PART 1 LORENZO MIGUEL COLUMBA

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} , & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} , & -\frac{1}{\sqrt{2}} \end{bmatrix} \qquad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad 0 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

1. $H \cdot Y$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} \left(\cancel{\frac{1}{\sqrt{2}} \times 0} + \frac{1}{\sqrt{2}} \times i\right), & \left(\frac{1}{\sqrt{2}} \times -i + \cancel{\frac{1}{\sqrt{2}} \times 0}\right) \\ \left(\cancel{\frac{1}{\sqrt{2}} \times 0} + \left(-\frac{1}{\sqrt{2}}\right) \times i\right), & \left(\frac{1}{\sqrt{2}} \times -i + \cancel{\left(-\frac{1}{\sqrt{2}}\right) \times 0}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} \times i & \frac{1}{\sqrt{2}} \times -i \\ -\frac{1}{\sqrt{2}} \times i & \frac{1}{\sqrt{2}} \times -i \end{bmatrix} = \boxed{\begin{bmatrix} \frac{1}{\sqrt{2}}i & -\frac{1}{\sqrt{2}}i \\ -\frac{1}{\sqrt{2}}i & -\frac{1}{\sqrt{2}}i \end{bmatrix}}$$

Magnitude

$$\left\| \begin{bmatrix} \frac{1}{\sqrt{2}}i & -\frac{1}{\sqrt{2}}i \\ -\frac{1}{\sqrt{2}}i & -\frac{1}{\sqrt{2}}i \end{bmatrix} \right\|_F = \sqrt{\begin{bmatrix} \left(\frac{1}{\sqrt{2}}i\right)^2 + \left(-\frac{1}{\sqrt{2}}i\right)^2 + \\ -\left(\frac{1}{\sqrt{2}}i\right)^2 + \left(-\frac{1}{\sqrt{2}}i\right)^2 \end{bmatrix}} = \sqrt{\left(\frac{1}{\sqrt{2}}i\right)^2 + \left(-\frac{1}{\sqrt{2}}i\right)^2 + \left(-\frac{1}{\sqrt{2}}i\right)^2 + \left(-\frac{1}{\sqrt{2}}i\right)^2}$$

$$\boxed{= 1.41421356}$$

Angle

$= \cancel{\tan^{-1}\left(\frac{A'}{A}\right)}$

$= \cancel{\tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}i}{0}\right) = 90^\circ} \qquad = \cancel{\begin{bmatrix} 90^\circ & -90^\circ \\ -90^\circ & -90^\circ \end{bmatrix}}$

$= \tan^{-1}\left(\frac{-\frac{1}{\sqrt{2}}i}{0}\right) = \boxed{-90^\circ}$

$= \cancel{\tan^{-1}\left(\frac{-\frac{1}{\sqrt{2}}i}{0}\right) = -90^\circ}$

$= \cancel{\tan^{-1}\left(\frac{-\frac{1}{\sqrt{2}}i}{0}\right) = -90^\circ}$

2. Y . H

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \cancel{(0 \times \frac{1}{\sqrt{2}})} + (-i \times \frac{1}{\sqrt{2}}) & \cancel{(0 \times \frac{1}{\sqrt{2}})} + (-i \times -\frac{1}{\sqrt{2}}) \\ (i \times \frac{1}{\sqrt{2}}) + \cancel{(0 \times \frac{1}{\sqrt{2}})} & (i \times \frac{1}{\sqrt{2}}) + \cancel{(0 \times -\frac{1}{\sqrt{2}})} \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} -\frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i \\ \frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i \end{bmatrix}}$$

Magnitude

$$\left\| \begin{bmatrix} -\frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i \\ \frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i \end{bmatrix} \right\|_F = \sqrt{\left(-\frac{1}{\sqrt{2}}i\right)^2 + \left(\frac{1}{\sqrt{2}}i\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$\boxed{= 1.4141356}$$

Angle

$$\cancel{= \tan^{-1}\left(\frac{-\frac{1}{\sqrt{2}}i}{0}\right) = -90^\circ}$$

$$= \tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}i}{0}\right) \boxed{= 90^\circ}$$

$$\cancel{\begin{bmatrix} -90^\circ & 90^\circ \\ 90^\circ & 90^\circ \end{bmatrix}}$$

$$\cancel{= \tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}i}{0}\right) = 90^\circ}$$

$$\cancel{= \tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}}{0}\right) = 90^\circ}$$

3. H·H

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right) & \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} \times -\frac{1}{\sqrt{2}}\right) \\ \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right) & \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} \times -\frac{1}{\sqrt{2}}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 + 0.5 & 0.5 + (-0.5) \\ 0.5 + (-0.5) & 0.5 + (0.5) \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

$\cancel{\begin{bmatrix} 0.99999999999966 \\ 0 \end{bmatrix}}$ Magnitude

$$= \left\| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\|_F = \sqrt{(1)^2 + (0)^2 + (0)^2 + (1)^2}$$

$$= \sqrt{2}$$

$$\boxed{= 1.414213562}$$

Angle

$\cancel{\tan^{-1}\left(\frac{0}{1}\right)}$ $\boxed{\tan^{-1}\left(\frac{0}{1}\right) = 0^\circ}$

$\cancel{\tan^{-1}\left(\frac{0}{0}\right)}$ $\cancel{\tan^{-1}\left(\frac{0}{0}\right) = 0^\circ}$

$\cancel{\tan^{-1}\left(\frac{0}{0}\right)}$ $\cancel{\tan^{-1}\left(\frac{0}{0}\right) = 0^\circ}$

$\cancel{\tan^{-1}\left(\frac{0}{1}\right)}$ $\cancel{\tan^{-1}\left(\frac{0}{1}\right) = 0^\circ}$

