Problem Chosen	2022	Team Control Number
C	**MCM/ICM Summary Sheet**	**2214695**

Many a Little Make a Mickle

Summary

In modern times, it is increasingly difficult to gain a significant competitive edge in trading business just by being faster than others, which means relying on sophisticated trading signals, predictive models, and strategies. So, we should know how to go about designing, building and operating all the components required to build a profitable algorithmic trading model in live markets.

As for task I, we first perform missing value completions and feature mining, such as analyzing **simple moving averages** and **seasonal features**, to explore the surface patterns of bitcoin and gold and provide a basis for subsequent parameter setting tasks. Since we can only use price data as of the day of the investment process, a **predictive** model is an essential part of our first build. We perform **lag and error analysis** based on linear regression forecasts. Subsequently, in order to refine our model, we perform the forecasting task on **differential series** and propose the **LR-LSTM model** after combining it with the LSTM model that performs well on long series with sufficient amount of data. Finally, after error analysis, we can infer that the model achieves good prediction results.

In the decision model section, we innovatively try to **quantify the purchase intention** of bitcoin and gold in each day as a **purchase index PI**, which we divide into two parts for a comprehensive consideration, the expected profit and **risk index RI**. we build a linear model of RI with three key influencing factors: price deviation, market boom condition, and magnitude increase rate. After obtaining the purchase index PI, we implement a quantitative investment strategy based on the purchase index PI by making several attempts to find the optimal threshold to control buying or selling. We end up with a maximum total asset of $13,931,321 on the last day.

As for task II, we consider whether the decision model perfectly achieves the predicted profit. We construct the ideal profit index using the monthly rise of gold and bitcoin and compare whether the real monthly profit trend after the end of the decision process is similar to it, so as to get the superiority and reliability of the model.

As for task III, we add a series of **perturbations** to the transaction costs of gold and bitcoin, and consider the **magnitude of the change** in profitability of the decision after adding the perturbations. The final results show that the jitter in the results is about 4.0% when the bitcoin transaction costs are changed in steps of 0.5%, and 10.9% for gold. We then visualize the change in total assets in the decision-making process after changing the transaction costs, and the difference from the original **trend** in total assets is small, just a change in the size of the value.

Finally, we provide a detailed memorandum for the trader with a description of the strategy and expected results to help him get ridiculously rich overnight.

Keywords: LR-LSTM; Purchase Index; Quantitative Investment; Sensitivity Analysis.

Contents

1 Introduction

1.1 Background

Nowadays, most of the financial investment products market is often not in line with the Strong-Form Market Efficiency, the price of the transaction will reflect the historical information. As a trader, we need to keep an eye on the historical information of investment products during the trading process in the financial markets in order to make the most optimal investment decisions.

Bitcoin and gold are representatives of emerging virtual currencies and valuable traditional wealth. One of them is a clever interplay of high risk and high reward; the other is a stable and suitable long-term investment. We always need to calmly and thoughtfully analyze the market direction and make the best investment choice in the trading market. For a trader, he needs to be careful and prudent in the market and wrestle a future for himself.

1.2 Problems Restatement

A trader asked us to develop a model for him to apply to the gold-bitcoin market for the time period of 9/11/2016 to 9/10/2021. The model requires a reasonable buying and selling strategy for each day to maximize the return on the 5 years investment. Due to commissions, too frequent trading seems unlikely to be possible here. In order to achieve the trader's requirements, we will use gold and bitcoin trading data over a 5-year period to accomplish the following tasks.

1. Obtain the decision model for the transaction and find the maximum asset value that can be achieved under the strategy provided by the model.
2. Sensitivity analysis is performed for the model to ensure a certain degree of model adaptability.

For the above task, we accomplished the following.

1. Use of financial indicators to assist in the analysis of investment product price data characteristics.
2. Modeling of time series analysis of two investment prices. We use each day's price data to build forecasting models that predict as accurately as possible the prices of investments over time, thereby helping us to make consensual decisions.
3. Combining forecast data and available data to make a risk assessment and use it as one of the decisive indicators for the purchase of investments.
4. An evaluation model combining multiple characteristics, including risk indicators, to derive a quantitative purchase propensity index for each day for both investment products.
5. A sensitivity analysis of the model is performed to evaluate the superiority and robustness of the model by perturbing the commission factor and other profit-influencing factors.

Our workflow is shown in Figure 1 .

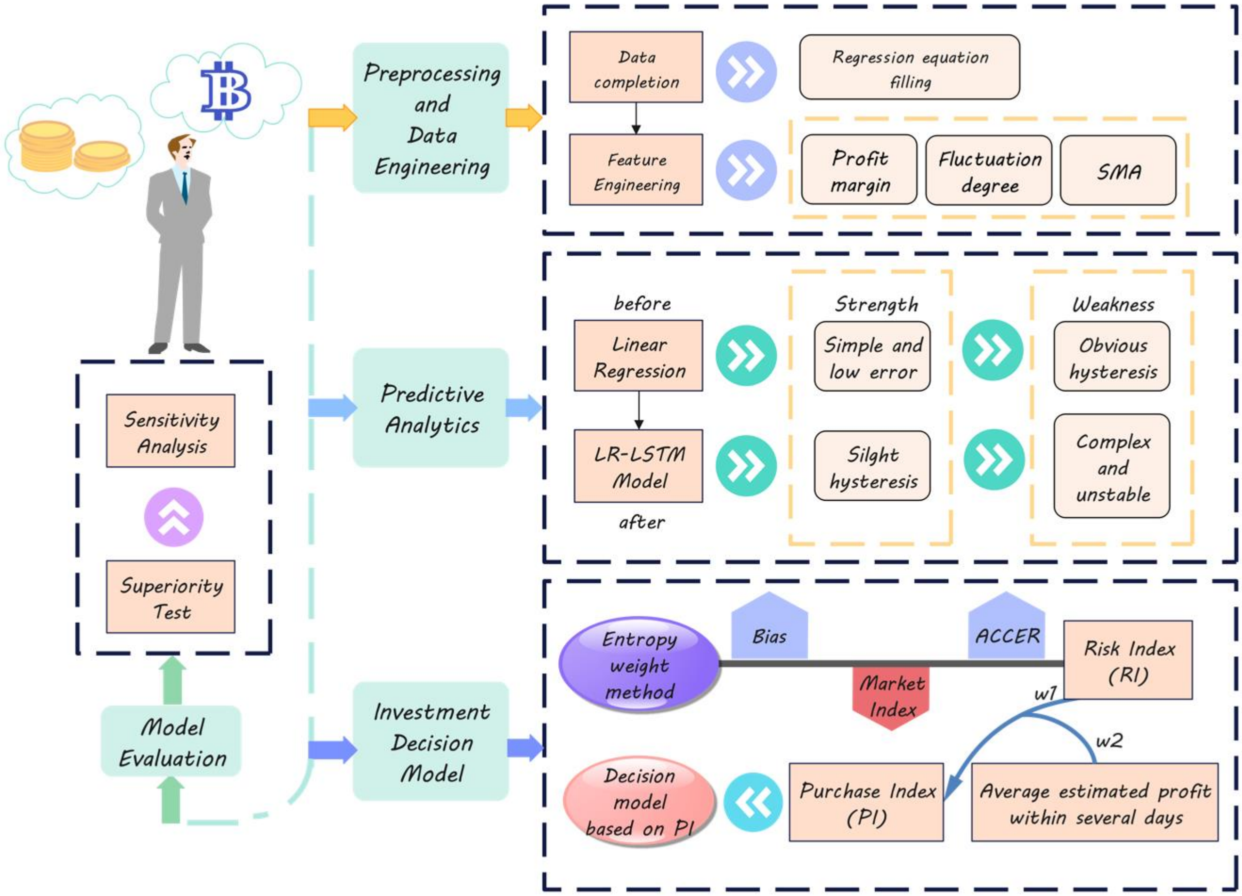

Figure 1 Working Structure Schematic

2 Notations and Signs

The primary notations used in this paper are listed in Table 1 Notations and Signs.

Table 1 Notations and Signs

Symbol	Abbreviation	Unit
P_{b_i}	Bitcoin USD Price on Day i	\$
P_{g_i}	Gold USD Price on Day i	\$
E_{b_i}	Bitcoin Daily Yield on Day i	
E_{g_i}	Gold Daily Yield on Day i	
$\overline{E_{b_i}}$	Bitcoin Volatility Index on Day i	
$\overline{E_{g_i}}$	Gold Volatility Index on Day i	
L_{bfive}	Bitcoin 5-day SMA	\$
$L_{gtwenty}$	Gold 20-day SMA	\$
σ^2	Variance	
MI	Market Index	
RI	Risk Index	
D	Deviation Rate	
Ω	Monthly Rates	
L	Expected Monthly Return Index Vector	
H	Real Monthly Return Index Vector	

3 Assumptions and Justifications

By adequate analysis of the problem, to simplify our model, we make the following well-justified assumptions.

