Stochastic Methods for Finance: Report 4

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Abstract

Analysis of the Greeks from a theoretical point of view and and empirical one in the case of Western Digital Corporation. Effects of an increase of the volatility on the market through the Greeks.

Introduction

This report is composed of three parts: the first is a theoretical part in which we implemented a VBA code for the Black Scholes model and the Greeks of this model, we drew the 3D graphs of the Greeks for different times to maturity and different stock prices. We studied a theoretical shock to the market increasing the volatility. The second part was empirical: we chose an asset and calculated the Greeks out of the data we harvested from Yahoo Finance. We checked the presence of the smile effect, which is the skewness of the implied volatility graph. The third part was still empirical: we performed a comparison between the market price of Call options at the money and the price provided by the Black Scholes model. In conclusion we compared historical volatility with the quoted at the money implied volatility.

1. First Section: Greek's Theory

We wanted to study the Greeks by a theoretical point of view. The Greeks are the quantities representing the sensitivity of the price of derivatives such as options to a change in underlying parameters. From the Black Scholes equation governing the price evolution of a European Call or European Put

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

we can recall the price formula for an European Call option:

$$C\left(S_t, t\right) = N\left(d_1\right) S_t - N\left(d_2\right) K e^{-r(T-t)}$$
$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right]$$
$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Now we can define the Greeks as follows

$$\Delta = \frac{\partial V}{\partial S} = N\left(d_1\right)$$
$$\Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{N'\left(d_1\right)}{S\sigma\sqrt{T-t}}$$
$$\nu = \frac{\partial V}{\partial \sigma}$$
$$\Theta = \frac{\partial V}{\partial t} = -\frac{SN'\left(d_1\right)\sigma}{2\sqrt{T-t}} - rKe^{-r(T-t)}N\left(d_2\right)$$
$$\rho = \frac{\partial V}{\partial r} = K(T-t)e^{-r(T-t)}N\left(d_2\right)$$

1.1. Results with Visual Basic

We implemented a VBA script to compute the Greeks for a Black Scholes model. We set the stock price to vary from 60 to 140 with steps of 5 and the time to maturity to be $0.1, 0.2, \ldots, 1, 2, 3, 4, 5$. We chose the strike price to be 100, the risk free rate of 1%, a volatility of 20% and absence of dividends. The graphs of the Greeks over time to maturity and stock prices are following. In the right side we performed a shock of the market increasing the volatility up to 70%.

