#ml-papers September 2019

The empirical risk-return relation: a factor analysis approach

Section 1: Intro - Criticisms

Criticisms of current literature re: Risk Return relation

a. **Small amount of conditioning information** used to model the conditional mean and conditional volatility of excess stock market returns.

b. Estimated relation between the conditional mean and conditional volatility of excess returns often **depends on the parametric model of volatility**, which has restrictive assumptions.

c. Reliance on a small number of conditioning variables exposes existing analyses to problems of **temporal instability** in the underlying forecasting relations being modeled.

Section 1: Current State

Dynamic Factor Analysis - turn dimensionality from a curse into a blessing

a. **A large amount of information** can be effectively summarized by a relatively few common factors

b. **Combine DFA with a non-parametric model of volatility** avoids restrictive parametric structures

c. **Robustness against the temporal instability** that often plagues low-dimensional forecasting regressions

Section 1: Intro - Data

Data: Two quarterly post-war datasets from 1960Q1 to 2002Q4

1. **Macroeconomic** (209 Timeseries including Real Income, Employment, Consumer Spending,etc.)

   a. **Volatility Factor:** First principal component of the Financial Indicators dataset.

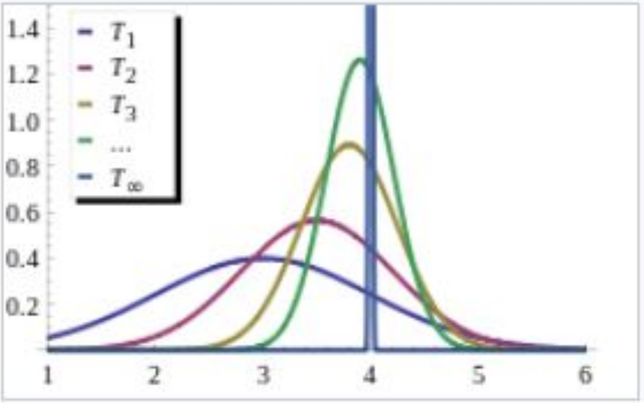
   b. **Risk Premium Factor:** Third principal component of financial indicators (highly correlated with linear combination of the Market returns and the Fama-French Factors)

2. **Financial** (172 Timeseries including valuation ratios, corporate bonds yields, etc.)

   a. **Real Factor:** First principal component from the macroeconomic dataset. Highly correlated with real output and employment but not prices.

Statistical properties of the models

- The presumption of the dynamic factor model (with **principal components analysis** as a specific application of this) is that the covariation among lots of economic time series can be captured by just a few unobserved (latent) common factors
- Are model estimates consistent when "both the number of economic time series used to construct common factors, ***N***, and the number of time periods, ***T***, are large and converge to ∞" ?
- Consistent estimates of the space spanned by the common factors can be constructed by principal components analysis (Stock and Watson 2002b).
- Least squares estimates from factor-augmented forecasting regressions are consistent and asymptotically normal. Pre-estimation of the factors does not affect the consistency of the least squares regression estimates (Bai and Ng 2005).



$\{T_1, T_2, T_3, \ldots\}$ is a sequence of estimators for parameter θ_0, the true value of which is 4. This sequence is consistent: the estimators are getting more and more concentrated near the true value θ_0; at the same time, these estimators are biased. The limiting distribution of the sequence is a degenerate random variable which equals θ_0 with probability 1.

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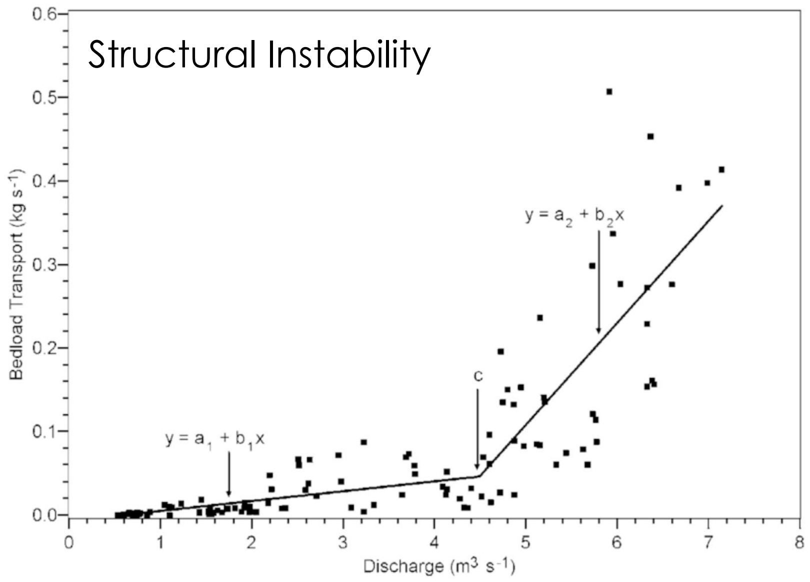
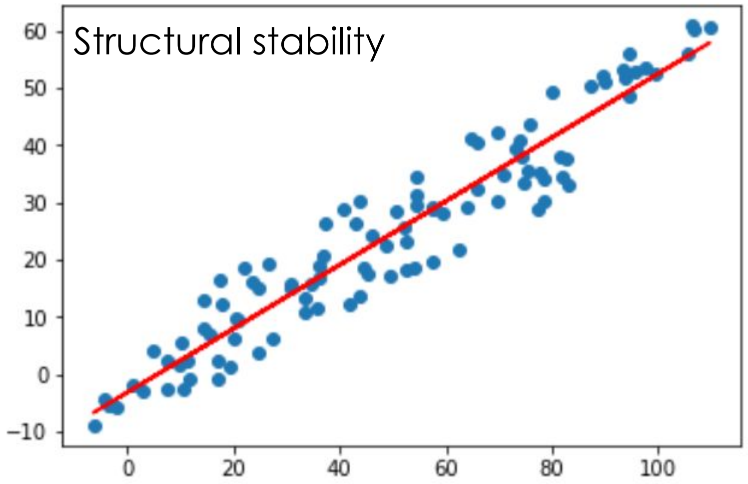
Factor-augmented models lead to better predictions