1. The data used in this paper are realistic and accurate to a certain extent.
2. No changes in social factors such as policy changes or changes in people's preferences occur during the forecast time period.
3. Only the closing price of the day is considered for both investment products, and no price changes during the day are considered.
4. From an economic perspective, the current market is not a Strong-Form Market Efficiency, which makes our forecast based on historical data valid.
5. It is assumed that the propensity to purchase an investment in the short run can be represented by a linear combination of key influences obtained by virtue of the evaluation model.

4 Data Engineering

4.1 Data Cleaning

Because gold is only traded on days when the market is open, there is missing data in the price of gold. We use linear regression for data completion instead of using statistical value filling (e.g. recent price averages, etc.). This is because using linear regression for data completion allows us to use as much information as possible from the original data.

4.2 Feature Engineering

For the bitcoin and gold data, we need a variety of indicators to help make investment decisions.

The first is the daily returns of bitcoin and gold, as follows

$$E_{b_i} = \frac{P_{b_i} - P_{b_{i-1}}}{P_{b_{i-1}}} \tag{1}$$

$$E_{g_i} = \frac{P_{g_i} - P_{g_{i-1}}}{P_{g_{i-1}}} \tag{2}$$

In addition, we use the n-day average, a common indicator in stocks, as a feature. We calculate the average daily return of bitcoin and gold as a measure of their volatility. The average daily return is calculated as follows:

$$\overline{E_b} = \frac{\sum_{i=2}^{N} E_{b_i}}{N} \tag{3}$$

$$\overline{E_g} = \frac{\sum_{i=2}^{N} E_{g_i}}{N} \tag{4}$$

The calculation gives the results shown in Table 2:

Table 2 Daily Return Average

$\overline{E_b}$	$\overline{E_g}$
0.0266551	0.0041929

In financial market trading, the five-day SMA is usually used as a reference line for investments with large increases or decreases, and the twenty-day SMA is usually used as a reference line for investments with small price fluctuations.[1] The data in Table 2 indicates that $\overline{E_b} > \overline{E_g}$, which means that the price of bitcoin is relatively volatile and the price of gold is relatively stable. Here we refer to the five-day SMA for bitcoin and the twenty-day SMA for gold. Two simple moving averages are calculated as follows:

$$L_{bfive_i} = \frac{\sum_{j=i-4}^{i} P_{b_j}}{5},\ i \geq 5 \tag{5}$$

$$L_{gtwenty_i} = \frac{\sum_{j=i-19}^{i} P_{g_j}}{20},\ i \geq 20 \tag{6}$$

4.3 Seasonal factor

Since it is a time series, we should study the stationary factors, such as mean, variance over time, and seasonal effect also affects the stationary.

There are multiple seasonality effects: weekend, monthly, and holidays. The prices are time dependent and can have increasing or decreasing trends and seasonality trends, in other words, variations specific to a particular time frame.[2]

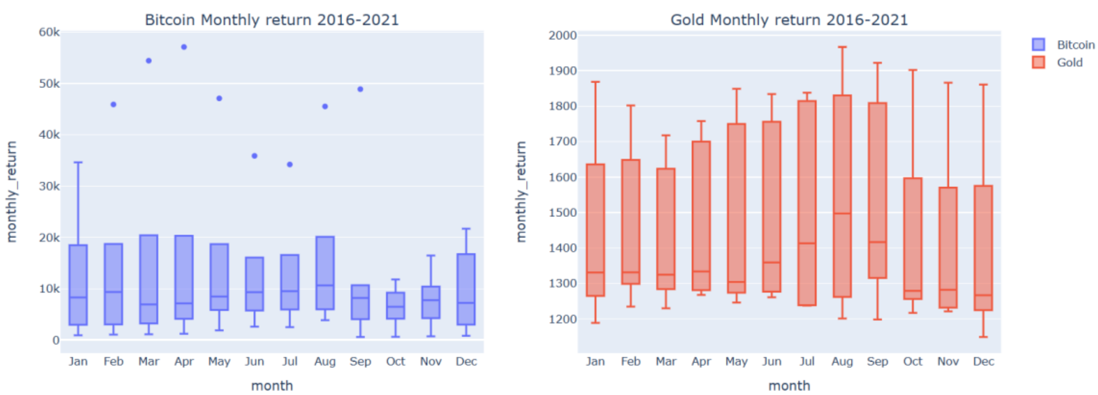

Figure 2 Monthly Revenue Trends

As shown in Figure 2, we observe a repeating pattern. October, November and December appear to be the months with the lowest returns. They are different from July and August, where we observe a decrease in returns.

To avoid seasonal profit decay, we use special markers to detect and adapt to seasonal market conditions and different trading relationships. We thus profit from seasonal trends. Seasonal profit decay is part of a trading strategy dealing with asset classes and trading instruments that have seasonal trends in their behavior and cross-asset relationships. We build analyses and strategies that take seasonal trends into account to maximize profits.

5 Predictive Model

5.1 Base Model

Trader can only use price data up to the current day, next we will use the current data to predict the time series of bitcoin price and gold price.

Our underlying forecasting model is based on linear regression.[3] Linear regression assumes that there is a linear relationship between our variable to be predicted and the independent variable so that a linear equation can be built to predict the price of the investment for the next day or days. We use the bitcoin price for the previous 5 days as a characteristic of bitcoin for that day, i.e., the eigenvector db_i which shown in (7), and the gold price for the previous 20 days as a characteristic of gold, i.e., the eigenvector dg_i which shown in (8).

$$db_i = [Pb_{i-5}, Pb_{i-4}, Pb_{i-3}, Pb_{i-2}, Pb_{i-1}] \tag{7}$$

$$dg_i = [Pg_{i-20}, Pg_{i-19}, Pg_{i-18}, \cdots, Pg_{i-4}, Pg_{i-3}, Pg_{i-2}, Pg_{i-1}] \tag{8}$$

5.1.1 Linear Regression on Bitcoin data

Assume that the current has m days of features as well as the destination variable & price, and use them as training data. Let X be a vector matrix representing m days of data and Y denote the coefficients in a linear equation containing m days of real price data.

$$X \in R_{m\times 5}, \beta = [\beta_1, \beta_2, \cdots, \beta_5]^T \tag{9}$$

$$Y \in R_{m\times 1} \tag{10}$$

Establishing the regression equation yields:

$$Y = X\beta + \epsilon \tag{11}$$

The optimization objective function of linear regression is as follows:

$$J = \frac{1}{2m}\sum_{i=1}^{m}(\hat{y}_i - y_i) \tag{12}$$

The data of the first m days are used to solve equation (12) to obtain the appropriate β. When it is necessary to forecast the stock price Y for the next few days, it is sufficient to substitute the characteristic matrix into X.

It seems that our prediction can only be done day by day, because when our operator marches to day A, we can provide data from day $(A-4)$ to day A as features for day $(A+1)$, but not enough feature data for day $(A+2)$.

But in fact, predicting prices for multiple days simultaneously is just a stack of feature vectors for predicting subsequent single days individually. Considering the efficiency of the algorithm, we still use the training results of the previous A days when predicting the commodity prices on day $(A+2)$, using the same β.

There are similar methods and conclusions for Gold price prediction.

5.1.2 Error Analysis

The partial prediction data obtained by linear regression and the error analysis are shown in Table 3 & Figure 3, Figure 4.

Table 3 Error Analysis

	MAE	MSE	RMSE
Bitcoin	390.219798	676047.840609	822.221284
Gold	7.724287	172.029360	13.115996

