Stochastic Methods for Finance: Report 1

Leonardo Schiavo

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Abstract

We want to compare the results given by using the binomial model to price a call option and the price provided by Yahoo Finance.

1. Introduction

We have chosen a publicly traded company from finance.yahoo.com and a call option with maturity around 3 months and strike price at the money. We considered the mean between bid and ask, the mid-price, as the target price for our model. We implemented a binomial model with the historical data and compared the mid-price with the price of the call option suggested by the binomial model. We repeated for a 7 month maturity call option. Note that the currency is USD throughout the article.

2. Western Digital Corporation (WDC)

We have chosen Western Digital Corporation. WDC develops, manufactures, and sells data storage devices and solutions in the United States, China, Hong Kong, Europe, the Middle East, Africa, rest of Asia, and internationally. It offers client devices, including hard disk drives (HDDs) and solid state drives (SSDs) for computing devices, such as desktop and notebook personal computers (PCs), smart video systems, gaming consoles, and set top boxes; flash-based embedded storage products for mobile phones, tablets, notebook PCs, and other portable and wearable devices, as well as automotive, Internet of Things, industrial, and connected home applications; and flash-based memory wafers. The company also provides data center devices and solutions comprising enterprise helium hard drives; enterprise SSDs consisting of flash-based SSDs and software solutions for use in enterprise servers, online transactions, data analysis, and other enterprise applications; data center solutions for data storage systems and tiered storage models; and data storage platforms. In addition, it offers client solutions, such as external HDD storage products in mobile and desktop form; client portable SSDs; removable cards that are used in consumer devices comprising mobile phones, tablets, imaging systems, and cameras and smart video systems; universal serial bus flash drives for use in the computing and consumer markets; and wireless drive products used in-field back up of created content, as well as wireless streaming of high-definition movies, photos, music, and documents to tablets, smartphones, and PCs. The company sells its products under the G-Technology, SanDisk, and WD brands to original equipment manufacturers, distributors, dealers, resellers, and retailers. Western Digital Corporation was founded in 1970 and is headquartered in San Jose, California.

Previous Close	48.86
Open	48.31
Bid	49.01 x 800
Ask	49.38 x 800
Day's Range	48.09 - 49.36
52 Week Range	43.85 - 78.19
Volume	4,227,365
Avg. Volume	4,177,827
Market Cap	15.427B
Earnings Date	Apr 27, 2022 - May 02, 2022
Ex-Dividend Date	Apr 02, 2020

3. Target price

The study is done on March 20, 2022. This is important since we used the knowledge we had since that day. We selected the call option for June 17, 2022: WDC220617C00047500 with strike price 47.50, last price 5.30, bid 5.35 and ask 5.80. This option is at the money since its strike is close to the current value of the stock. The mid-price is the mean of bid and ask, thus our target price is 5.575.

4. Binomial model: procedure and calculations

We downloaded the "Historical Data" from the last 3 months of daily data from finance.yahoo.com and considered the Adjusted Closing Prices. We noticed that the company provides no dividend and we can take it into account simplifing our model. We computed the daily return as follows:

$$\textbf{return}(t) = \frac{\textbf{price}(t) - \textbf{price}(t-1)}{\textbf{price}(t-1)}$$

The sigma deviation of the daily returns and the annual volatility are

$$\sigma_d = 0,02908$$
$$\sigma_y = \sigma_d \times \sqrt{252} \approx 0,46168$$

From this we could compute the parameters of the binomial model related to σ and $T = 3/12 = 0.25$, which is 3 months with respect of one year.

$$u = e^{\sigma_y \sqrt{T}} \approx 1.25966$$
$$d = e^{-\sigma_y \sqrt{T}} \approx 0.79387$$

We used the Libor rate we found in global-rates.com corresponding to 3 months that is USD LIBOR 0.934% at 03-18-2022. Let $r = 0.00934$, R the simple compounding and $1/R$ the simple discounting, then

$$R = 1 + rT \approx 1.00233$$
$$\frac{1}{R} = \frac{1}{1+rT} \approx 0.99767$$

This allowed us to compute the risk neutral probability weight q:

$$q = \frac{R-d}{u-d} \approx 0.44756$$

Finally we could use the pricing formula for the price of a call option without dividends.

$$\textbf{price} = \frac{1}{R}\left(q(Su-K)^+ + (1-q)(Sd-K)^+\right) \approx 6.520$$

Where we used S as the last adjusted price, about 49.300, and K = 47.50.

5. Comparison: three months maturity

Our target for a maturity of 3 months was 5.575. By the binomial model we stated the price of a call option at 6.520. Thus, we had a relative error of 16.95%.

6. Results with seven months maturity

For the second attempt we chose a call option with maturity 7 months since there were no call options with maturity 6 months. We chose a Calls option for October 21, 2022: WDC221021C00047500 with strike price 47.50, last price 7.43, bid 7.80 and ask 8.30. Thus the mid-price is 8.05.

Repeating the procedure for the price of a call option with the binomial model we set $T = 7/12$, $K = 47.50$ and we encountered a problem: there is no Libor 7 month US Dollar LIBOR interest rate. We decided to use the 6 months one, which at march 18, 2022 is 1.28757%, since we think that our model is not very sensitive with respect to LIBOR rate. With the binomial model we priced a call option for 8.804. The relative error is 9.37%.

7. Table of results and conclusions

	Yahoo	B.M.	Error (%)
3 Months	5.575	6.520	16.95%
6 Months	8.050	8.804	9.37%

In conclusion this study provides us some good results since the relative error is not so low but still considerably low with respect to the simplicity of the binomial model. A question one may ask is how the error is distributed among different strike prices. The only pattern we encountered with WDC is that the relative error is moderate for the in the money options, precisely under 20%, and it explodes out of the money and clearly then stabilizing at 100% when the price

of the call option collapses to 0. The graph of the error could resem an oscillation damped by an exponential but it is not sure at all and need further studies, which require analysis over more options and possibly more interpolation points, that means more call options available.

References

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"Options, Futures, and Other Derivatives", John C. Hull, 11th Edition, Pearson 2022.