- Predictions of economic activity are greatly improved relative to low-dimensional forecasting regressions when the forecasts are based on the estimated factors of large datasets.
- The use of common factors can provide robustness against the structural instability that often impacts the effectiveness of low-dimensional forecasting regressions. The relationship between LHS and RHS can often change over time and linear regressions with a limited number of observable explanatory variables can't capture these changes (Stock and Watson 2002a).
- Principal components factor estimates are consistent even in the face of temporal instability in the individual time series used to construct the factors. The reason is that such instabilities can "average out" in the construction of common factors if the instability is sufficiently dissimilar from one series to the next.





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Advantages with modeling realized volatility

- Nonparametric volatility measures such as realized volatility benefit from being free of tightly parametric functional form assumptions (e.g., GARCH and Stochastic Volatility) and provide a consistent estimate of ex-post return variability (Andersen et al. 2002 and 2003)
- Realized volatility permits the use of traditional time-series methods for modeling and forecasting and makes possible the employment of estimated common factors from large datasets to measure conditional, or expected, volatility.
- ***I'm not really sure why it's important to point this out. This seems kind of obvious.***

$$VOL_t = \sqrt{\sum_{k \in t} (R_{sk} - \overline{R}_s)^2},$$

Realized volatility for quarter ***t***

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This study is connected to a large literature on the relationship between conditional return and conditional volatility

- Some authors have found a positive risk-return relation (as expected based on theory) while others find a negative relationship.
- Other authors find that the risk-return relation varies over time.
- Most of these studies use a small number of predetermined, potentially arbitrary, and observable conditioning variables to form estimates of the conditional mean and the conditional volatility, potentially subjecting the findings to omitted information bias.
- One study identified does not rely on predetermined conditioning variables where the conditional mean and conditional volatility are modeled as latent state variables extracted only from the history of returns data (Brandt and Kang 2004).
    - An advantage of this approach is that it eliminates the reliance on a few arbitrary conditioning variables in forming estimates
    - A potential weakness is that potentially useful information is still discarded as latent variables are in practice modeled as simple low-order, linear time series (e.g., first-order Gaussian VAR processes) when the true representation might be a higher order, more complex model (e.g., higher order, non-Gaussian).

Section 3: Econometric framework

Sharpe Ratio

mean return

volatility

$$\frac{\mu_t}{\sigma_t}$$

Are assumptions present in this formula accurate?

Goal: generalize to

Assess the Sharpe ratio and possibly improve on it

$$\mu_t = \delta + \beta_1 \sigma_t + \beta_2 \sigma_{t-1} + \alpha \mu_{t-1} + \varepsilon_t$$

volatility in mean

lag volatility in mean

Section 3: Returns and Volatility

Historical time periods:

$$t = 1, \ldots, T$$

Mean excess returns: *(vs. treasuries)*

$$m_{t+1}$$

Volatility of excess returns (*realized volatility*):

$$VOL_t = \sqrt{\sum_{k \in t} \left(R_{sk} - \overline{R}_s \right)^2}$$

day

daily returns v. treasury yield

mean of R_{sk}

Conditional mean:

$$E_t m_{t+1} = E(m_{t+1} | \text{ historical data through } t)$$

Conditional volatility:

$$E_t VOL_{t+1} = E(VOL_{t+1} | \text{ historical data through } t)$$

Section 3: Derivation for Returns

Standard model:

$$m_{t+1} = \beta' Z_t + \epsilon_{t+1}$$

K *predetermined* conditioning variables in vector Z_t

“standard” metrics Z

Estimated conditional mean:

$$E_t m_{t+1} \approx \hat{m}_{t+1|t} = \beta' Z_t$$

"Obvious" generalization:

$$m_{t+1} = \gamma' x_t + \beta' Z_t + \epsilon_{t+1}$$

$T \times N$ dimensional *panel of data* $x_{i,t}$

$N \gg K$

Degrees-of-freedom problem!