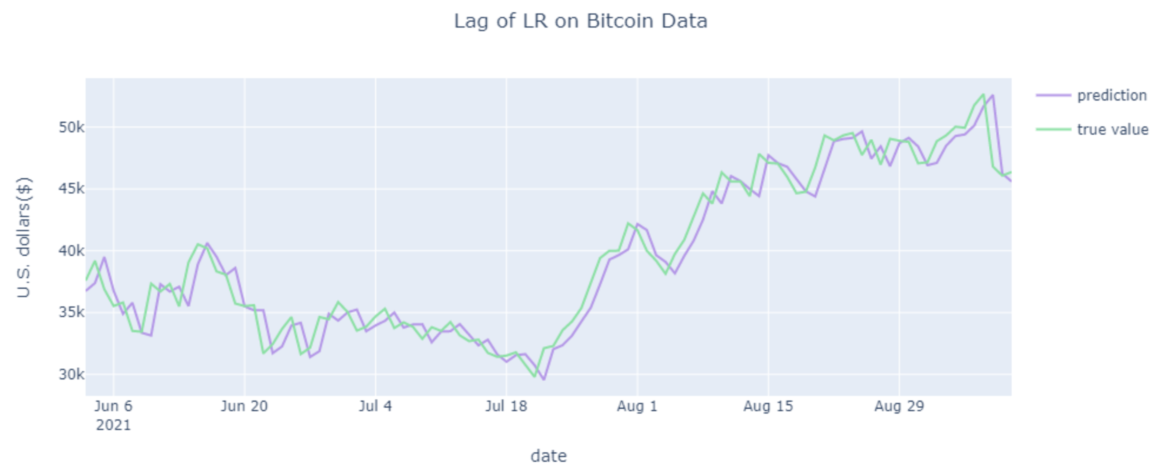

Figure 3 Preliminary Predictions from Linear Regression on Bitcoin Price Data

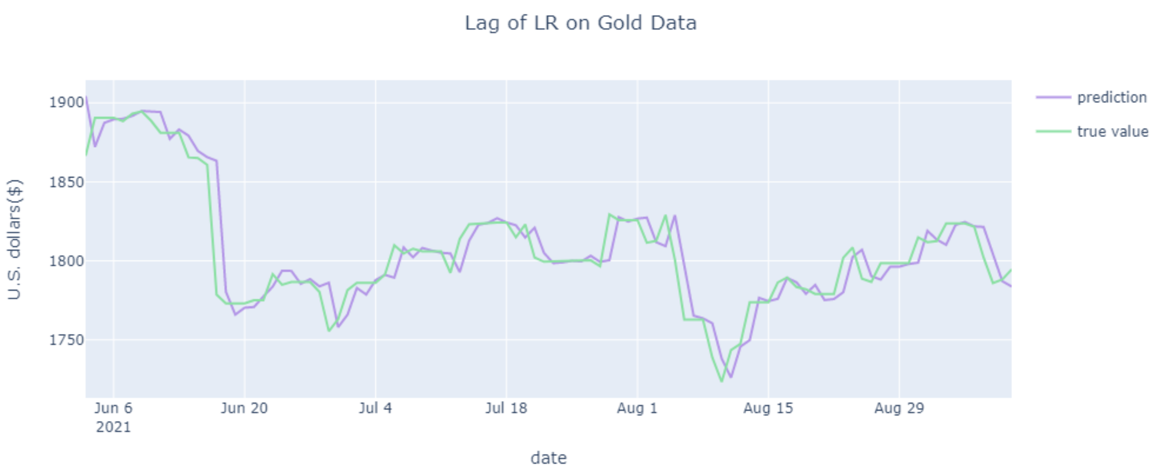

Figure 4 Preliminary Predictions from Linear Regression on Gold Price Data

As shown in Table 3, the linear regression accomplishes the prediction task relatively well just from error metrics such as mean absolute error and root mean squared deviation and from a visual view of the goodness of fit. Combined with Figure 3 & Figure 4, the results obtained from linear regression have a significant lag, and the prediction curve seems to be the true value curve shifted backward by one or two days.

The model does not capture changes in data trends more acutely, which makes the linear regression model not a very good forecasting model.

5.2 Optimization of the Model

5.2.1 Augmented Dickey-Fuller test (ADF test)

In 5.1 we use an autoregressive model as a model for price forecasting. For the autoregressive model, we need to consider whether there is a unit root in the time series. The ADF test is used to determine whether there is a unit root in the series: if the series is smooth, there is no unit root; otherwise, there is a unit root.

The original hypothesis of ADF test is the existence of unit root, as long as the ADF test value is less than the number at 1% level can significantly reject the original hypothesis, if the significance test statistic obtained is less than three confidence levels $(10\%, 5\%, 1\%)$, it corresponds to a $(90\%, 95\%, 99\%)$ possibility to reject the original hypothesis.

We performed ADF tests on the original series of bitcoin price and gold price, partial difference series, and obtained the following results:

Table 4 ADF Test - Bitcoin

Sequence	test statistic	p_value	critical values		
			1%	5%	10%
Original Sequence	-0.237743	0.933871	-3.433986	-2.863146	-2.567625
First order difference sequence	-8.537736	9.973934e-14	-3.433984	-2.863145	-2.567624
Second order difference sequence	-7.659254	1.705973e-11	-3.433986	-2.863146	-2.567625

Table 5 ADF Test - Gold

Sequence	test statistic	p_value	critical values		
			1%	5%	10%
Original Sequence	-0.507949	0.890456	-3.435626	-2.863870	-2.568010
First order difference sequence	-7.957540	3.009055e-12	-3.435621	-2.863868	-2.568009
Second order difference sequence	-8.180001	8.179822e-13	-3.435622	-2.863868	-2.568009

The results of the ADF test in Table 4 & Table 5 show that the original time series of bitcoin price and gold price are not smooth, while the first-order difference series are both smooth. Based on this finding, the following optimization is proposed for the original model.

5.2.2 Steps for Improvement

1.Fitting and Predicting Models on Differenced Series

- Because the smoothness of the series requires that the fitted curves obtained from the sample time series continue inertially with the existing pattern over a future period, we expect the regression model to learn this feature of the degree of variability of the time series.

2.Additional Features

- Adding weekly features. One day is which day of the week, denoted by $day\_of\_week_i, i \in [0,6]$. For example, $day\_of\_week_2 = 1$ means that the day is Tuesday, and $day\_of\_week_2 = 0$ means that the day is not Tuesday;
- Add whether the day is a holiday or not as a feature
- Add the simple moving average data obtained in feature engineering

3.Adaptation to Data Conditions

- Use linear regression to do short-term forecasting, and use LSTM model for forecasting when time lapses to a larger amount of data

5.3 Three-Layer LSTM

The LSTM model can be divided into three parts, **the input module**, **the LSTM module**, and **the output module**, and the main flow is shown in Figure 5.

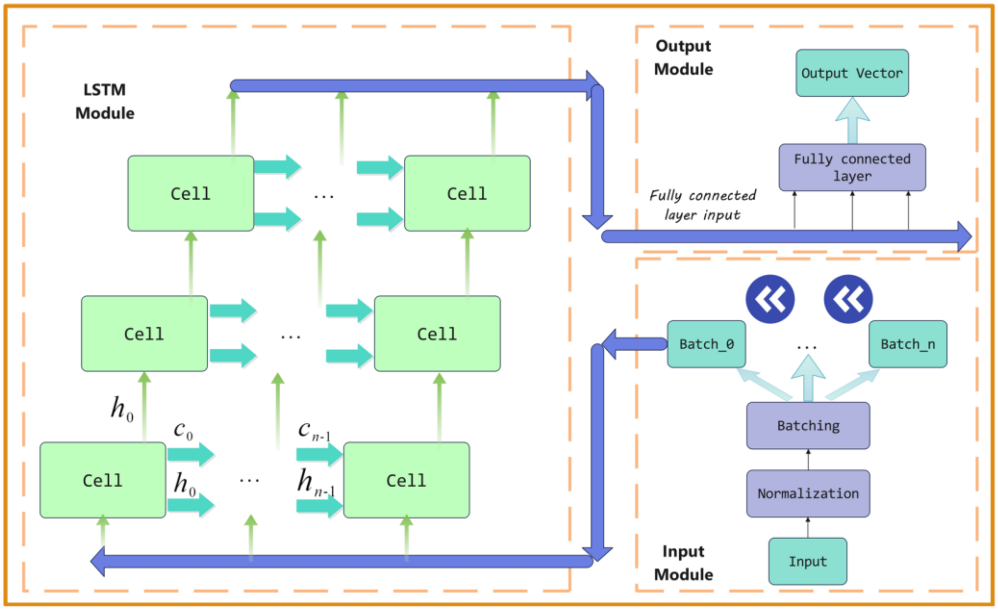

Figure 5 Structure of Three-Layer LSTM Model

5.3.1 Input Module

The input module mainly performs data normalization and data batching tasks.

