$
$= \mathcal{L}_1(x) + \cdots \mathcal{L}_k(x) + \cdots \mathcal{L}_n(x) = 1 \quad$ for $x = x_i \quad i = 1 \cdots n$
since $\begin{array}{l} L_k(x_i) = 1 \quad if \quad i = k \\ L_k(x_i) = 0 \quad if \quad i \neq k \end{array}$

Problem 3

x	0	1	2	3
3	4	9	15	8

$\deg(L_h) = 3$

$p(x) = \frac{4\cdot(x-1)\cdot(x-2)(x-3)}{(0-1)(0-2)(0-3)} + 4\cdot\frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)} + 15\frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)} + 8\frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}$
$= \frac{-7x^3}{3} + \frac{15}{2}x^2 - \frac{x}{6} + 4$

b)

$basis\,\{1, \quad (x-x_1), \quad (x-x_1)(x-x_2), \quad (x-x_1)(x-x_2)(x-x_3)\}$

$$\begin{array}{c|cccc} 0 & 4 & & & \\ & & 5 & & \\ 1 & 9 & & 0.5 & \\ & & 6 & & -7/3 \\ 2 & 15 & & -6.5 & \\ & & -7 & & \\ 3 & 8 & & & \end{array}$$

$4 + 5(x-x_1) + \frac{1}{2}(x-x_1)(x-x_2) - \frac{7}{3}(x-x_1)(x-x_2)(x-x_3)$
$= 4 + 5x + \frac{1}{2}x(x-1) - \frac{7}{3}x(x-1)(x-2)$
$= -\frac{7x^3}{3} + \frac{15}{2}x^2 - \frac{x}{6} + 4$
The results are the same

Problem 4

$$\begin{array}{c|ccc}
1 & 0 & & \\
 & & \ln 2 & \\
2 & \ln 2 & & -\frac{ln2}{6} \\
 & & \frac{ln2}{2} & \\
4 & \ln 4 & &
\end{array}$$

$\ln 2(x-1) + \frac{-\ln 2}{6}(x-1)(x-2)$
$P(3) = 2 \cdot \ln 2 - \frac{\ln 2}{3} = \frac{5}{3} \ln 2$

c)

$f(x) - p(x) = \frac{(x-x_1)(x-x_2)(x-r_2)}{n!} f^n(c)$
$= \frac{(x-4)(x-2)(x-1)}{3 \cdot 2 \cdot 1} \cdot 2 \cdot C^{-3}$
$|f(x) - p(x)| \leqslant |\frac{1 \cdot 1 \cdot 2}{3 \cdot 2 \cdot 1}^2 \cdot C^{-3}|$
$|f(x - p(x)| \leqslant \frac{2}{3}$ when c = 1

d)

$\frac{5}{2} ln(2) - ln(3) = 0.056633 < \frac{2}{3}$

Problem 5

```
%Program 0.1 Nested multiplication
%Evaluates polynomial from nested form using Horners Method
%Input: degree d of polynomial,
% array of d+1 coefficients c (constant term first),
% x-coordinate x at which to evaluate, and
% array of d base points b, if needed
%Output: value y of polynomial at x
function y=nest(d,c,x,b)
    if nargin<4, b=zeros(d,1); end
    y=c(d+1);
    for i=d:-1:1
        y = y.*(x-b(i))+c(i);
    end
end
```

```
%Use with nest.m to evaluate interpolating polynomial
function c = newtdd(x,y,n)
    for j=1:n
        v(j,1)=y(j); % Fill in y column of Newton triangle
    end
```

```
for i=2:n % For column i,
    for j=1:n+1-i % fill in column from top to bottom
        v(j,i)=(v(j+1,i-1)-v(j,i-1))/(x(j+i-1)-x(j));
    end
end
for i=1:n
    c(i)=v(1,i); % Read along top of triangle
end % for output coefficients

end
```

Input

```
x0 = zeros(10,1);
for i = 0:9
    x0(i+1) = -1+i*2/9;
    y0(i+1) = 1/(1+12*x0(i+1)^2);
end
c = newtdd(x0,y0,10);
x = -1:0.01:1;
y = nest(9,c,x,x0);
plot(x0,y0,'o',x,y);
```

output