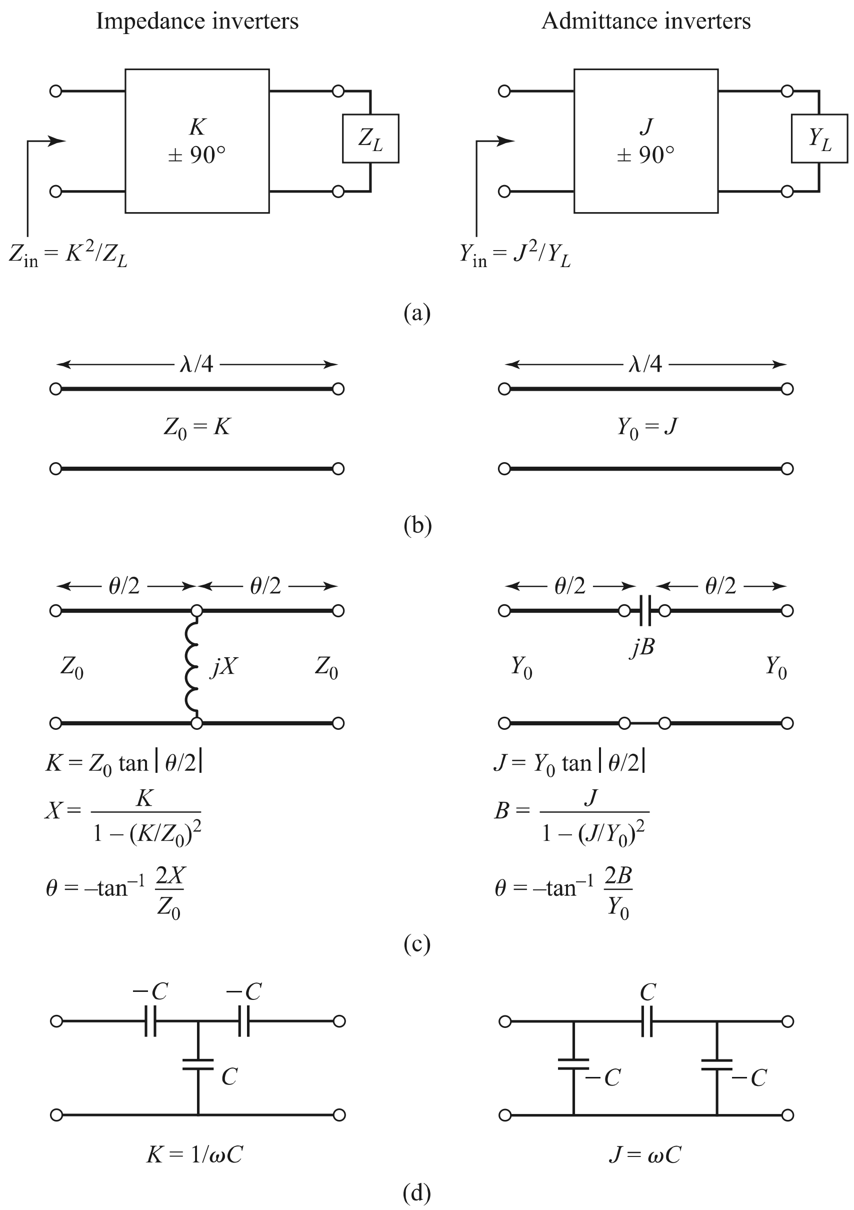

FIGURE 8.38 Impedance and admittance inverters. (a) Operation of impedance and admittance inverters. (b) Implementation as quarter-wave transformers. (c) Implementation using transmission lines and reactive elements. (d) Implementation using capacitor networks.

8.6 STEPPED-IMPEDANCE LOW-PASS FILTERS

A relatively easy way to implement low-pass filters in microstrip or stripline is to use alternating sections of very high and very low characteristic impedance lines. Such filters are usually referred to as *stepped-impedance*, or hi-Z, low-Z filters, and are popular because they are easier to design and take up less space than a similar low-pass filter using stubs. Because of the approximations involved, however, their electrical performance is not as good, so the use of such filters is usually limited to applications where a sharp cutoff is not required (for instance, in rejecting out-of-band mixer products).