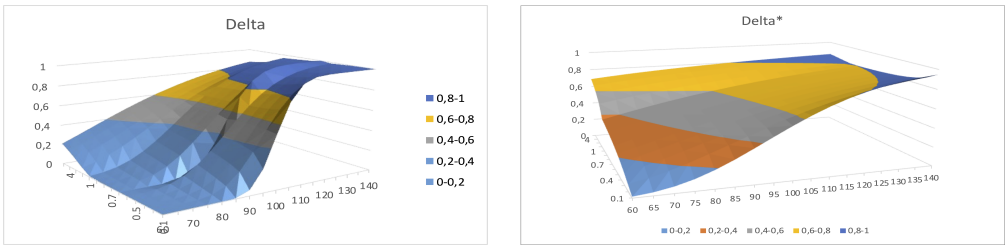


Figure 1: Delta 20% vol - Delta 70% vol

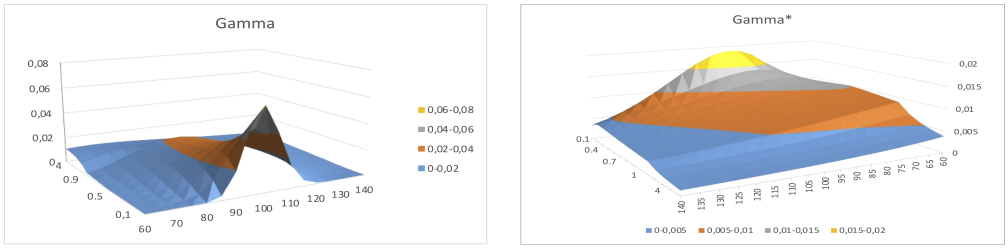


Figure 2: Gamma 20% vol - Gamma 70% vol

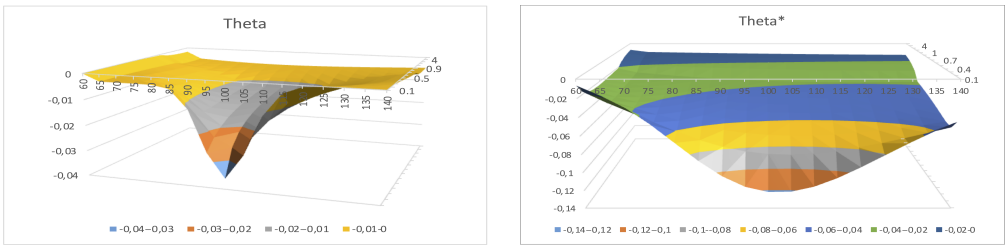


Figure 3: Theta 20% vol - Theta 70% vol

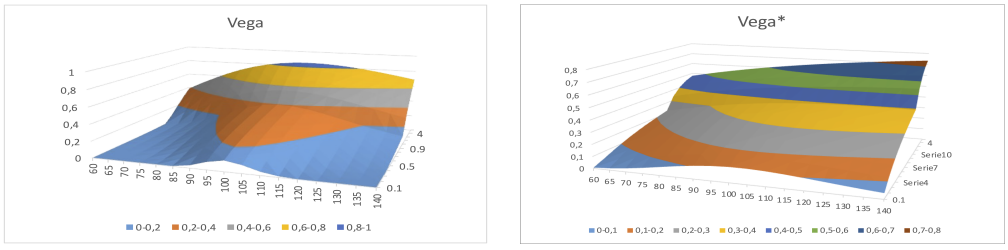


Figure 4: Vega 20% vol - Vega 70% vol

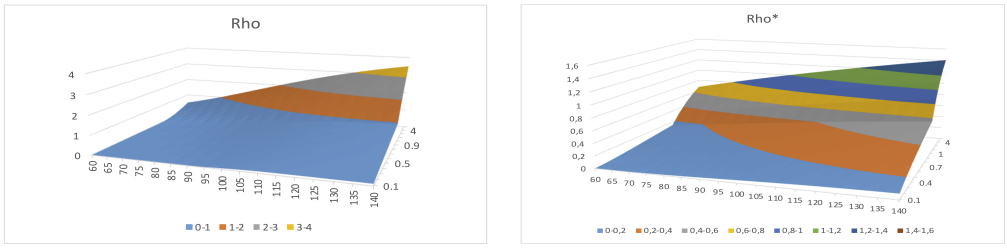


Figure 5: Rho 20% vol - Rho 70% vol

We can see that the an increase of the volatility is reflected in the graphs of the Greeks. With a low volatility there is no need to worry of high oscillation of the value of our stock; on the other hand if there is a huge volatility even tho our stock is now very in the money there is a concrete possibility that it rapidly goes out of the money. This is reflected in the Gamma, in particular if the volatility is low we need to buy and sell stocks a lot near the strike price; but if the volatility is high we need to do it always and not only near the strike price. Furthermore the smoothness of the Greeks can be seen as the time to maturity increases: all the Greeks become flatter and flatter as the time to maturity increases except for Rho since it measures the impact of the interest rate on the stock and it becomes higher as the time to maturity increases, as expected. This is the reflection of the urgency to hedge near the maturity and the not urgency to hedge when the maturity is still far in time.

2. Second Section: Empirical study

We chose in Yahoo an asset which does not pay dividends. We would like to find one with a book of European options but they are not shared on Yahoo. At the end we were satisfied with American options. This introduced a relevant error in this study, nevertheless we could not do in any other way. We chose Western Digital Corporation.