Different features in the input data will have different magnitudes, so we need to normalize the input vectors. We use the following normalization operation for each kind of features:

$$x_i = \frac{x_i - \min(x_i)}{\max(x_i) - \min(x_i)} \tag{13}$$

In the task of deep learning, we artificially set the training batch to avoid the problem of difficult convergence caused by too small a batch. A large batch size will lead to a decrease in the number of iterations required for the neural network to complete one cycle of the training task, which leads to a decrease in accuracy, and therefore requires a larger number of cycles to complete the training. The batch size we choose in this task is 32, which is a moderate batch size based on the training effect, and this is where the batch gradient descent method is applied.[4]

5.3.2 LSTM Module

LSTM is a neural network commonly used in natural language processing to extract semantic-grammatical information from text, and is a variant of Recurrent Neural Networks. It performs well in time-series tasks, and the use of forgetting gates, input gates and output gates in the individual cell of LSTM to control the transfer of information makes it perform better than most other models in long sequence tasks. The basic unit of LSTM is shown in Figure 6.[5]

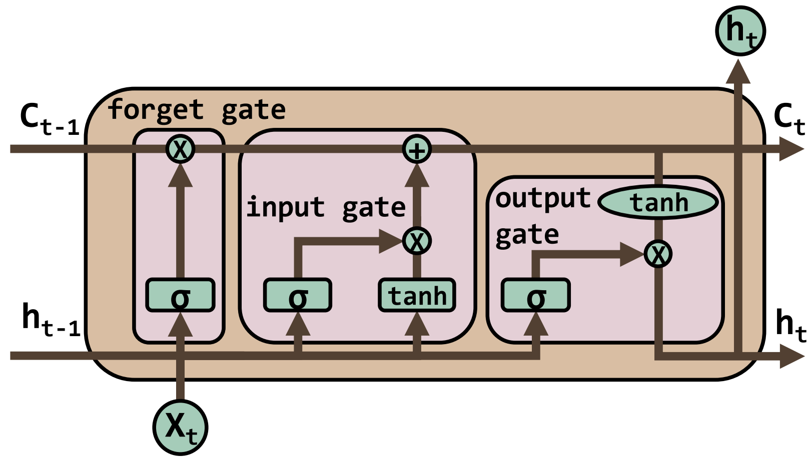

Figure 6 Basic Unit of Three-Layer LSTM

$$\tilde{c}^{<t>} = \tan h(W_c[h^{<t-1>}, x^{<t>}] + b_c) \tag{14}$$
$$\Gamma_u = \sigma(W_u[h^{<t-1>}, x^{<t>}] + b_u) \tag{15}$$
$$\Gamma_f = \sigma\left(W_f[h^{<t-1>}, x^{<t>}] + b_f\right) \tag{16}$$
$$\Gamma_f = \sigma\left(W_f[h^{<t-1>}, x^{<t>}] + b_f\right) \tag{17}$$
$$\Gamma_o = \sigma(W_o[h^{<t-1>}, x^{<t>}] + b_o) \tag{18}$$
$$c^{<t>} = \Gamma_u * \tilde{c}^{<t>} + \Gamma_f * c^{<t-1>} \tag{19}$$
$$h^{<t>} = \Gamma_o * tanh(c^{<t>}) \tag{20}$$

Table 6 Notations for one-layer LSTM

Symbol	Definition
$\boldsymbol{W}$	The weight matrix for transitions
$\boldsymbol{b}$	The bias for transitions
$\boldsymbol{\sigma}$	The sigmoid function
$\boldsymbol{h}$	Hidden state which memory the features in the transmission flow
$\boldsymbol{C}$	Cell state
$\boldsymbol{\Gamma_f}$	Forgetting gate, accepting a long-term memory $\boldsymbol{c^{<t-1>}}$ and deciding which parts to discard
$\boldsymbol{\Gamma_u}$	Input gate, determining what new information is stored in the cell state
$\boldsymbol{\Gamma_o}$	Output gate, determining output values based on cell status

As shown in Figure 6, the LSTM unit selectively accepts long-term memory through a gate mechanism and merges long-term memory with short-term memory, thus making the training and prediction of the LSTM unit on long-term sequences more effective.[6]

5.3.3 Output Module

In the output module, we predict the results by feeding the hidden layer data output from the LSTM module into a linear layer (i.e., a fully connected layer). The calculation process is as follows.

$$y_{pred} = W_{fc}h + b_{fc} \tag{21}$$

Where $W_{fc}h$ is the weight matrix and b_{fc} is the bias.

5.4 LR-LSTM Model

Combining the characteristics of the two models mentioned above, we use a combined model of linear regression and LSTM for forecasting. We assume a timestamp T. Before T we use linear regression for sliding window type forecasting and after T we use LSTM for forecasting. We obtained good results when the critical point T is about 0.58 after several calculations. Both predictions satisfy $\frac{Test\ Set}{Training\ Set} = 5\%$, and the training set is divided into 20% of the validation set.

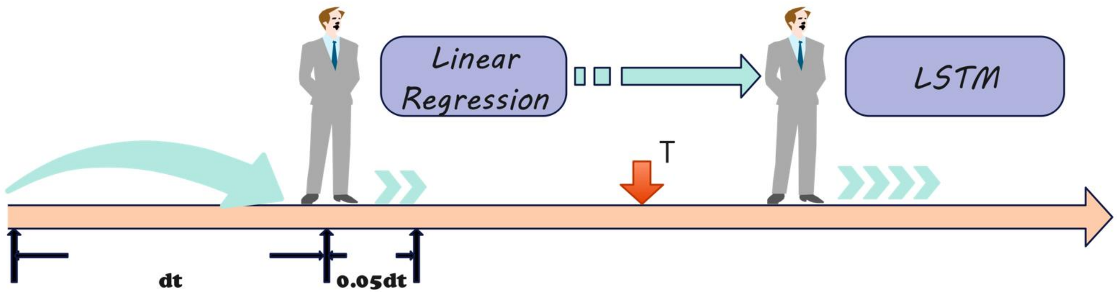

Figure 7 Prediction Process

We obtain forecasts for the price of bitcoin and the price of gold, plotting the last few months of forecast data as follows.

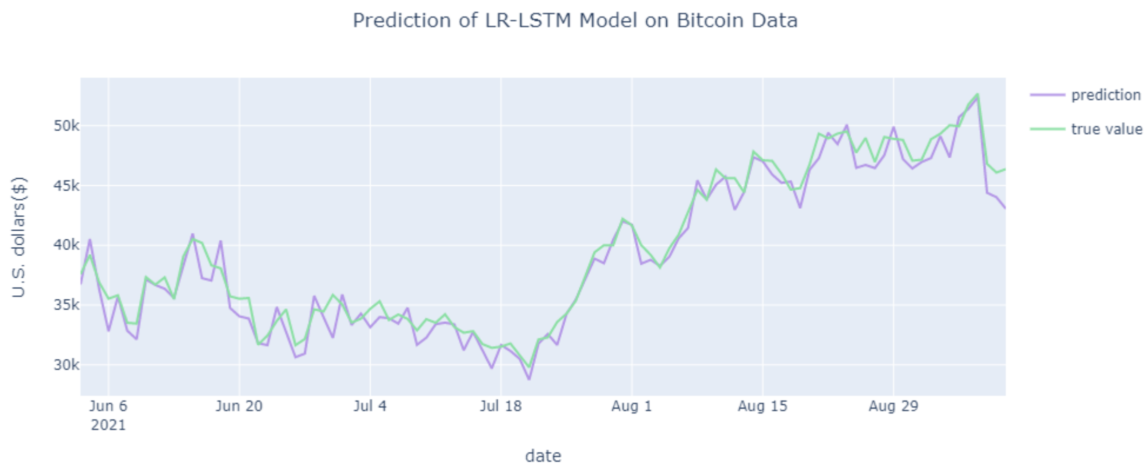

Figure 8 Prediction of LR-LSTM Model on Partial Bitcoin Data

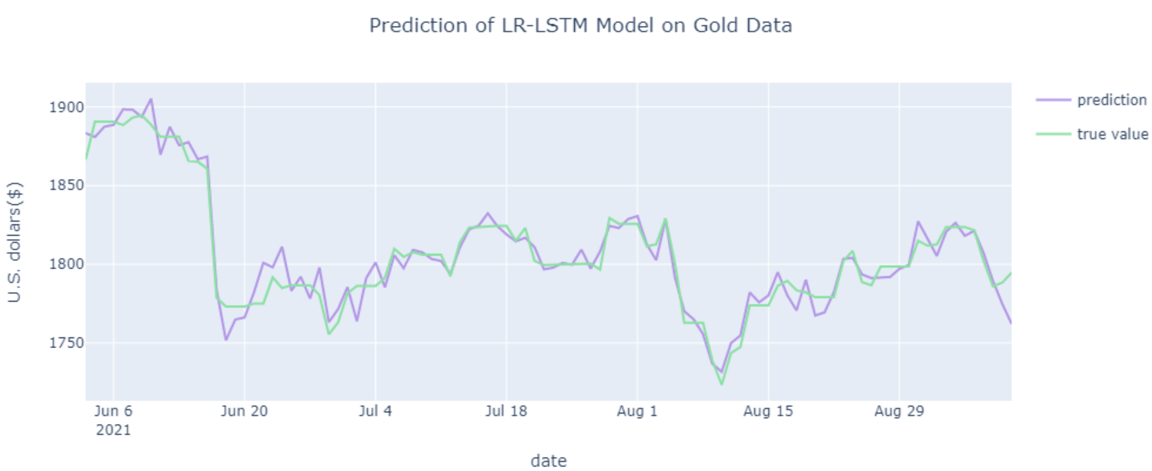

Figure 9 Prediction of LR-LSTM Model on Partial Gold Data

We used the linear regression, LR-LSTM model for predicting the prices of bitcoin and gold, respectively, and obtained the following errors.