4.Y.H.O

$$= \begin{bmatrix} -\frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i \\ \frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \cancel{\left(-\frac{1}{\sqrt{2}}i \times 1\right) + \left(\frac{1}{\sqrt{2}}i \times 1\right)} & \cancel{\left(-\frac{1}{\sqrt{2}}i \times 0\right)} + \left(\frac{1}{\sqrt{2}}i \times 1\right) \\ \left(\frac{1}{\sqrt{2}}i \times 1\right) + \left(\frac{1}{\sqrt{2}}i \times 1\right) & \cancel{\left(\frac{1}{\sqrt{2}}i \times 0\right)} + \left(\frac{1}{\sqrt{2}}i \times 1\right) \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 0 & \frac{1}{\sqrt{2}}i \\ \sqrt{2}i & \frac{1}{\sqrt{2}}i \end{bmatrix}}$$

Magnitude

$$\left\| \begin{bmatrix} 0 & \frac{1}{\sqrt{2}}i \\ \sqrt{2}i & \frac{1}{\sqrt{2}}i \end{bmatrix} \right\|_F = \sqrt{(0)^2 + \left(\frac{1}{\sqrt{2}}i\right)^2 + (\sqrt{2}i)^2 + \left(\frac{1}{\sqrt{2}}i\right)^2}$$

$$\boxed{= 1.732050808}$$

Angle

$$\tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}}{0}\right) = \boxed{90^0}$$

5. H · Y · H · O

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{1}{\sqrt{2}}i \\ \sqrt{2}i & \frac{1}{\sqrt{2}}i \end{bmatrix} = \begin{bmatrix} \cancel{(\frac{1}{\sqrt{2}} \times 0)} + (\frac{1}{\sqrt{2}} \times \sqrt{2}i) & (\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}i) + (\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}i) \\ \cancel{(\frac{1}{\sqrt{2}} \times 0)} + (-\frac{1}{\sqrt{2}} \times \sqrt{2}i) & (\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}i) + (\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}i) \end{bmatrix}$$

$$= \begin{bmatrix} 1i & 1i \\ -1i & 0 \end{bmatrix}$$

Magnitude

$$= \left\| \begin{bmatrix} 1i & 1i \\ 1i & 0 \end{bmatrix} \right\|_F = \sqrt{(1)^2 + (1)^2 + (-1)^2 + (0)^2} = \boxed{1.732050808}$$

Angle

$$\tan^{-1}\left(\frac{1}{0}\right) \boxed{= 90^\circ}$$

PART 2.

$$H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = \boxed{-1}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = (0)(0) - (-i)(i) = \boxed{-i}$$

PART 3

$$1.\ \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 5 \\ 2 & 7 & 6 \\ 6 & 4 & 7 \end{pmatrix} = \begin{bmatrix} (5\times1)+(0\times2)+(0\times6) & (5\times0)+(0\times7)+(0\times4) & (5\times5)+(0\times6)+(0\times7) \\ (0\times1)+(5\times2)+(0\times6) & (0\times0)+(5\times7)+(0\times4) & (0\times5)+(5\times6)+(0\times7) \\ (0\times1)+(0\times2)+(5\times6) & (0\times0)+(0\times7)+(5\times4) & (0\times5)+(0\times6)+(5\times7) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 25 \\ 10 & 35 & 30 \\ 30 & 20 & 35 \end{bmatrix}$$

D

Determinant

$$= 5\begin{bmatrix} 35 & 30 \\ 20 & 35 \end{bmatrix} - 0\begin{bmatrix} 10 & 30 \\ 30 & 35 \end{bmatrix} + 25\begin{bmatrix} 10 & 35 \\ 30 & 20 \end{bmatrix}$$

$$= (35)(35) - (30)(20) = \cancel{328}\ 625$$

$$= (10)(35) - (30)(30) = -650$$

$$= (10)(20) - (35)(30) = -850$$

$$= 5(625) - 0(-650) + 25(-850)$$

$$\boxed{= -18125}$$

The vectors are linearly Independent.

2. $\begin{pmatrix} 1 & 2 & 6 \\ 3 & 15 & 4 \\ 2 & 10 & 3 \end{pmatrix} \cdot \begin{pmatrix} 5 & 2 & 4 \\ 6 & 2 & 4 \\ 0 & 1 & 1 \end{pmatrix}$

$$= \begin{bmatrix} (1\times5)+(2\times6)+(6\times0) & (1\times2)+(2\times2)+(6\times1) & (1\times4)+(2\times4)+(6\times1) \\ (3\times5)+(15\times6)+(4\times0) & (3\times2)+(15\times2)+(4\times1) & (3\times4)+(15\times4)+(4\times1) \\ (2\times5)+(10\times6)+(3\times0) & (2\times2)+(10\times2)+(3\times1) & (2\times4)+(10\times4)+(3\times1) \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 12 & 18 \\ 105 & 40 & 76 \\ 70 & 27 & 51 \end{bmatrix} = 17\begin{bmatrix} 40 & 76 \\ 27 & 51 \end{bmatrix} - 12\begin{bmatrix} 105 & 76 \\ 70 & 51 \end{bmatrix} + 18\begin{bmatrix} 105 & 40 \\ 70 & 27 \end{bmatrix}$$

$$= \cancel{17(-12) - 12(35) + 18}$$

$$(40)(51) - (76)(27) = -12$$

$$(105)(51) - (76)(70) = 35$$

$$(105)(27) - (40)(70) = 35$$

$$= 17(-12) - 12(35) + 18(35)$$

$\boxed{= 6}$ The vectors are linearly independent