Numerical Computing HW7

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Problem 1

$[4, 12]$ with $n = 6$
Interpolation nodes: $\frac{b-a}{2}\cos(\frac{(2i-1)\pi}{2n}) + \frac{b+a}{2}$
$8+4\cos\frac{\pi}{12}$ $\quad 8+4\cos\frac{\pi}{4}$ $\quad 8+4\cos\frac{5\pi}{12}$ $\quad 8+4\cos\frac{7\pi}{12}$ $\quad 8+4\cos\frac{3\pi}{4}$ $\quad 8+4\cos\frac{11\pi}{12}$

b)

$[-0.3, 0.7]$ n = 5
$0.2+0.5\cos(\frac{\pi}{10})$ $\quad 0.2+0.5\cos(\frac{3\pi}{10})$ $\quad 0.2+0.5\cos(\frac{5\pi}{10})$ $0.2+0.5\cos(\frac{7\pi}{10})$ $\quad 0.2+0.5\cos(\frac{9\pi}{10})$

Problem 2

$[3, 4]$ with $n = 5$
$\frac{7}{2}+\frac{1}{2}\cos\frac{\pi}{8}$ $\quad \frac{7}{2}+\frac{1}{2}\cos\frac{3\pi}{8}$ $\quad \frac{7}{2}+\frac{1}{2}\cos\frac{5\pi}{8}$ $\quad \frac{7}{2}+\frac{1}{2}\cos\frac{7\pi}{8}$

3.96194	0.01608			
		-0.01405		
3.69134	0.019881		0.009403	
		-0.020192		-0.00568
3.30866	0.027609		0.014651	
		-0.029763		
3.03806	0.035662			

$Q(x) = 0.01608+(-0.01405)(x-3.96194)+0.009403(x-3.96144)(x-3.649134)+(-0.00568)(x-3.96194)(x-3.69134)(x-3.30866)$

$Q(3.96194) = 0.0160796$
$Q(3.69134) = 0.0198814$
$Q(3.30866) = 0.0276086$
$Q(3.0380) = 0.0356626$

$|f(x) - Q(x)| = |\frac{(x-x_1)(x-x_4)}{4!}f^4(a)|$
$f^4(x) = -60x^{-6}$
$\max\left|f^4(x)\right| = 360x^{-7}$

$\max \mid (x-x_1)\cdots(x-x_4) \mid \leqslant \frac{\left(\frac{b-a}{2}\right)^n}{2^{n-1}} = 2^{-7}$
$|f(x)-Q(x)| \leqslant \frac{360\cdot 3^{-6}\cdot 2^{-7}}{4!} = 0.000054$

Problem 3

$f(x) = \begin{cases} x^3+x-1 & [0,1] \\ -(x-1)+3(x-1)^2+3(x-1)+1 & [1,2] \end{cases}$
$S_1(1)=1$
$S_2(1)=1$
$S_1'(1) = 3x^2+1|_{x=1} = 4$
$S_2'(1) = -3(x-1)^2+6(x-1)+3|_{x=1} = 3$
$S_2'(1) \neq S_1'(1)$
Thus not cubic spline

Problem 4

$(0,0) \quad (1,1) \quad (2,1) \quad (3,1)$

x	0	1	2	3
y	0	1	1	1

$S_1(x) = b_1(x-0)+C_1(x-0)^2+d_1(x-0)^3$
$S_2(x) = 1+b_2(x-1)+C_2(x-1)^2+d_2(x-1)^3$
$S_3(x) = 1+b_3(x-1)+C_3(x-2)^2+d_3(x-2)^3$
$S_1(x_2) = y_2 \quad S_2(x_3) = y_3 \quad S_3(x_4) = y_4$

$$\begin{aligned} b_1x_2+c_1(x_2)^2+d_1(x_2)^3 &= 1 \\ 1+b_2(x_3-1)+c_2(x_3-1)^2+d_2(x_2-1)^3 &= 1 \\ 1+b_3(x_4-2)+c_3(x_4-2)^2+d_3(x_4-2)^3 &= 1 \end{aligned}$$
$$\left.\begin{aligned} b_1+c_1+d_1 &= 0 \\ 1+b_2+c_2+d_2 &= 1 \\ 1+b_3+c_3+d_3 &= 1 \end{aligned}\right\}$$
$S_1'(x_2) = S_2'(x_2) \quad S_2'(x_3) = S_3'(x_3)$
$b_1+2c_1x_2+3d_1x_2^2 = b_2$
$b_2+2c_2(x_3-1)+3d_2(x_3-1)^2 = b_3+2c_3(x_3-2)+3d_3(x_3-2)^2$
$b_2+2c_2+3d_2 = b_3$
$S_1''(x_2) = S_2''(x_2) \quad S_2''(x_3) = S_3''(x_3)$
$2c_1+6d_1x_2 = 2c_2+6d_2(x_2-1) \quad 2c_2+6d_2(x_3-1) = 2c_3+6d_3(x_3-2)$
$\begin{cases} b_1+2c_1+3d_1 = b_2 \\ b_2+2c_2+3d_2 = b_3 \end{cases} \quad \begin{cases} 2c_1+6d_1 = 2c_2 \\ 2c_2+6d_1 = 2c_3 \end{cases}$
$S_1''(x_1) = 0 \quad S_3''(x_4) = 0$
$\begin{cases} 2c_1+0 \\ 2c_3+6d_3 = 0 \end{cases}$