Approximate Equivalent Circuits for Short Transmission Line Sections

We begin by finding the approximate equivalent circuits for a short length of transmission line having either a very large or a very small characteristic impedance. The *ABCD*

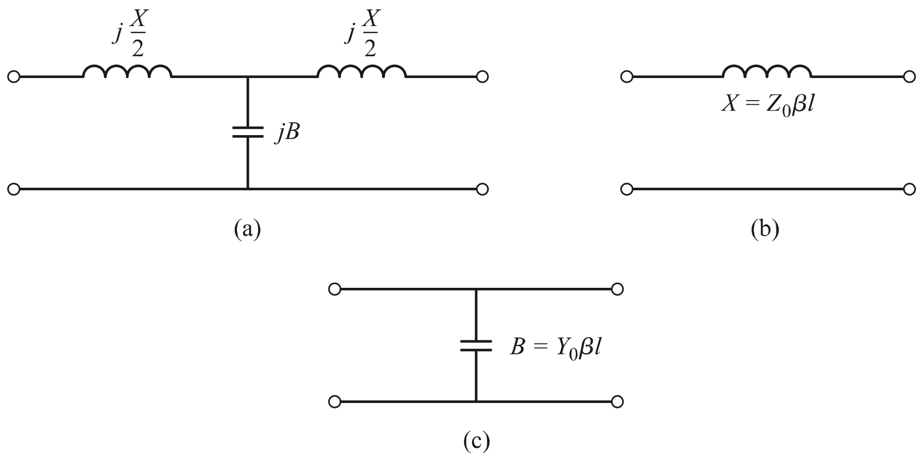

FIGURE 8.39 Approximate equivalent circuits for short sections of transmission lines. (a) T-equivalent circuit for a transmission line section having $\beta\ell \ll \pi/2$. (b) Equivalent circuit for small $\beta\ell$ and large Z_0. (c) Equivalent circuit for small $\beta\ell$ and small Z_0.

parameters of a length ℓ of line having characteristic impedance Z_0 are given in Table 4.1; the conversion in Table 4.2 can then be used to find the impedance parameters as

$$Z_{11} = Z_{22} = \frac{A}{C} = -jZ_0 \cot\beta\ell, \tag{8.81a}$$

$$Z_{12} = Z_{21} = \frac{1}{C} = -jZ_0 \csc\beta\ell. \tag{8.81b}$$

The series elements of the T-equivalent circuit are

$$Z_{11} - Z_{12} = -jZ_0\left(\frac{\cos\beta\ell - 1}{\sin\beta\ell}\right) = jZ_0 \tan\left(\frac{\beta\ell}{2}\right), \tag{8.82}$$

while the shunt element of the T-equivalent is Z_{12}. If $\beta\ell < \pi/2$, the series elements have a positive reactance (inductors), while the shunt element has a negative reactance (capacitor). We thus have the equivalent circuit shown in Figure 8.39a, where

$$\frac{X}{2} = Z_0 \tan\left(\frac{\beta\ell}{2}\right), \tag{8.83a}$$

$$B = \frac{1}{Z_0}\sin\beta\ell. \tag{8.83b}$$

Now assume a short length of line (say $\beta\ell < \pi/4$) and a large characteristic impedance. Then (8.83) approximately reduces to

$$X \simeq Z_0\beta\ell, \tag{8.84a}$$

$$B \simeq 0, \tag{8.84b}$$

which implies the equivalent circuit of Figure 8.39b (a series inductor). For a short length of line and a small characteristic impedance, (8.83) approximately reduces to

$$X \simeq 0, \tag{8.85a}$$

$$B \simeq Y_0\beta\ell, \tag{8.85b}$$

which implies the equivalent circuit of Figure 8.39c (a shunt capacitor). So the series inductors of a low-pass prototype can be replaced with high-impedance line sections ($Z_0 = Z_h$), and the shunt capacitors can be replaced with low-impedance line sections ($Z_0 = Z_\ell$). The ratio Z_h/Z_ℓ should be as large as possible, so the actual values of Z_h and Z_ℓ are usually set to the highest and lowest characteristic impedance that can be practically fabricated. The lengths of the lines can then be determined from (8.84) and (8.85); to get the best response near cutoff, these lengths should be evaluated at $\omega = \omega_c$. Combining the results of (8.84) and (8.85) with the scaling equations of (8.67) allows the electrical lengths of the inductor sections to be calculated as

$$\beta\ell = \frac{LR_0}{Z_h} \quad \text{(inductor)} \tag{8.86a}$$

and the electrical length of the capacitor sections as

$$\beta\ell = \frac{CZ_\ell}{R_0} \quad \text{(capacitor)}, \tag{8.86b}$$

where R_0 is the filter impedance and L and C are the normalized element values (the g_k) of the low-pass prototype.

EXAMPLE 8.6 STEPPED-IMPEDANCE FILTER DESIGN

Design a stepped-impedance low-pass filter having a maximally flat response and a cutoff frequency of 2.5 GHz. It is desired to have more than 20 dB insertion loss at 4 GHz. The filter impedance is 50 Ω; the highest practical line impedance is 120 Ω, and the lowest is 20 Ω. Consider the effect of losses when this filter is implemented with a microstrip substrate having $d = 0.158$ cm, $\epsilon_r = 4.2$, $\tan\delta = 0.02$, and copper conductors of 0.5 mil thickness.

Solution
To use Figure 8.26 we calculate

$$\frac{\omega}{\omega_c} - 1 = \frac{4.0}{2.5} - 1 = 0.6;$$

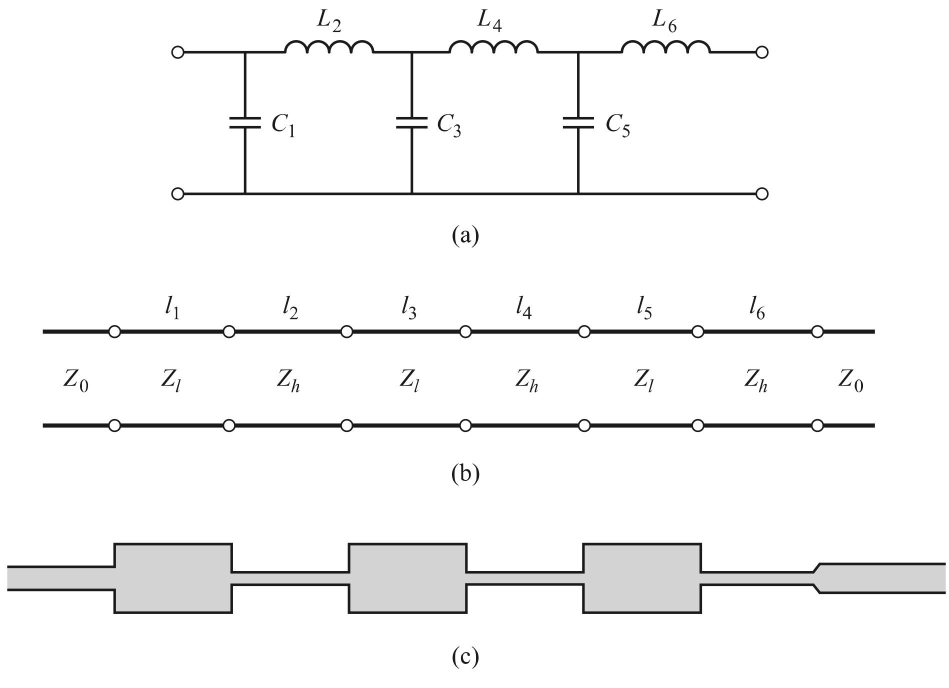

FIGURE 8.40 Filter design for Example 8.6. (a) Low-pass filter prototype circuit. (b) Stepped-impedance implementation. (c) Microstrip layout of final filter.

then the figure indicates $N = 6$ should give the required attenuation at 4.0 GHz. Table 8.3 gives the low-pass prototype values as

$$\begin{aligned} g_1 &= 0.517 = C_1, \\ g_2 &= 1.414 = L_2, \\ g_3 &= 1.932 = C_3, \\ g_4 &= 1.932 = L_4, \\ g_5 &= 1.414 = C_5, \\ g_6 &= 0.517 = L_6. \end{aligned}$$

The low-pass prototype filter is shown in Figure 8.40a.

Next, (8.86a) and (8.86b) are used to replace the series inductors and shunt capacitors with sections of low-impedance and high-impedance lines. The required electrical line lengths, $\beta\ell_i$, along with the physical microstrip line widths, W_i, and lengths, ℓ_i, are given in the table below.

Section	$Z_i = Z_\ell$ or $Z_h (\Omega)$	$\beta\ell_i$ (deg)	W_i (mm)	ℓ_i (mm)
1	20	11.8	11.3	2.05
2	120	33.8	0.428	6.63
3	20	44.3	11.3	7.69
4	120	46.1	0.428	9.04
5	20	32.4	11.3	5.63
6	120	12.3	0.428	2.41

The final filter circuit is shown in Figure 8.40b, with $Z_\ell = 20\ \Omega$ and $Z_h = 120\ \Omega$. Note that $\beta\ell < 45°$ for all but one section. The microstrip layout of the filter is shown in Figure 8.40c.

Figure 8.41 shows the calculated amplitude response of the filter, with and without losses. The effect of loss is to increase the passband attenuation to about

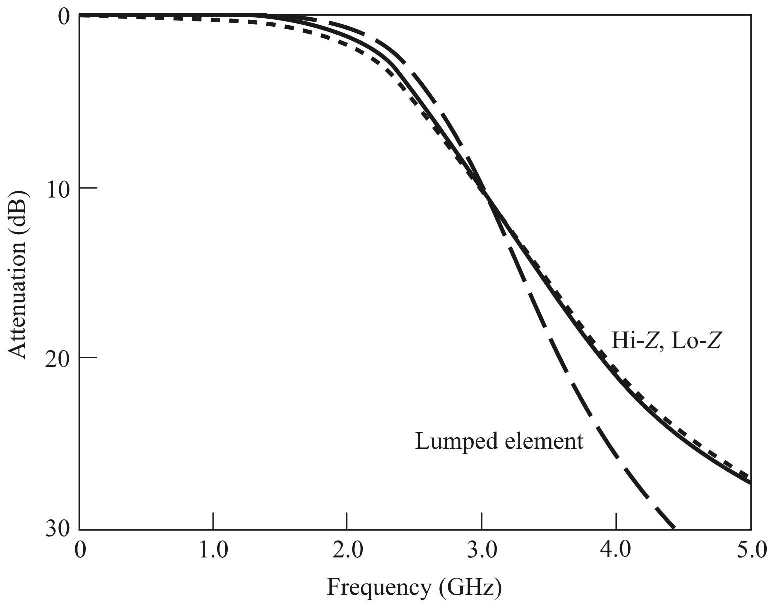

FIGURE 8.41 Amplitude response of the stepped-impedance low-pass filter of Example 8.6, with (dotted line) and without (solid line) losses. The response of the corresponding lumped-element filter is also shown.

1 dB at 2 GHz. The response of the corresponding lumped-element filter is also shown in Figure 8.41. The passband characteristic is similar to that of the stepped impedance filter, but the lumped-element filter gives more attenuation at higher frequencies. This is because the stepped-impedance filter elements depart significantly from the lumped-element values at higher frequencies. The stepped-impedance filter may have other passbands at higher frequencies, but the response will not be perfectly periodic because the lines are not commensurate. ■

8.7 COUPLED LINE FILTERS

The parallel coupled transmission lines discussed in Section 7.6 (for directional couplers) can be used to construct many types of filters. Fabrication of multisection bandpass or bandstop coupled line filters is particularly easy in microstrip or stripline form for bandwidths less than about 20%. Wider bandwidth filters generally require very tightly coupled lines, which are difficult to fabricate. We will first study the filter characteristics of a single quarter-wave coupled line section, and then show how these sections can be used to design a bandpass filter [7]. Other filter designs using coupled lines can be found in reference [1].

Filter Properties of a Coupled Line Section

A parallel coupled line section is shown in Figure 8.42a, with port voltage and current definitions. We will derive the open-circuit impedance matrix for this four-port network by considering the superposition of even- and odd-mode excitations [8], which are shown in Figure 8.42b. Thus, the current sources i_1 and i_3 drive the line in the even mode, while i_2 and i_4 drive the line in the odd mode. By superposition, we see that the total port currents, I_i, can be expressed in terms of the even- and odd-mode currents as

$$I_1 = i_1 + i_2, \tag{8.87a}$$

$$I_2 = i_1 - i_2, \tag{8.87b}$$

$$I_3 = i_3 - i_4, \tag{8.87c}$$

$$I_4 = i_3 + i_4. \tag{8.87d}$$

First consider the line as being driven in the even mode by the i_1 current sources. If the other ports are open-circuited, the impedance seen at port 1 or 2 is

$$Z_{\text{in}}^e = -jZ_{0e} \cot \beta\ell. \tag{8.88}$$

The voltage on either conductor can be expressed as

$$\begin{aligned} v_a^1(z) = v_b^1(z) &= V_e^+[e^{-j\beta(z-\ell)} + e^{j\beta(z-\ell)}] \\ &= 2V_e^+ \cos \beta(\ell - z), \end{aligned} \tag{8.89}$$

so the voltage at port 1 or 2 is

$$v_a^1(0) = v_b^1(0) = 2V_e^+ \cos \beta\ell = i_1 Z_{\text{in}}^e.$$

This result and (8.88) can be used to rewrite (8.89) in terms of i_1 as

$$v_a^1(z) = v_b^1(z) = -jZ_{0e} \frac{\cos \beta(\ell - z)}{\sin \beta\ell} i_1. \tag{8.90}$$