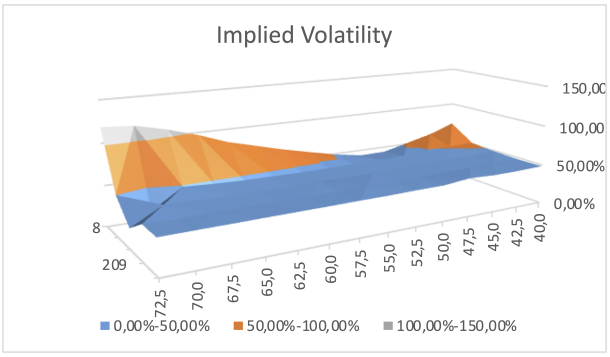
2.1. Western Digital Corporation (WDC)

We have chosen Western Digital Corporation. WDC develops, manufactures, and sells data storage devices and solutions in the United States, China, Hong Kong, Europe, the Middle East, Africa, rest of Asia, and internationally. It offers client devices, including hard disk drives (HDDs) and solid state drives (SSDs) for computing devices, such as desktop and notebook personal computers (PCs), smart video systems, gaming consoles, and set top boxes; flash-based embedded storage products for mobile phones, tablets, notebook PCs, and other portable and wearable devices, as well as automotive, Internet of Things, industrial, and connected home applications; and flash-based memory wafers. The company also provides data center devices and solutions comprising enterprise helium hard drives; enterprise SSDs consisting of flash-based SSDs and software solutions for use in enterprise servers, online transactions, data analysis, and other enterprise applications; data center solutions for data storage systems and tiered storage models; and data storage platforms. In addition, it offers client solutions, such as external HDD storage products in mobile and desktop form; client portable SSDs; removable cards that are used in consumer devices comprising mobile phones, tablets, imaging systems, and cameras and smart video systems; universal serial bus flash drives for use in the computing and consumer markets; and wireless drive products used infield back up of created content, as well as wireless streaming of high-definition movies, photos, music, and documents to tablets, smartphones, and PCs. The company sells its products under the G-Technology, SanDisk, and WD brands to original equipment manufacturers, distributors, dealers, resellers, and retailers. Western Digital Corporation was founded in 1970 and is headquartered in San Jose, California.

Previous Close	46.59
Open	47.23
Bid	47.01 x 800
Ask	47.38 x 800
Volume	4,227,365
Avg. Volume	4,177,827
Market Cap	15.427B
Ex-Dividend Date	Apr 02, 2020

2.2. Smile Effect

The smile effect is the skewness of the curve of the volatility over different maturities and strike prices. The following surface is the smile effect coming from the implied volatility of the Call options in Yahoo Finance. The time is measured in "weekdays to the maturity" (8, 54, 74, 209, 469).



The smile effect reflects the reliability of the Black Scholes model to predict the right prices of the options in the real market. Indeed more the skew is prominent less the Black Scholes model is reliable. To use safely the BS model we need to have an almost horizontal surface (neutral face).

2.3. Smoothness of the Greeks

In this last part of the section we wanted to verify that the surfaces of the Greeks become smoother and smoother as the time to maturity increases. We just showed it plotting the empirical Greeks. To calculate the points of the following surfaces we used the current stock price 46.49; the implied volatility with respect to any time and strike couple; the rate obtained by spline interpolation of the data downloaded from Bloomberg of the US market. Note that the following graphs are not from the exact same sample: sometimes data are missing or filtered to ensure more readability and the scale is not respected as the distance from one maturity to the following one is set to one unit. Reminder that the time to maturity is measured in weekdays to maturity.