Figure 10 Comparison of the Errors of the Two Prediction Methods

The MAE and RMSE are smaller when predicting the price of Bitcoin with the LR-LSTM model. In contrast, the prediction performance is relatively poor when the LR-LSTM model is used to predict the gold price data. Evaluated together with the prediction graphs, the LR-LSTM model is able to attenuate the lag more significantly and achieve relatively good prediction results on both data. In summary, we decided to use the prediction results made by the LR-LSTM model as the data basis for the subsequent work.

6 Decision-Making Models

6.1 Risk Index

Once we have the complete prediction data, we can use the prediction data to help in decision making. In the decision model, we define the purchase index $PI_i^t(i = 1,2)$ to represent our propensity to purchase two investments, bitcoin and gold, on day t. When buying stocks, we not only consider whether a particular stock can bring us the desired profit, but also what risks the stock will have later on. Therefore, we consider both factors together and propose a return-risk decision model.[7]

6.1.1 Risk-related Feature Mining

6.1.1.1 Deviation rate

In the feature engineering we extracted the five-day average data from the bitcoin price series and the twenty-day average data from the gold price series. We further consider the use of the deviation, i.e. the degree of deviation between the stock price and the average moving line, as one of the factors of the risk index. the n-day average deviation D_t is calculated as follows:

$$\overline{x_t} = \frac{\sum_{i=t-n+1}^{t} x_i}{n} \tag{22}$$

$$D_t = \frac{x_t - \overline{x_t}}{\overline{x_t}} \times 100\% \tag{23}$$

where x_t is the price of the investment on day t and $\overline{x_t}$ is the average value of the price of the investment on day n before day t.

6.1.1.2 Bull and Bear Markets

A bull market means that the stock market is expected to be stable and bullish, and the outlook is optimistic; a bear market means that the stock market is expected to be bearish and the outlook is pessimistic. The concepts of bull market and bear market broadly describe the current development of the stock market. The laws of the market reflect various environmental factors such as the economy and society to a certain extent. We design the market index MI_i^t to help judge the current market situation. When $i = 1$, it represents the market index of Bitcoin, and when $i = 2$, it represents the market index of gold. We set the size of the market index to be related to the mean value of the stock's profitability over time and the variance

$$\sigma^{2^t} = \frac{\sum_{i=t-n+1}^{t}(x_t - \overline{x_t})^2}{n} \tag{24}$$

After normalizing the mean and variance data separately, the market index equation is established

$$MI^t = l_1\overline{x_t} + l_2\sigma^{2^t} \tag{25}$$

For bitcoin we consider a 30-day observation period, i.e., $n = 30$; for gold we consider a 90-day observation period, i.e., $n = 90$. After adjusting the parameters, we find that the market index performs more consistently when $l_1 = 0.7, l_2 = 0.3$

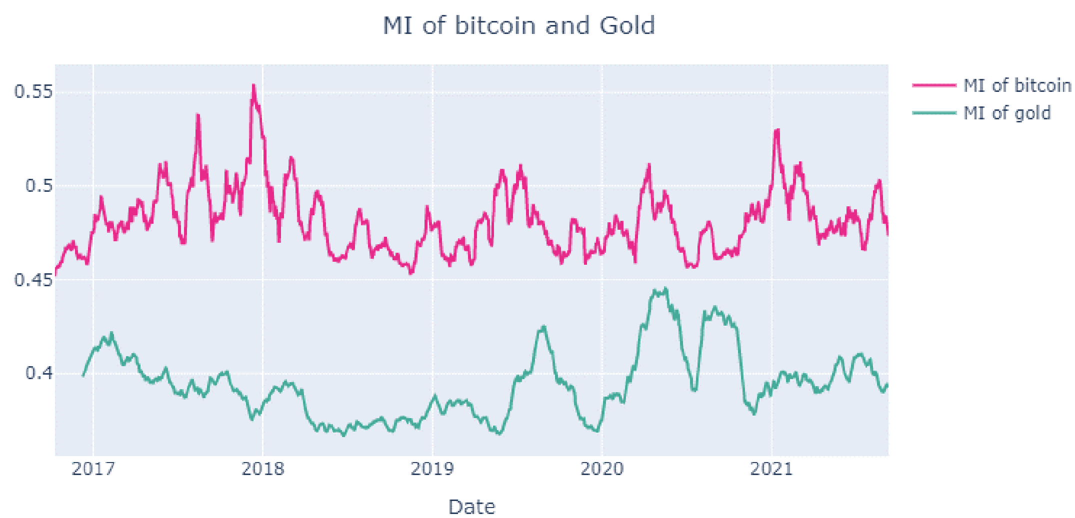

Figure 11 MI of Bitcoin & Gold

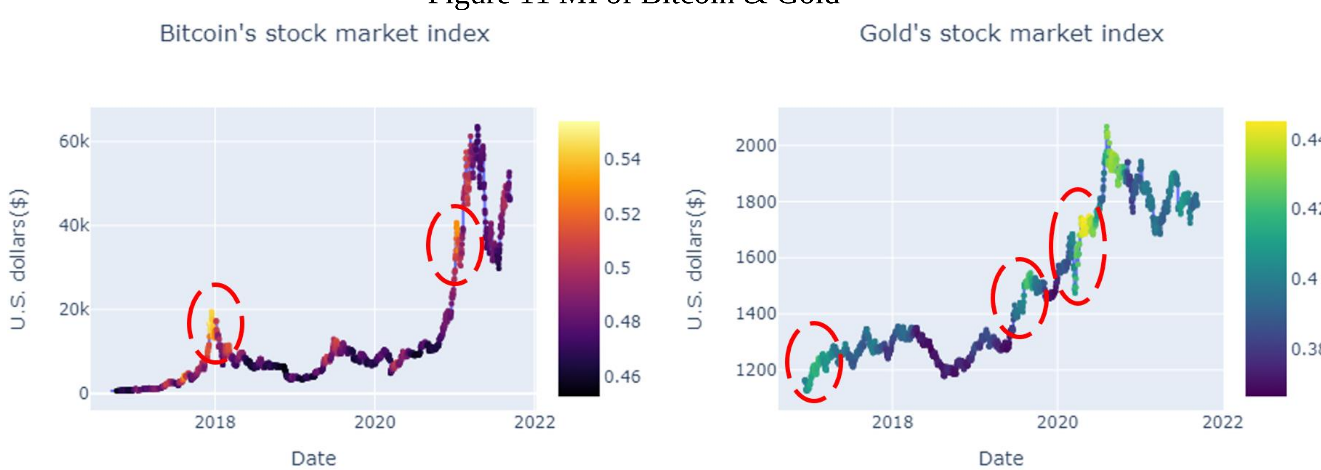

Figure 12 Visualization of the MI on the Original Price Series

The upward trend in the circled portion of Figure 12 suggests that the Market Index $(MI)$is a good simulation of the current market conditions in the stock market

Based on the MI chart and the performance of the market indices in the time series, we can roughly determine the dividing line between bull and bear markets for bitcoin is $MI = 0.48$. The dividing line between bull and bear markets for gold is $MI = 0.4$.

6.1.1.3 ACCER

ACCER is a measure of the rate at which stock prices rise and fall by calculating the slope of the N-day linear regression of the closing price.

The speed at which a stock rises or falls V reflects riskiness to some extent. Stocks that rise rapidly in the short term are very promising, but may also be in danger of a stock market pullback. The calculation of the stock's upward velocity comes from the optimal slope of a linear fit of the stock price for the last few days divided by the number of days n, where the optimal slope can be calculated by least squares. Bitcoin uses a 5-day observation period due to the frequent ups and downs in the price of bitcoin. A 10-day observation period is used for gold.

6.1.2 Normalization

All of the above indicators should be normalized before weight assignment. Due to the presence of deviations, stock speed contains negative values and is an intermediate indicator. The stock speed is normalized in absolute terms before being normalized.

The market index, as a very large indicator, is normalized directly. Normalization method using maximum and minimum values.

$$x = \frac{x - min(x)}{max(x) - min(x)} \tag{26}$$

6.1.3 Entropy Weighting Method to Determine the Weights

After obtaining the above indicators, we establish the risk index model.

$$RI = \mu_1 D + \mu_2 MI + \mu_3 V \tag{27}$$

We use the entropy weighting method based on information theory to assign weights to the three influencing factors. The advantage of using this method is that it has the advantage of avoiding the influence of subjectivity on the results as an objective assignment method that judges the amount of information contained based on the degree of variability of the indicators, and thus assigns weights quantitatively.[8]

For the feature matrix $X \in Rn \times 3$ that has been normalized, the normalization matrix $Z \in Rn \times 3$ is first obtained, where $z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}, j = 0,1,2$

We perform sample weighting calculations for each of the three evaluation metrics to obtain the probability matrix $P \in R_{n\times 3}$ i.e.$p_{ij} = \frac{z_{ij}}{\sum_{i=1}^{n} z_{ij}}, j = 0,1,2$.

