Numerical Computing HW9

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Problem 1

$y_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ $r_{11} = \sqrt{2^2 + 2^2 + 1} = 3$

$q_1 = \frac{y_1}{\|y_1\|_2} = \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$

$y_2 = A_2 - q_1 q_1^\top A_2 = \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix}$

$r_{22} = \|y_0\|_2 = 3$

$q_2 = \frac{y_2}{\|y_2\|} = \begin{bmatrix} -\frac{1}{3} \\ -\frac{2}{3} \\ -\frac{2}{3} \end{bmatrix}$ $r_1 = q_1^\top A_2 = \begin{bmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} = 6$

$\begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{2}{3} & -\frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 3 & 6 \\ 0 & 3 \end{bmatrix}$

$y_3 = A_3 - q_1 q_1^\top A_3 - q_2 q_2^\top A_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{9} \\ \frac{-4}{9} \\ \frac{2}{9} \end{bmatrix} - \begin{bmatrix} \frac{1}{9} \\ \frac{2}{9} \\ \frac{2}{9} \end{bmatrix} = \begin{bmatrix} \frac{4}{9} \\ \frac{2}{9} \\ -\frac{4}{9} \end{bmatrix}$

$q_1 = \frac{y_2}{\|r_3\|} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}$

$A = QR = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 3 & 6 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix}$

Problem 2

$$\begin{bmatrix} 3 & 6 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 3 \\ -3 \\ 6 \end{bmatrix}$$
$$\begin{bmatrix} 3 & 6 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix} \Rightarrow \begin{matrix} x_1 = 4 \\ x_2 = -1 \end{matrix}$$

Problem 3

$f(x+\frac{h}{2}) = f(x) + \frac{1}{2}hf'(x) + \frac{1}{8}h^2f^2(x) + \frac{1}{48}h^3f^3(x) + \frac{1}{384}h^4f^4(x) + O\left(h^5\right)$
$f(x-\frac{h}{2}) = f(x) - \frac{1}{2}hf'(x) + \frac{1}{8}h^2f^2(x) - \frac{1}{48}h^3f^3(x) + \frac{1}{384}h^4f^4(x) + O\left(h^5\right)$
$f(x+h) = f(x) + hf'(x) + \frac{1}{2}h^2f^2(x) + \frac{1}{6}h^3f^3(x) + \frac{1}{24}h^4f^4(x) + O\left(h^5\right)$
$f(x-h) = f(x) + hf'(x) + \frac{1}{2}h^2f^2(x) + \frac{1}{6}h^3f^3(x) + \frac{1}{24}h^4f^4(x) + O\left(h^5\right)$
$f\left(x_0-h\right) - 8f\left(x_0-h/2\right) + 8f\left(x_0+h/2\right) - f\left(x_0+h\right) = 6hf'(x) + O\left(h^5\right)$
$f'\left(x_0\right) = \frac{f(x_0-h)-8f(x_0-h/2)+8f(x_0+h/2)-f(x_0+h)}{6h} + O\left(h^4\right)$

b)

$f\left(x_i+h\right) = f\left(x_i\right) + hf'\left(x_i\right) + \frac{1}{2}h^2f''\left(x_i\right) + \frac{h^3}{3!}f'''\left(x_i\right) + \frac{h^4}{4!}f^{(4)}\left(c_1\right)$ $f\left(x_i-h\right) = f\left(x_i\right) - hf'\left(x_i\right) + \frac{1}{2}h^2f''\left(x_i\right) - \frac{h^3}{3!}f'''\left(x_i\right) + \frac{h^4}{4!}f^{(4)}\left(c_2\right)$
$f\left(x_i+h\right) + f\left(x_i-h\right) = 2f\left(x_i\right) + h^2f''\left(x_i\right) + \frac{h^4}{4!}\left(f^{(4)}\left(c_1\right) + f^{(4)}(c_2)\right.$
$\frac{f(x_i+h)-2f(x_i)+f(x_i-h)}{h^2} = f''\left(x_i\right) + \frac{h^2}{4!}\left(f^{(4)}\left(c_1\right) + f^{(4)}(c_2)\right)$

Problem 4

x	$f(x)$
0.5	0.4794
0.6	0.5646
0.7	0.6442

$f'(0.6) = \frac{f(0.7)-f(0.6)}{0.7-0.6} = \frac{0.6442-0.5646}{0.1}$
$|0.796 - \cos(0.6)| \quad = 0.029336$
range:$|\frac{1}{2}f''(c)\frac{b-a}{n-1}| = \begin{cases} 0.028232 & c = 0.6 \\ 0.032211 & c = 0.7 \end{cases}$
$f'(0.6) = \frac{f(0.6)-f(0.5)}{0.6-0.5} = \frac{0.5646-0.4749}{0.1} = |0.852 - \cos(0.6)| = 0.026664$
range:$|\frac{0.1}{2}\sin c| = \begin{cases} 0.02832 & c = 0.6 \\ 0.023971 & c = 0.5 \end{cases}$
$f'(0.6) = \frac{0.6442-0.4794}{2.0.1} = 0.824$
$error = 0.001336$

range:$|\frac{1}{3!}cosc * 0.1^2| = \begin{cases} 0.001463 & C = 0.5 \\ 0.001275 & C = 0.7 \end{cases}$

Problem 5

```
% modified Gram-Schmidt orthogonalizatoin
function [Q,R]=modifiedgs(A)
[m,n]=size(A);
for j=1:n
    y=A(:,j);
    for i=1:j-1
        R(i,j)=Q(:,i)'*y;
        y=y-R(i,j)*Q(:,i);
    end
    R(j,j)=norm(y);
    Q(:,j)=y/R(j,j);
end
```

Input and output

```
A =

     3    -1     2
     4     1     0
    -3     2     1
     1     1     5
    -2     0     3

>> [Q,R]=modifiedgs(A)

Q =

    0.4804   -0.2697    0.4057
    0.6405    0.5494   -0.2236
   -0.4804    0.6592   -0.0310
    0.1601    0.4295    0.6914
   -0.3203   -0.0799    0.5535


R =

    6.2450   -0.6405    0.3203
         0    2.5670    2.0277
         0         0    5.8980
```

Input and output

```
>> [Q,R] = qr(A);
>> y = [10;10;-5;15;0];
>> b = Q'*y;
>> c = R\b

c =

    2.5246
    0.6616
    2.0934

>> norm(y-A*c)

ans =

    2.4135
```