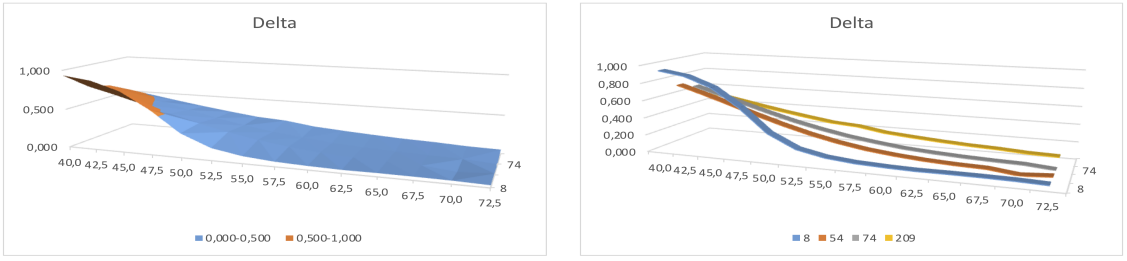


Figure 6: Surface Delta - Lines Delta

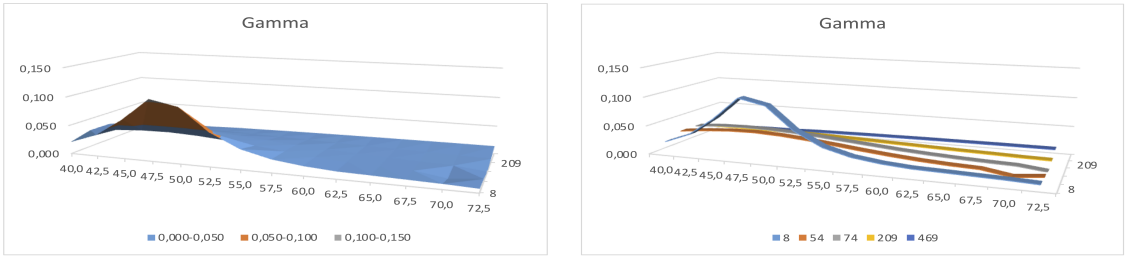


Figure 7: Surface Gamma - Lines Gamma

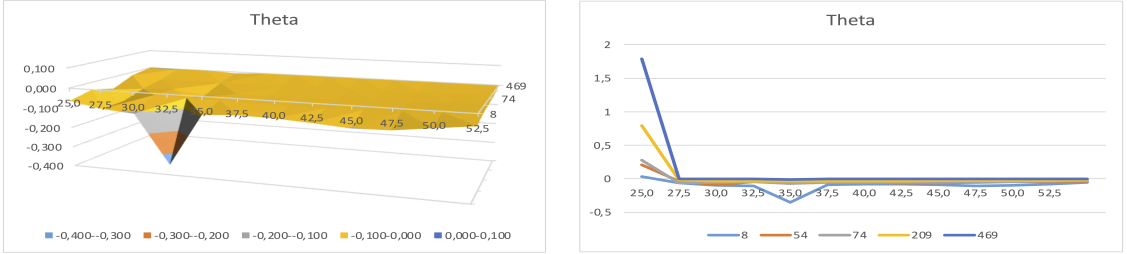


Figure 8: Surface Theta - Lines Theta

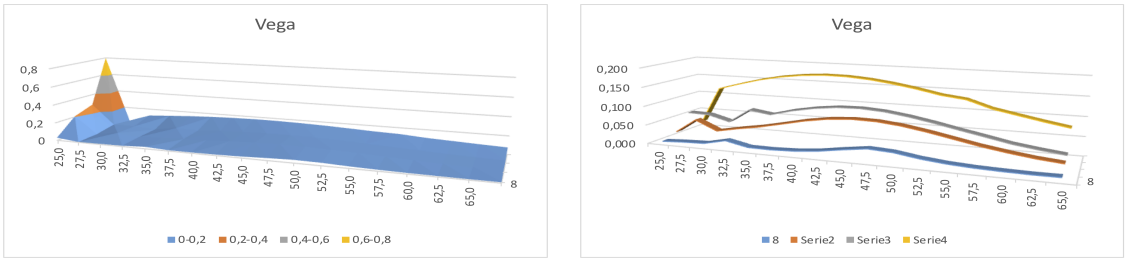


Figure 9: Surface Vega - Lines Vega

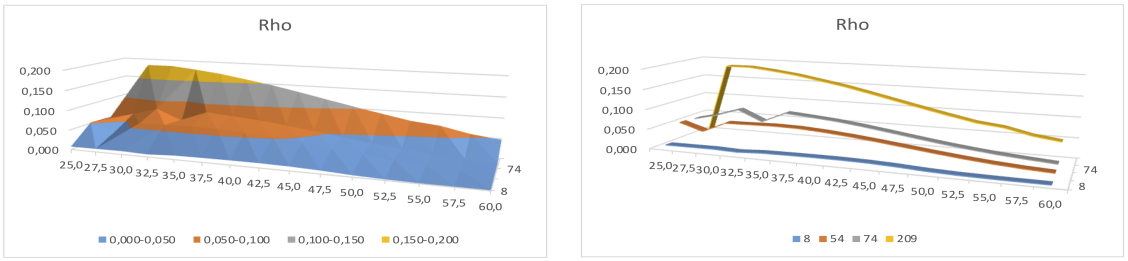


Figure 10: Surface Rho - Lines Rho

3. Third Section: The empirical study continues

In this section we wanted to compare the Black Scholes prevision about the price of a Call option and the estimated one in the previous section. Last we compared the implied volatility with the historical volatility.

