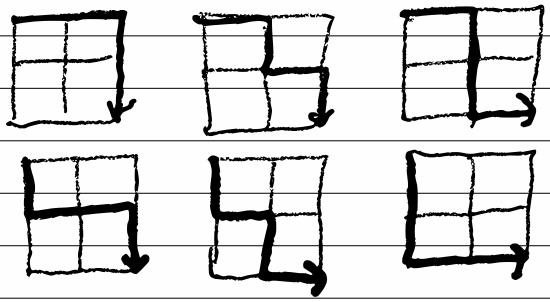
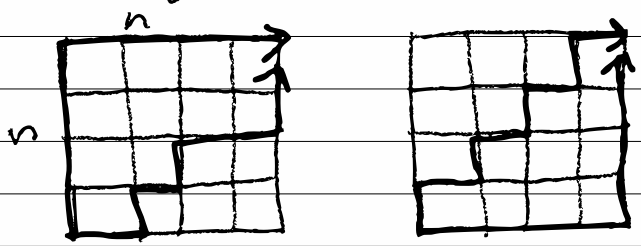
15. Starting in the top left corner of a 2×2 grid, and only being able to move right and down, there are exactly 6 routes to the lower right corner.



How many routes are there through a 20×20 grid?

For an $n \times n$ square, the longest 'up' path and longest 'right' path are n long –



Every path that doesn't backtrack is $2n$ long. Each unique path has a unique set of 'up' paths.
How many way are there to choose n from $2n$?

$$\binom{n+n}{n} \to \binom{2n}{n} = \frac{(2n)!}{n!(2n-n)!}$$

$$\frac{(2\cdot 2)!}{2!(2\cdot 2-2)!} = 6$$

$\boxtimes$