$$
\left[\begin{array}{ccccccccc}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 2 & 3 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 2 & 3 & -1 & 0 & 0 \\
0 & 2 & 6 & 0 & -2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 6 & 0 & -2 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 12
\end{array}\right]^{-1} [1;0;0;0;0;0;0;0;0]
$$

$S_1(x) = \frac{19}{15}x - \frac{4}{15}x^3$
$S_2(x) = 1 + \frac{7}{15}(x-1) - \frac{4}{5}(x-1)^2 + \frac{1}{3}(x-1)^3$
$S_3(x) = 1 - \frac{2}{15}(x-2) + \frac{1}{5}(x-2)^2 - \frac{1}{15}(x-2)^3$

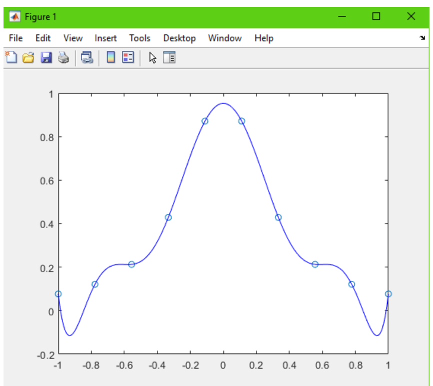
Problem 5

Input

```
x0 = zeros(10,1);
for i = 0:9
    x0(i+1) = -1+i*2/9;
    y0(i+1) = 1/(1+12*x0(i+1)^2);
end
c = newtdd(x0,y0,10);
x = -1:0.01:1;
y = nest(9,c,x,x0);
plot(x0,y0,'o',x,y, 'blue');
hold on
```

Output

```
y0 =

  Columns 1 through 4

   0.076923076923077   0.121076233183857   0.212598425196850   0.428571428571429

  Columns 5 through 8

   0.870967741935484   0.870967741935484   0.428571428571429   0.212598425196850

  Columns 9 through 10

   0.121076233183857   0.076923076923077

>> x0

x0 =

  -1.000000000000000
  -0.777777777777778
  -0.555555555555556
  -0.333333333333333
  -0.111111111111111
   0.111111111111111
   0.333333333333333
   0.555555555555556
   0.777777777777778
   1.000000000000000
```



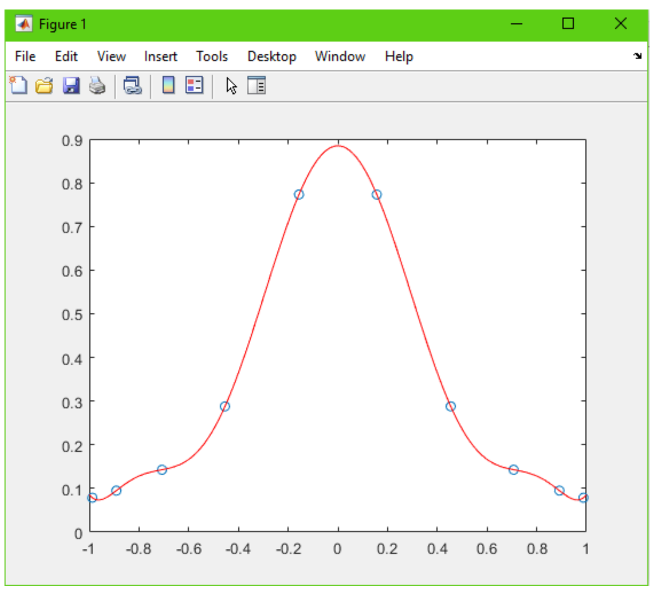
b)

Input

```
x0 = zeros(10,1);
for i = 0:9
    x0(i+1) = cos((2*i+1)*pi/20);
    y0(i+1) = 1/(1+12*x0(i+1)^2);
end
c = newtdd(x0,y0,10);
x = -1:0.01:1;
y = nest(9,c,x,x0);
plot(x0,y0,'o',x,y, 'red');
```