According to the formula of information entropy, the information entropy e and information utility value d of the three indicators are obtained.

$$e_j = -\frac{1}{ln\, n}\sum_{i=1}^{n} p_{ij}\, ln(p_{ij}), j = 0,1,2 \tag{28}$$

$$d_j = 1 - e_j, j = 0,1,2 \tag{29}$$

The information utility values are normalized to obtain the entropy weights W corresponding to each indicator:

$$W_j = \frac{d_j}{\sum_{j=0}^{2} d_j}, j = 0,1,2 \tag{30}$$

Obtain the parameters in the expression for the risk index of bitcoin and gold:

Table 7 Risk Index of Bitcoin & Gold

	μ_1	μ_2	μ_3
Bitcoin	0.31689368	0.34626313	0.33684319
Gold	0.32732319	0.34240056	0.33027624

The risk factor of the two is calculated:

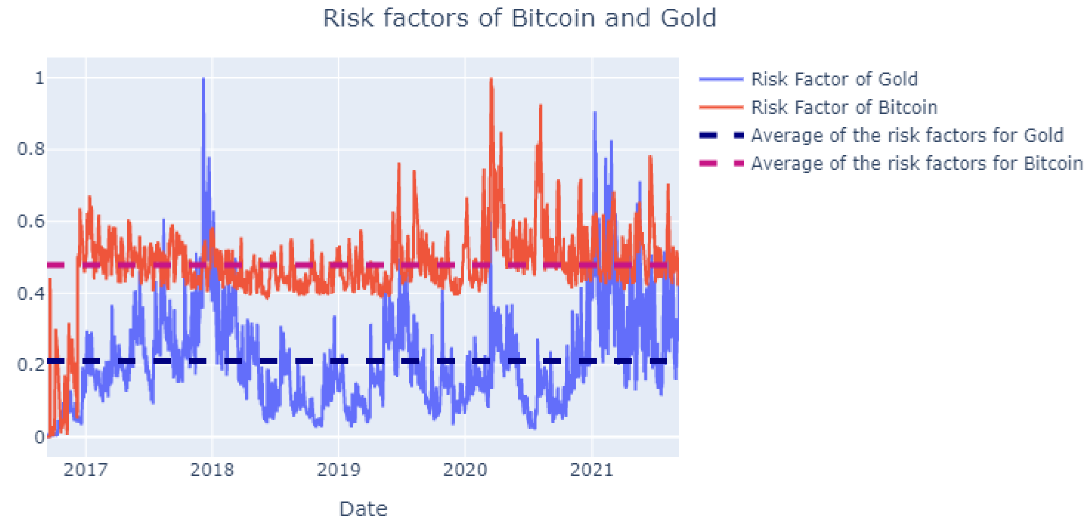

Figure 13 RI of Bitcoin & Gold

6.2 Purchasing Power Evaluation Based on RI

For traders' willingness to buy, traders tend to buy investments that are profitable and low risk. Therefore, the smaller the risk index, the better the positive.

$$x_i = \frac{max(x) - x_i}{max(x) - min(x)} \tag{31}$$

For the previously defined purchase index PI^t can be expressed as (32)

$$PI^t = w_1 \Pi^t + w_2 RI^t \tag{32}$$

Because Bitcoin and gold have different characteristics, we use different data to predict profits for both. We consider the mean of the predicted profit for bitcoin over 5 days, using the normalized mean of the existing interest rate data, and substituting it into Π. Similarly, we consider the predicted profit for gold over 10 days of data. At the same time, we give Bitcoin an additional skewed portion of the risk factor weights, w_1, w_2 determined as in Table 8:

Table 8 Weight of Risk Factors

	w_1	w_2
Bitcoin	0.7	0.3
Gold	0.8	0.2

6.3 Decision-Making Model Based on Purchase Index

We construct decision models based on the daily purchase indices (PI) of bitcoin and gold obtained above. For both purchase indices PI_b, PI_g, we explore two thresholds $P, Q(Q > P)$ respectively. We have sufficient incentive to purchase the investment when $PI > Q$, we tend to sell the investment when $PI < P$, and do not operate on individual investments when the value is in the middle, with the overall idea as follows.

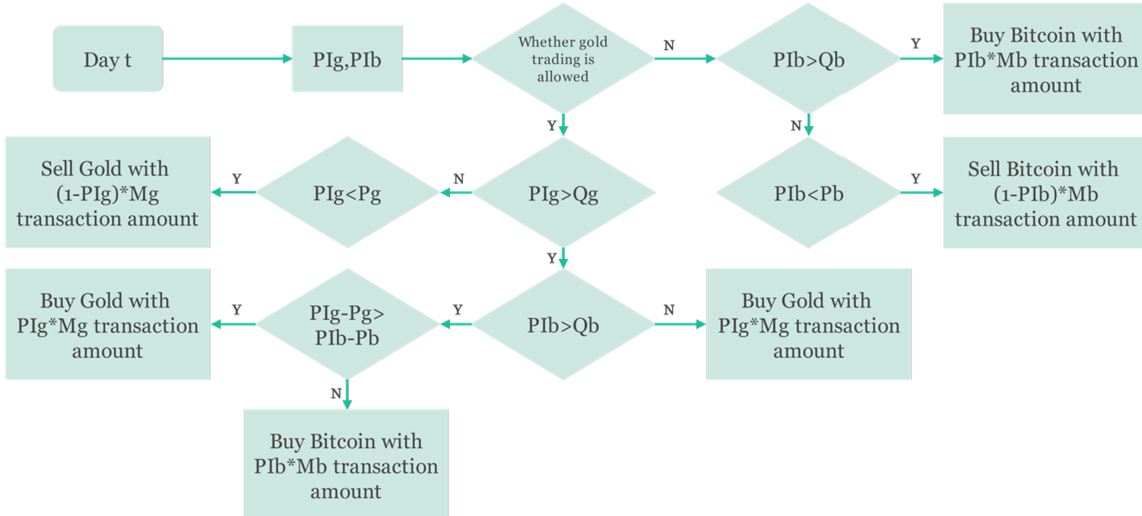

Figure 14 Flow of Decision

In the Figure 14, M_b, M_g denote the day traders hold bitcoin and gold respectively; PI_b, PI_g represent the purchase index of the two investments; P_b, Q_b are the division boundaries of bitcoin investment and Pg, Qg denote the division boundaries of gold investment.

The purchase index measures our tendency to want to own the commodity. The lower the purchase index, the less we want to hold the investment. So, we use $1 - PI$ as the selling portion of the ratio.

After resizing P, Q several times, we get the optimal result when $\mathrm{P_g} = 0.41, \mathrm{Q_g} = 0.61, \mathrm{P_b} = 0.43, \mathrm{Q_b} = 0.67$. The total assets of cash, gold, and bitcoin in this case at 9/10/2021 are \$13931321.3740. The property distribution process diagram and the volume of bitcoin and gold transactions at the time of occurrence are shown in Figure 15 & Figure 16.

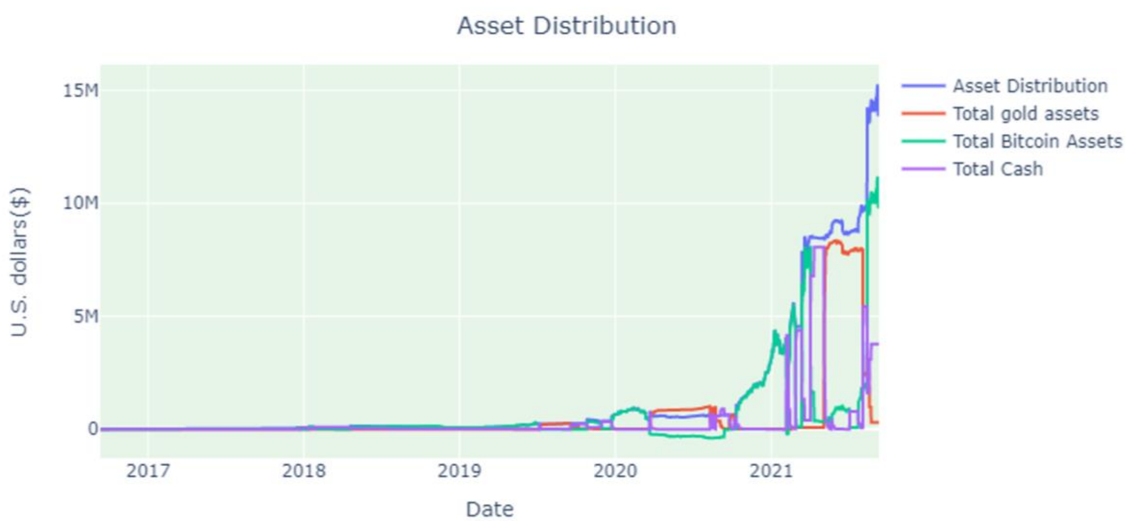

Figure 15 Visualization of Various Assets

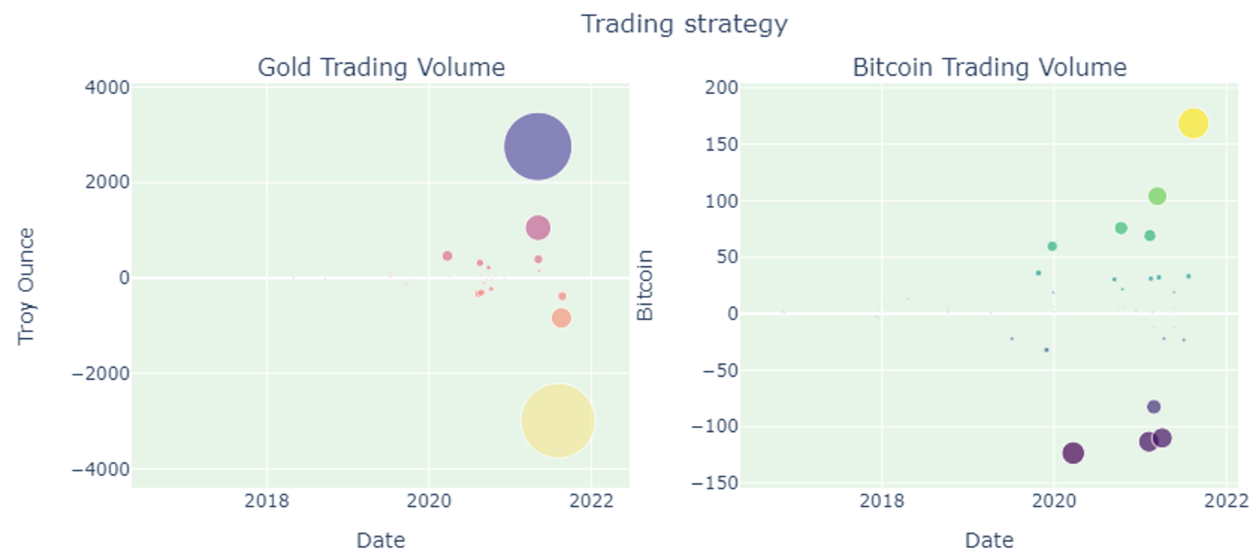

Figure 16 Trade Strategy on Time Series

7 Program Evaluation

7.1 Proof of Optimality

We believe that the optimality of the solution depends on two aspects.

One is the accuracy of the price prediction, which directly affects the purchase index of each decision step by affecting the expected return. We tested the MAE and RMSE of the prediction results in the LR-LSTM prediction model and obtained small deviations of the predicted results from the actual ones.

The other one is whether the decision model achieves the desired profitability result. We take the predicted monthly profitability as the reference indicator and the true monthly return is noted as $H = (h_1, h_2, \cdots, h_n), n = 61$.

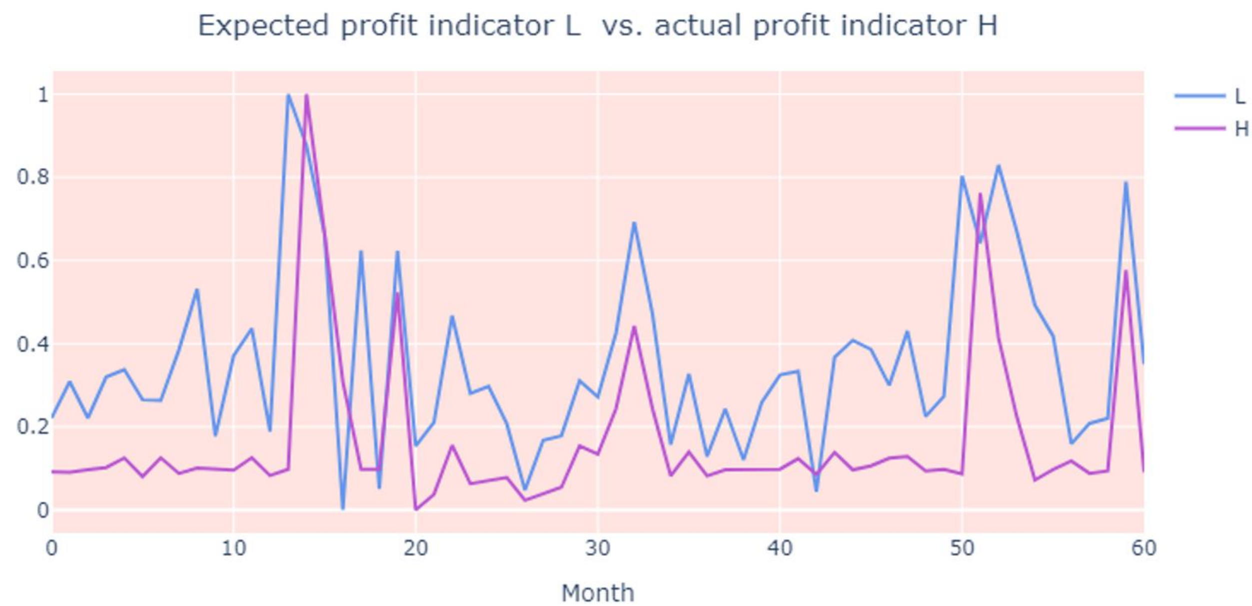

Figure 17 Comparison of L-index and H-index

We obtain the monthly growth rates of bitcoin and gold values based on the forecast data of bitcoin and gold $\Omega_b = (\omega_{b_1}, \omega_{b_2}, \cdots, \omega_{b_n}), \Omega_g = (\omega_{g_1}, \omega_{g_2}, \cdots, \omega_{g_n}), n = 61$. Normalizing the growth rates gives $\widetilde{\Omega_b}, \widetilde{\Omega_g}$. Assigning the same weight to the normalized results gives the expected return $L = 0.5\widetilde{\Omega_b} + 0.5\widetilde{\Omega_g}$. L is a vector of length 61. The normalized L and the expected monthly return are shown in Figure 17. Figure 17 clearly shows the trend of the expected return and the true return.

The mean value of the offset is 0.214806. As can be seen in Figure A, our expected profit trend is similar to the actual profit trend. The monthly interest rate indicator indicates that our decision model basically achieves the desired profitability goal.

7.2 Sensitivity Analysis

To determine the impact of Transaction Costs on the final returns, we analyze the sensitivity of the model. We add a certain degree of perturbation to Transaction Costs, respectively set $\alpha_{gold} \in [1\%, 5\%]$, $\alpha_{bitcoin} \in [1\%, 5\%]$, in steps of 0.5%, and observe the magnitude and degree of statistical change in the final return.[9]

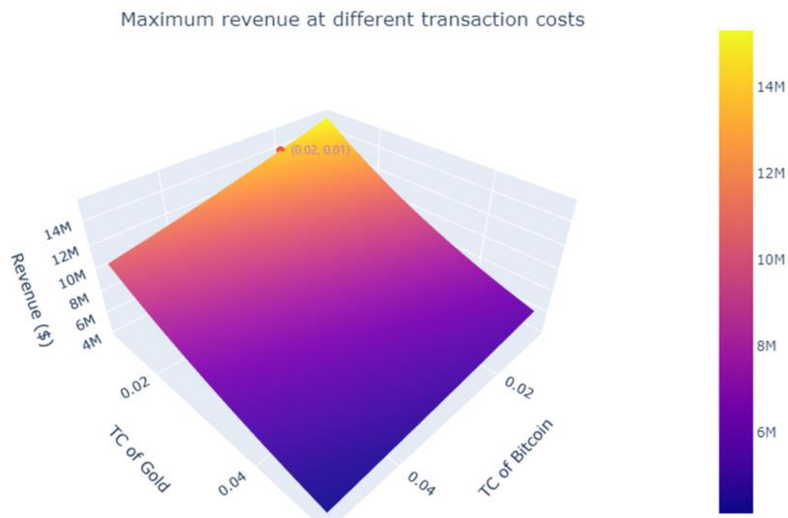

Figure 18 Final Total Assets with Different Transaction Costs

As shown in Figure 18, the trend of change is relatively smooth, and we calculate the effect of the results caused by a 5% change in $\alpha_{bitcoin}$ for each fixed α_{gold}. The trend matrix $E_1 \in R_{9*8}$, the mean variance of each row is shown in Table 9 & Table 10:

Table 9 Analysis of the Impact of the Change in Bitcoin Transaction Costs on the Final Asset

α_{gold}	1.0%	1.5%	2.0%	2.5%	3.0%
	3.5%	4.0%	4.5%	5.0%	
$\bar{x}$	-0.04109	-0.04107	-0.04105	-0.04103	-0.04101
	-0.04099	-0.04097	-0.04094	-0.04092	
σ^2	0.000341	0.000340	0.000339	0.000339	0.000339
	0.000339	0.000338	0.00034	0.00034	

Table 10 Analysis of the Impact of the Change in Gold Transaction Costs on the Final Asset

α_{gold}	1.0%	1.5%	2.0%	2.5%	3.0%
	3.5%	4.0%	4.5%	5.0%	
$\bar{x}$	-0.10928	-0.10927	-0.10927	-0.10927	-0.10926
	-0.10926	-0.10926	-0.10925	-0.10925	
σ^2	0.001556	0.001556	0.001557	0.001558	0.001558
	0.001558	0.001559	0.001559	0.001560	

As shown in Table 9 & Table 10, after the Transaction Costs of gold are fixed, the rate of change of the final return is around $4\% - 4.1\%$ as $\alpha_{bitcoin}$ changes, and the variance shows that $\bar{x}$

performs very consistently. After the Transaction Costs of Bitcoin are fixed, as α_{gold} changes, the rate of change of the final return is around 10.92%, which is relatively large, but $\bar{x}$ also performs very stable.