3.1. Using Black Scholes to price a call ATM

We downloaded the historical data from Yahoo Finance, calculated the daily returns with the formula:

$$\textbf{return}(t) = \frac{\textbf{price}(t) - \textbf{price}(t-1)}{\textbf{price}(t-1)}$$

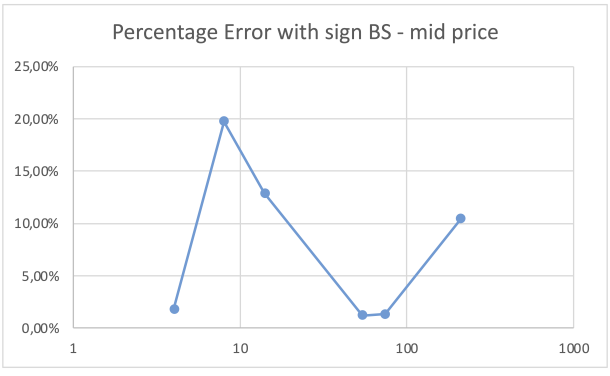
and then sigma deviation of the daily returns over the maturities backwards and annualizing the results

$$\sigma_y = \sigma_d \times \sqrt{252}$$

Now recalling that the current stock price is 46.49 for each triplet time to maturity, strike, rate we computed the price of an European call option by the Black Scholes model. Then we took the mid price, which is the mean of ask and bid, for each call and compare the results. Note that the error is a percentage error with sign centered in the mid price.

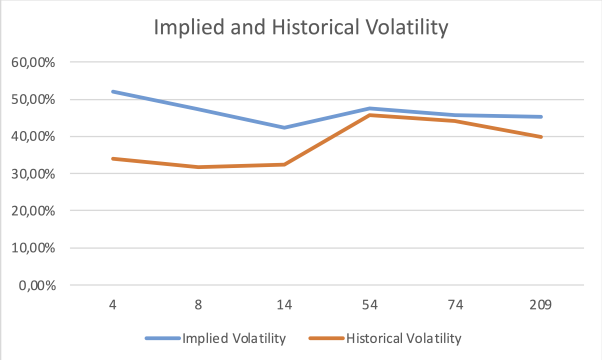
Wks	Mid price	BS price	Error
4	0.80	0.79	1.82%
8	1.30	1.04	19.76%
14	1.91	1.66	12.90%
54	3.50	3.46	1.19%
74	4.03	3.97	1.32%
209	6.95	6.23	8.98%

As we expected the prices predicted by the Black Scholes model are smaller than the prices on Yahoo since the Black Scholes model prices European options and we are dealing with American options which are more expensive than the European counterpart. The following graph shows how the error (percentage error with sign) increases as time to maturity passes. Note that we chose to not take the absolute value as the sign is a valuable information.



3.2. Implied and Historical Volatility

Now we can visualise the difference between the quoted ATM implied volatility and the historical volatility:



Here the difference between them is due to the fact we are using the Black Scholes model which is not good on American options. It is enough the volatility are comparable.

4. Conclusions

We have studied the Greeks both from a theoretical point of view and an empirical one. The skewness of the volatility was noticeable and the trend to the smothness as the time to maturity increases. The errors we encountered are caused by the choice to use the Black Scholes model to operate with American options, instead of European options, and the lack of meaningful data on this company. With more data all the essay would have been more meaningful.

References

"Arbitrage Theory in Continuous Time", Tomas Björk, 4th Edition, Oxford Univeristy Press 2020.

"Options, Futures, and Other Derivatives", John C. Hull, 11th Edition, Pearson 2022.