Not even feasible for N+K > T

Section 3: Derivation for Returns

Posited (theoretical) factor structure:

$$x_{it} = \lambda_i' f_t + e_{it}$$

$r \ll N$ *unobserved* factors:

$$f_t$$

Refined model:

$$m_{t+1} = \alpha' F_t + \beta' Z_t + \epsilon_{t+1}$$

$$F_t \subset f_t$$

Subset F_t *of most predictive* factors for m_{t+1}

Section 3: Derivation for Returns

Estimated factors (via PCA) converging to same subspace

$$\hat{f}_t$$

Assumptions:

$$N, T \to \infty, \sqrt{T}/N \to 0$$

$$\hat{F}_t \subset \hat{f}_t \cup \{\text{monomials, "other" functions of } \hat{f}_{it}\}$$

Final *mean returns* model:

$$m_{t+1} = \alpha' \hat{F}_t + \beta' Z_t + \epsilon_{t+1}$$

(chosen to minimize BIC)

Fitted conditional mean:

$$\mu_t \equiv \hat{m}_{t+1|t} = \hat{a}' \hat{F}_t + \hat{\beta}' Z_t$$

Section 3: Regression on Fitted

Similar derivation for vol's:

Final *volatility* model:

$$VOL_{t+1} = \alpha' \hat{F}_t + \beta' Z_t + u_{t+1}$$

(same letters, but different values)

Fitted conditional volatility:

$$\sigma_t \equiv \widehat{VOL}_{t+1|t} = \hat{a}' \hat{F}_t + \hat{\beta}' Z_t$$

Now all set up to do the linear regression:

$$\boxed{\mu_t = \delta + \beta_1 \sigma_t + \beta_2 \sigma_{t-1} + \alpha \mu_{t-1} + \varepsilon_t}$$

Empirical Implementation & Data (1 of 4)

Authors estimate the following factor models:

$$m_{t+1} = \alpha_1' \hat{F}_t + \alpha_2' \hat{G}_t + \beta_t Z_t + \epsilon_{t+1}$$
$$VOL_{t+1} = \alpha_1' \hat{F}_t + \alpha_2' \hat{G}_t + \beta_t Z_t + \epsilon_{t+1}$$

Where *m* is the excess market returns and *VOL* is the volatility of returns

They then take μ = *E*[*m*] and σ = *E*[*VOL*] and estimate:

$$\mu_t = \delta + \beta_1 \sigma_t + \beta_2 \sigma_{t-1} + \alpha \mu_{t-1} +_{t-1} + \epsilon_t$$

Empirical Implementation & Data (2 of 4)

Excess Returns *m* is measured as the log return on the Center for Research in Security Prices (CRSP) value-weighted prices for NYSE/AMEX/NASDAQ - 3-month T-Bill rate

Volatility *VOL* is measured as the daily CRSP - implied *daily* return on 3-month T-Bill

The factor models include three sets of features:

- *F* => r factors from a dataset of 209 macroeconomic indicators
- *G* => r factors from a dataset of 172 financial indicators
- *Z* => controls from prior studies on risk/return

The data is quarterly from 1960:1 to 2002:4

Empirical Implementation & Data (3 of 4)

Authors estimate the time-*t* common factors using the *full* sample of data

Model selection is done on the basis of minimizing the Bayesian Information Criterion (BIC) using one-period ahead forecasting, where

$$BIC = ln(n)k - 2ln(\hat{L})$$

The top five factors from the macroeconomic indicators account for 60% of the variance of the dataset and the top five factors from the financial indicators account for 80% of the variance of the dataset. These are the factors selected for use in the model

Empirical Implementation & Data (4 of 4)

Can the factors be interpreted?

No detailed interpretation possible, but we can make statements based on the loadings with the input features

G^2_{1t} has a high correlation with squared stock market returns ("volatility factor")

G_{3t} has a high correlation with a linear combination of three state variables used in empirical asset pricing ("risk premium factor")

Main conclusions

- Common factors constructed from a large amount of macro and financial markets data using PCA have meaningful predictive power for one quarter ahead (excess) stock returns and volatility. Out-of-sample results showed stability over multiple sub-periods of time.
- These factors are more likely to span the information that sets of financial market participants.
- Two of these factors stand-out in particular:
  - volatility factor that is highly correlated with squared returns
  - risk premium factor that is highly correlated with well established risk factors for explaining the cross-section of expected returns for stocks (over time, small companies outperform big companies, cheap stocks outperform expensive ones)
- The estimated factors used as predictors in the regressions are strongly statistically significant and remarkably stable over time. This is important as commonly used predictors used in past studies were shown to be unstable over the same time periods that authors analyzed in this study (especially during the last half of the 1990s). These results provide evidence that dynamic factor analysis can provide robustness against temporal instability.
- If you deal with the omitted information bias problem by using the information in the estimated factors and once you control for the lags of these variables, you're able to empirically support the theoretical prediction of a contemporaneous positive relationship between volatility and return.

Possible directions for “future” research

- Investigate potential role of non-linearities in risk-return relation
- Assess degree of time variation in risk-return relation
- Using the same dynamic factor analysis techniques to model conditional covariances and conditional betas.
- Done. *Phew!*