In addition, we select four sets of data ($\alpha_{bitcoin} = 1.0\%, \alpha_{bitcoin} = 1.5\%, \alpha_{gold} = 1.0\%, \alpha_{gold} = 1.5\%$) to obtain the trend of asset changes under different conditions, taking some time visualizations to observe:

Figure 19 Trends in Assets Under Different Transaction Costs

We can clearly observe that the four curves in Figure 19 have almost the same trend, with only a small difference in values. We can assume that Transaction Costs has very little impact on our decision. To sum up, although Transaction Cost α is a sensitive parameter, the robustness of our decision model is very high.

8 Strength and Weakness

8.1 Strengths

8.1.1 Time Series Forecasting Models

1. We combine the advantages and disadvantages of linear regression and LSTM neural network forecasting. When the amount of data is small in the pre-investment period, we aim at error minimization and allow for lags. When the data volume increases over time, we build Three-layer LSTM models for more accurate forecasting, which can more powerfully extract information on long series, and the final model prediction results have the advantages of reduced lag and lower error.
2. We consider several metrics such as seasonality for larger scale data mining, thus making greater use of limited data and avoiding the reliance of the dispersion model on a single feature.

8.1.2 Decision-Making Models Based on Purchase Index

1. We innovatively propose a quantitative representation of the propensity to buy commodities by considering a variety of economic indicators and the characteristics of the data itself to assist in decision making. We quantify trading by means of a purchase index, which from the results is very effective in characterizing the potential of an investment

on a given day. Through the model we are able to select reasonable thresholds to determine whether to trade.

2. We quantify the risk factor as one of the linear indicators involved in the formation of the purchase index. Among the risk factors, we consider both short-term indicators such as ACCER and long-term indicators. We innovatively propose a market index model to measure the overall performance of an investment in the market over a certain time horizon. We use the entropy weighting method to assign weights to the factors of risk factors, using an information-theoretic approach to circumvent subjectivity.

8.2 Weaknesses

8.2.1 Time Series Forecasting Models

1. The LR-LSTM model can weaken the lag to some extent, but the lag cannot be eradicated substantially. In the error analysis of gold price forecasting, it performs slightly worse than linear regression forecasting. The application of neural network models does not easily uncover the original characteristics of the time series.
2. The time cost of LR-LSTM prediction is high, so the optimization of the parameters is not optimal, so the prediction result is not necessarily the optimal result that can be achieved

8.2.2 Decision-Making Models Based on Purchase Index

1. The assignment of some of the weights is subjective, and even though the decision results perform well, a more accurate representation of the relevant index is not explored in depth.
2. There are potentially more valid economic indicators that can more accurately measure commodity purchase propensity and risk indicators.

9 Memorandum

(See next 2 pages)

Dear trader:

We have completed the model you need. Under limited data conditions, this model can help you determine whether you should buy, hold, or sell assets in your portfolio each day. We now present you with the strategy, model, and results of this model.

The strategy we propose is to either not trade for the first few days and wait for the bitcoin and gold markets to show a certain pattern before considering buying or selling, or to split the $1,000 in three equal parts, one for gold, one for bitcoin, and one to keep, thus spreading the risk. For a certain level of time accumulation, around 20 days, at this point we start to use existing data to forecast future data. In the pre-decision period, we use linear regression for forecasting, using the price of the previous number of hours, the one-hot vectorization of weekly features and whether the day is a holiday as features, and we keep the forecast step size not too large. However, the results of linear regression have a more significant lag. In the middle and late stages of decision making, we consider using the LSTM model to make more accurate predictions by training with a large amount of prior data, and the point at which our recommended prediction strategy changes is roughly September 2018, which is 60% of the entire time.

We need to make decisions while making forecasts, and we propose to quantify the propensity to buy both purchases, expressed using a purchase index. Since we have to consider not only the expected profit of the commodity, but also the risk associated with the investment. We therefore quantify the purchase index as a linear combination of the average value of the predicted interest rate for the next few days and the risk index for that day. Among the many indicators used to assess risk, we chose the five-day deviation rate for bitcoin and the twenty-day deviation rate for gold, as well as the range up rate, and the market index as reference factors. The reason for considering current market factors is that the overall market trends reflect to some extent the influence of external factors on the stock market and whether the economy and society are bullish on the stock. We use the market index to measure the current outlook for a particular commodity stock, and its size is measured by recent time returns and volatility. For bitcoin, which has a high rate of increase or decrease, a short-term observation is recommended, and for gold, a medium- to long-term observation strategy can be adopted. We use an information theoretic approach to assign weights to each of the three indices based on the size of the information they provide over time to obtain an expression for the risk index and ultimately the purchase index.

Once we get the buying index, we need to draw a buy lower bound and a sell upper bound for it. When the buying index for gold or bitcoin is above the buy lower bound, it means we have sufficient incentive to buy the commodity. And when the purchase index is below the upper sell boundary, it means we don't expect to keep the commodity. For the purchase index PI, when a buy is recommended, spend cash $C \times PI$ of the transaction amount to buy the commodity, otherwise sell $S \times (1 - PI)$ of the transaction amount to buy the commodity. If the purchase index lies between the two bounds, then press on. According to our projections, the lower bound for buying is 0.69 and the upper bound for selling is 0.56 for Bitcoin, and the lower bound for buying is 0.57 and the upper bound for selling is 0.28 for gold.

After our calculations, a quantitative investment following the above decision model could theoretically result in total assets of $13,931,321 in cash, bitcoin and gold by 9/10/2021 when holding $1,000 on 9/11/2016. This is a very surprising profit. Furthermore, after error analysis and sensitivity testing, our model has an almost constant investment trend for different commission ratios. The decrease in profit due to the increase in fees is easy to accept, so our proposed model has a good robustness.

We hope that our strategy, model, and results will be helpful to you in your investment planning. The model just described can help you determine whether you should buy, hold, or sell assets in your portfolio each day.

Best Wishes to You![1]

Sincerely yours
Team#2214695

Header image: https://unsplash.com/photos/S6cWf7TIk9U
https://unsplash.com/photos/sPLl8p04YKM
https://unsplash.com/photos/nAjil1z3eLk
Background image: https://www.pexels.com/zh-cn/photo/730547/
https://www.pexels.com/zh-cn/photo/47047/
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10 References

[1]. MITCHELL, C., How to Use a Moving Average to Buy Stocks. 2022. p. The moving average (MA) is a simple technical analysis tool that smooths out price data by creating a constantly updated average price.

[2]. Kunitomo, N. and S. Sato, Trend, seasonality and economic time series :the nonstationary errors-in-variables models, in CIRJE-F-977. 2015.

[3]. Ristanoski, G., W. Liu and J. Bailey, Time Series Forecasting Using Distribution Enhanced Linear Regression. Pacific-Asia Conference on Knowledge Discovery and Data Mining, 2013.

[4]. Hilton, J., K. Cobbe and J. Schulman, Batch size-invariance for policy optimization. 2021. 2022(2022-2-22).

[5]. Erdemir, E., P.L. Dragotti and D. Gunduz, Active Privacy-Utility Trade-off Against Inference in Time-Series Data Sharing. IEEE International Conference on Acoustics, Speech and Signal Processing, 2022. 2022(2022-2-22).

[6]. Sak, H.I., A. Senior and F.O. Beaufays, Long Short-Term Memory Recurrent Neural Network Architecturesfor Large Scale Acoustic Modeling, in Proc. Interspeech 2014. 2014.

[7]. Kumaradhas, A., A. Murugan and J. Bailey, Stock Market Risk Assessment Using Data Mining Techniques. 2020.

[8]. Chen, C., A Novel Multi-Criteria Decision-Making Model for Building Material Supplier Selection Based on Entropy-AHP Weighted TOPSIS. 2020. 2022(2022-2-22).

[9]. Iooss, B. and A. Saltelli, Introduction: Sensitivity Analysis. 2015.