Output

```
y0 =

  Columns 1 through 4

   0.078700874603247   0.094996428722620   0.142857142857143   0.287911586947119

  Columns 5 through 8

   0.773000094751972   0.773000094751973   0.287911586947119   0.142857142857143

  Columns 9 through 10

   0.094996428722621   0.078700874603247

>> x0

x0 =

   0.987688340595138
   0.891006524188368
   0.707106781186548
   0.453990499739547
   0.156434465040231
  -0.156434465040231
  -0.453990499739547
  -0.707106781186547
  -0.891006524188368
  -0.987688340595138
```

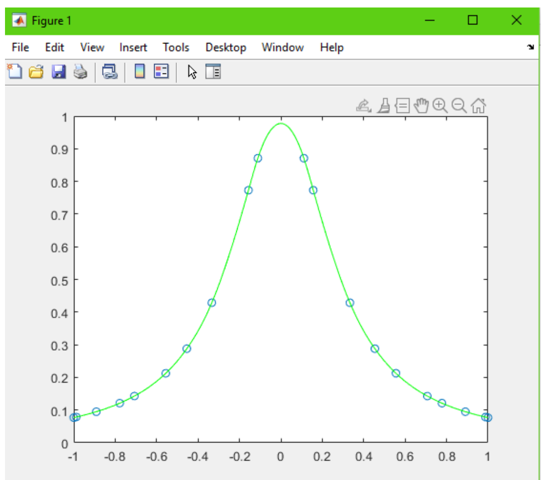


c)

Input

```
%cubic spline of two nodes combined (20 points)
for i = 0:9
    xx0(i+1) = cos((2*i+1)*pi/20);
    xx1(i+1) = -1+i*2/9;
end
aa = cat(2, xx0, xx1);
x0 = sort(aa);
for i = 1: length(aa)
    y0(i) = 1/(1+12*x0(i)^2);
end
xx = -1:0.01:1;
yy = spline(x0,y0,xx);
plot(x0,y0,'o',xx,yy, 'green')
```

Output



All code in one file: Comment out different sections of the code to see individual plots

```
close all; clear all;
%comment out differnet sections of the code to see individual
    ↪ plot


%polynomial interpolation using Chebyshev with n=0...9
%code is for the f(x) defined on interval [-1 1]
x0 = zeros(10,1);
for i = 0:9
    x0(i+1) = cos((2*i+1)*pi/20);
    y0(i+1) = 1/(1+12*x0(i+1)^2);
end
```

```
c = newtdd(x0,y0,10);
x = -1:0.01:1;
y = nest(9,c,x,x0);
plot(x0,y0,'o',x,y, 'red');
hold on

%the below is for equally spaced interpolatoin
x0 = zeros(10,1);
for i = 0:9
    x0(i+1) = -1+i*2/9;
    y0(i+1) = 1/(1+12*x0(i+1)^2);
end
c = newtdd(x0,y0,10);
x = -1:0.01:1;
y = nest(9,c,x,x0);
plot(x0,y0,'o',x,y, 'blue');
hold on


%the real plot of 1/(1+12x^2)
x = -1:0.01:1;
y = 1./(1+12.*x.^2);
plot(x,y,'magenta')
hold on

%{
%cubic spline chebyshev
for i = 0:9
    x0(i+1) = cos((2*i+1)*pi/20);
    y0(i+1) = 1/(1+12*x0(i+1)^2);
end
xx = -1:0.01:1;
yy = spline(x0,y0,xx);
plot(x0,y0,'o',xx,yy, 'black')
hold on


%cubic spline equally spaced interpolation
for i = 0:9
    x0(i+1) = -1+i*2/9;
    y0(i+1) = 1/(1+12*x0(i+1)^2);
end
xx = -1:0.01:1;
yy = spline(x0,y0,xx);
plot(x0,y0,'o',xx,yy, 'green')
hold on
```

```
%}

%cubic spline of two nodes combined (20 points)
for i = 0:9
    xx0(i+1) = cos((2*i+1)*pi/20);
    xx1(i+1) = -1+i*2/9;
end
aa = cat(2, xx0, xx1);
x0 = sort(aa);
for i = 1: length(aa)
    y0(i) = 1/(1+12*x0(i)^2);
end
xx = -1:0.01:1;
yy = spline(x0,y0,xx);
plot(x0,y0,'o',xx,yy, 'green')
hold off
```

Comparing with the real function (in magenta), the cubic splines (green) track the real function really well. The Chebyshev interpolation (in red) is okay, the evenly spaced interpolation (in blue) is the worst. As shown: