PMT

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Core Mathematics C34

Advanced

Monday 16 June 2014 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. $$f(x) = 2x^3 + x - 10$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[1.5, 2]$

(2)

The only real root of $f(x) = 0$ is α

The iterative formula

$$x_{n+1} = \left(5 - \frac{1}{2}x_n\right)^{\frac{1}{3}}, \quad x_0 = 1.5$$

can be used to find an approximate value for α

(b) Calculate x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that $\alpha = 1.6126$ correct to 4 decimal places.

(2)

1a) $f(x) = 2x^3 + x - 10$

$f'(x) = 6x^2 + 1$

$\Rightarrow f'(1.5)$ $\Rightarrow$

$= 6(1.5)^2 + 1$

$f'(2) = 6(2)^2 + 1$

1b) $x_0 = 1.5$

$x_1 = \left(5 - \left(\frac{1}{2}\right)(1.5)\right)^{\frac{1}{3}}$

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2. A curve C has the equation

$$x^3 - 3xy - x + y^3 - 11 = 0$$

Find an equation of the tangent to C at the point $(2, -1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

$3x^2 - 3y - 3x\frac{dy}{dx} - 1 + 3y^2\frac{dy}{dx}$

$= 0$

$3(2)^2 - (3)(-1) - (3)(2)\frac{dy}{dx} - 1$

$+ 3(2)^2\frac{dy}{dx} = 0$

$12 - 3 + 6\frac{dy}{dx} - 1 + 12\frac{dy}{dx} = 0$

$\Rightarrow -18\frac{dy}{dx} = 8$

$\Rightarrow \frac{dy}{dx} = \frac{-8}{18}$

$\Rightarrow \frac{dy}{dx} = \frac{-4}{9}$

$\Rightarrow y - y_1 = m(x - x_1)$

$\Rightarrow y + 1 = -\frac{4}{9}(x - 2)$

$9y + 9 = 8 - 4x$

$\Rightarrow 9y + 4x + 1 = 0$

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3. Given that

$$y = \frac{\cos 2\theta}{1+\sin 2\theta}, \quad \begin{matrix} u \\ v \end{matrix} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{a}{1+\sin 2\theta}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where a is a constant to be determined.

(4)

$\cos 2\theta = \sin^2\theta - \cos^2\theta$

$$\frac{\partial u}{\partial \theta} = -2\sin 2\theta$$

$$\frac{\partial v}{\partial \theta} = 2\cos 2\theta$$

$$\frac{(-2\sin 2\theta)(1+\sin 2\theta) - (2\cos 2\theta)(\cos 2\theta)}{(1+\sin 2\theta)^2}$$

$$\frac{(-2\sin 2\theta)(1+\sin 2\theta) - 2\cos^2 2\theta}{(1+\sin 2\theta)^2}$$

$$\frac{-(2\sin^2 2\theta + 2\cos^2 2\theta + 2\sin 2\theta)}{(1+\sin 2\theta)^2}$$

$$\Rightarrow \frac{-2\cancel{(1+\sin 2\theta)}}{(1+\sin 2\theta)^{\cancel{2}}}$$

$$\Rightarrow \frac{-2}{1+\sin 2\theta}$$

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4. Find

(a) $\int (2x + 3)^{12}\,dx$

(2)

(b) $\int \frac{5x}{4x^2 + 1}\,dx$

(2)

1a) $y = (2x+3)^{13}$

$\frac{dy}{dx} = (13)(2)(2x+3)^{12}$

$= 26(2x+3)^{12}$

$\Rightarrow \int (2x+3)^{12}\,dx = \frac{1}{26}(2x+3)^{13} + c$

1b) $\int \frac{5x}{4x^2+1}\,dx$

$\Rightarrow \frac{5}{8} \ln\left[\frac{5x}{4x^2+1}\right] + c$

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5. $$f(x) = (8 + 27x^3)^{\frac{1}{3}}, \quad |x| < \frac{2}{3}$$

Find the first three non-zero terms of the binomial expansion of f(x) in ascending powers of x. Give each coefficient as a simplified fraction.

(5)

$$(8+27x^3)^{\frac{1}{3}}$$

$$2\left(1+\frac{27x^3}{8}\right)^{\frac{1}{3}}$$

$$2\left[1+\left(\frac{1}{3}\right)\left(\frac{27x^3}{8}\right)+\left(\frac{1}{\cancel{3}}\right)\left(-\frac{\cancel{2}}{3}\right)\left(\frac{27x^3}{8}\right)^2\left(\frac{1}{3}\right)\right]$$

$$2\left(1+\frac{27x^3}{24}-\frac{81x^6}{64}+\ldots\right)$$

$$2+\frac{9x^3}{4}+\frac{81x^6}{32}+\ldots$$

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6. (a) Express $\dfrac{5-4x}{(2x-1)(x+1)}$ in partial fractions.

(3)

(b) (i) Find a general solution of the differential equation

$$(2x-1)(x+1)\frac{dy}{dx} = (5-4x)y, \quad x > \frac{1}{2}$$

Given that $y = 4$ when $x = 2$,

(ii) find the particular solution of this differential equation.
Give your answer in the form $y = f(x)$.

(7)

6a)

$$\frac{5-4x}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1}$$

$$\Rightarrow \quad 5-4x = A(x+1) + B(2x-1)$$

when $x = -1$:

$$9 = -3B \Rightarrow B = -3$$

when $x = \frac{1}{2}$

$$3 = \frac{3}{2}A \Rightarrow A = 2$$

$$\Rightarrow \frac{5-4x}{(2x-1)(x+1)} = \frac{2}{2x-1} - \frac{3}{x+1}$$

6bi)

$$\int \frac{2}{2x-1} - \frac{3}{x+1}\, dx = \int \frac{1}{y}\, dy$$

$$= \frac{2}{2}\ln|2x-1| - 3\ln|x+1| + C = \ln y$$

$$\Rightarrow \ln|2x-1| - 3\ln|x+1| + C = \ln|y|$$

6bii)

$$\ln 4 = \ln 3 - 3\ln 3 + C$$

$$\Rightarrow C = \ln(9)(4) = \ln 36$$

$$\ln y = \ln|2x-1| - 3\ln|x+1| + \ln 36$$

$$\ln y = \frac{\ln(2x-1)(36)}{(x+1)^3}$$

$$y = \frac{36(2x-1)}{(x+1)^3}$$

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Question 6 continued

$$y = \frac{36(2x-1)}{(x+1)^3}$$

Turn over

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7. The function f is defined by

$$f: x \mapsto \frac{3x-5}{x+1}, \quad x \in \mathbb{R}, x \neq -1$$

(a) Find an expression for $f^{-1}(x)$

(3)

(b) Show that

$$ff(x) = \frac{x+a}{x-1}, \quad x \in \mathbb{R}, x \neq -1, x \neq 1$$

where a is an integer to be determined.

(4)

The function g is defined by

$$g: x \mapsto x^2 - 3x, \quad x \in \mathbb{R}, 0 \leqslant x \leqslant 5$$

(c) Find the value of fg(2)

(2)

(d) Find the range of g

(3)

7a) $y = \frac{3x-5}{x+1}$

$\Rightarrow x = \frac{3y-5}{y+1}$

$\Rightarrow xy + x = 3y - 5$

$\Rightarrow xy - 3y = -x - 5$

$y(x-3) = -(x+5)$

$\Rightarrow f^{-1}(x) = \frac{-(x+5)}{x-3}$

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Question 7 continued

7b) $f(x) = \dfrac{3x-5}{x+1}$

$$ff(x) = \frac{3\left(\frac{3x-5}{x+1}\right)-5}{\left(\frac{3x-5}{x+1}\right)+1}$$

$$ff(x) = \frac{3(3x-5)-5(x+1))}{3x-5+x+1}$$

$$\frac{4x-20}{4x-4}$$

$$\frac{x-5}{x-1} \qquad a=-5$$

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8. The volume V of a spherical balloon is increasing at a constant rate of 250 cm^3 s^{-1}.
Find the rate of increase of the radius of the balloon, in cm s^{-1}, at the instant when the volume of the balloon is 12 000 cm^3.
Give your answer to 2 significant figures.

(5)

[*You may assume that the volume V of a sphere of radius r is given by the formula* $V = \frac{4}{3}\pi r^3$.]

$12{,}000 = \frac{4}{3}\pi r^3 \Rightarrow \sqrt[3]{\frac{9000}{\pi}}$

$\frac{dV}{dt} = 250 \text{ cm}^3\text{s}^{-1}$

$\frac{dV}{dr} = 4\pi r^2 \Rightarrow \frac{dr}{dV} = \frac{1}{4\pi r^2}$

$\Rightarrow \left(\frac{dV}{dt}\right)\left(\frac{dr}{dV}\right) = \left(\frac{1}{4\pi r^2}\right)(250)$

$= \frac{250}{4\pi r^2}$

$\Rightarrow \frac{250}{4\pi \sqrt[3]{\left(\frac{9000}{\pi}\right)^2}}$

$\Rightarrow \frac{dr}{dt} = 0.099$

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Question 8 continued

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9.

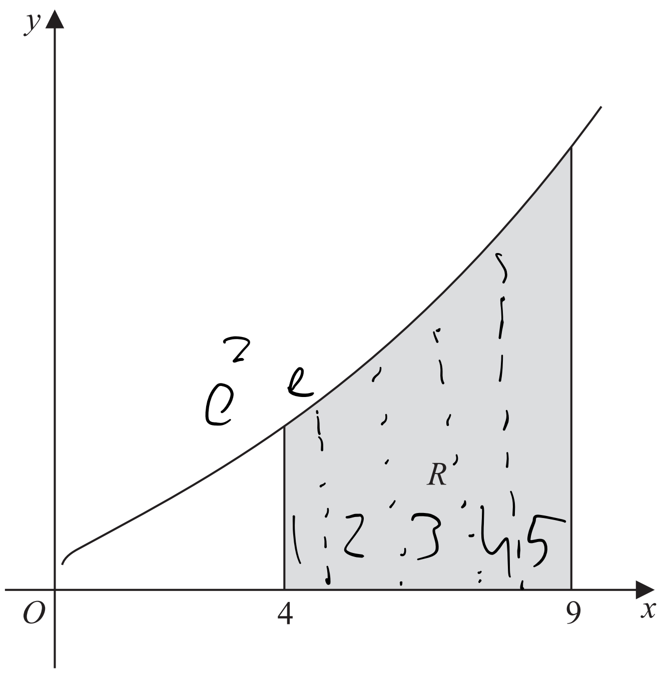

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = e^{\sqrt{x}}$, $x > 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines $x = 4$ and $x = 9$

(a) Use the trapezium rule, with 5 strips of equal width, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(b) Use the substitution $u = \sqrt{x}$ to find, by integrating, the exact value for the area of R.

(7)

9a) $\frac{1}{2}(a+b)h$

$\Rightarrow \frac{1}{2}(e^2 + e^{\sqrt{5}})$

$+ \frac{1}{2}(e^{\sqrt{5}} + e^{\sqrt{6}}) + \frac{1}{2}(e^{\sqrt{6}} + e^{\sqrt{7}})$

$+ \frac{1}{2}(e^{\sqrt{7}} + e^{\sqrt{8}}) + \frac{1}{2}(e^{\sqrt{8}} + e^{3})$

$\Rightarrow R = 65.69$ (2dp)

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Question 9 continued

9b) $u = x^{\frac{1}{2}}$, $\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$y = e^{\sqrt{x}}$

$2\int ue^{u}\,du$

$2\left(ue^{u} - \int e^{u}\,du\right)$

$\Rightarrow 2\left(ue^{u} - e^{u}\right)$

$\left[2\left(ue^{u} - e^{u}\right)\right]_{2}^{3}$

$\Rightarrow 2\left(3e^{3} - e^{3}\right) - 2\left(2e^{2} - e^{2}\right)$

$= \underline{\underline{4e^{3} - 2e^{2}}}$

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Turn over

P 4 4 9 6 9 A 0 2 5 4 8

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10. (a) Use the identity for $\sin(A + B)$ to prove that

$$\sin 2A \equiv 2\sin A \cos A$$

(2)

(b) Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\tan\left(\tfrac{1}{2}x\right)\right)\right] = \operatorname{cosec} x$$

(4)

A curve C has the equation

$$y = \ln\left(\tan\left(\tfrac{1}{2}x\right)\right) - 3\sin x, \qquad 0 < x < \pi$$

(c) Find the x coordinates of the points on C where $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$

Give your answers to 3 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

10a) $\sin(A+A)$

$= \sin A\cos A + \sin A\cos A$

$= 2\sin A\cos A$

10b) $$\frac{\frac{1}{2}\sec^2 x\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$$

$$\Rightarrow \frac{1}{2\tan\left(\frac{1}{2}x\right)\cos^2\left(\frac{1}{2}x\right)}$$

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Question 10 continued

$$\frac{1}{2\,\frac{\sin \frac{1}{2}x}{\cancel{\cos \frac{1}{2}x}}\,\cos^{\cancel{2}}\left(\frac{1}{2}x\right)}$$

$$\frac{1}{2\sin \frac{1}{2}x \cos \frac{1}{2}x}$$

$$\Rightarrow \frac{1}{\sin x}$$

$$\Rightarrow \operatorname{cosec} x$$

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11.

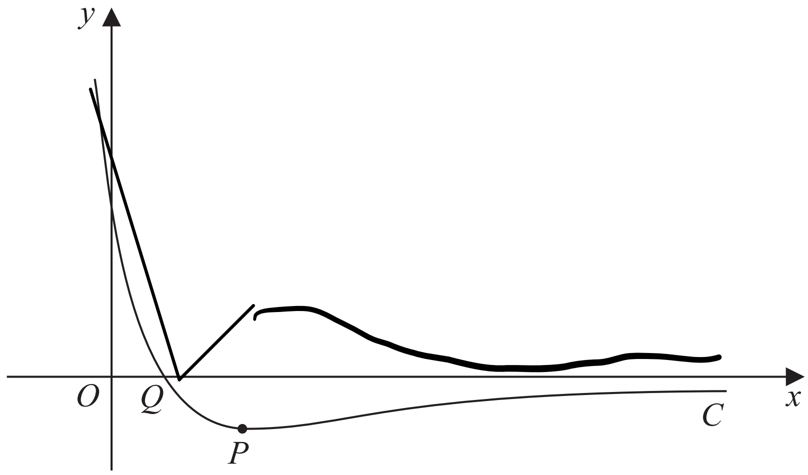

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = \mathrm{e}^{a-3x} - 3\mathrm{e}^{-x}, \quad x \in \mathbb{R}$$

where a is a constant and $a > \ln 4$

The curve C has a turning point P and crosses the x-axis at the point Q as shown in Figure 2.

(a) Find, in terms of a, the coordinates of the point P.

(6)

(b) Find, in terms of a, the x coordinate of the point Q.

(3)

(c) Sketch the curve with equation

$$y = \left|\mathrm{e}^{a-3x} - 3\mathrm{e}^{-x}\right|, \quad x \in \mathbb{R}, \ a > \ln 4$$

Show on your sketch the exact coordinates, in terms of a, of the points at which the curve meets or cuts the coordinate axes.

(3)

$y = e^{a-3x} - 3e^{-x}$

$\frac{dy}{dx} = 3e^{-x} - 3e^{a-3x}$

$= 3(e^{-x} - e^{a-3x})$

When $\frac{dy}{dx} = 0$

Question 11 continued

$0 = e^{-x} - e^{a-3x}$

$0 = -x - a + 3x$

$\Rightarrow x = \frac{1}{2}a$

$y = e^{-\frac{1}{2}a} - 3e^{-\frac{1}{2}a}$

$\Rightarrow y_p = -2e^{-\frac{1}{2}a}$

$\Rightarrow \left(\frac{1}{2}a, -2e^{-\frac{1}{2}a}\right)$

$0 = e^{a-3x} - 3e^{-x}$

$e^{a-3x} = 3e^{-x}$

$\Rightarrow e^{a-2x} = 3$

$a - 2x = \ln 3$

$\Rightarrow x = \frac{a - \ln 3}{2}$

$\Rightarrow \left(\frac{a - \ln 3}{2}, 0\right)$

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Q11

(Total 12 marks)

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12.

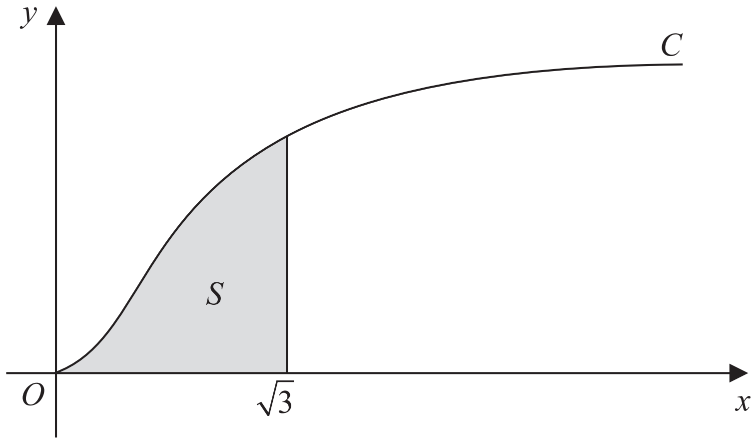

Figure 3

Figure 3 shows a sketch of part of the curve C with parametric equations

$$x = \tan t, \quad y = 2\sin^2 t, \quad 0 \leqslant t < \frac{\pi}{2}$$

The finite region S, shown shaded in Figure 3, is bounded by the curve C, the line $x = \sqrt{3}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume of the solid of revolution formed is given by

$$4\pi \int_0^{\frac{\pi}{3}} (\tan^2 t - \sin^2 t)\, dt$$

(6)

(b) Hence use integration to find the exact value for this volume.

(6)

12a)

$$\pi \int y^2 \frac{dx}{dt}\, dt$$

$$4\pi \int \sin^4 t \sec^2 t \, dt$$

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Question 12 continued

$$4\pi \int \tan^2 t \sin^2 t \, dt$$

$$4\pi \int (\tan^2 t)(1 - \cos^2 t)$$

$$4\pi \left(\tan^2 t - \tan^2 t \cos^2 t \right)$$

$$4\pi \int_0^{\frac{\pi}{3}} \tan^2 t - \sin^2 t \, dt$$

$$x = \sqrt{3} \rightarrow t = \frac{\pi}{3}$$

Leave blank

13. (a) Express $2\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give your value of α to 2 decimal places. **(3)**

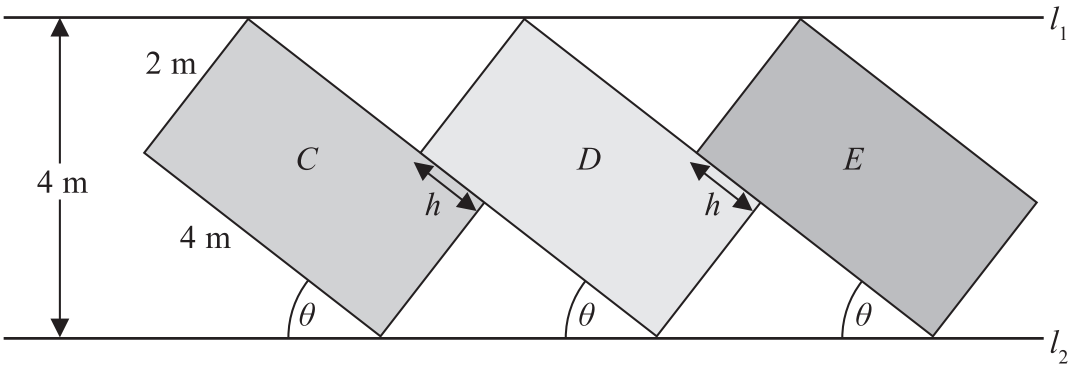

Figure 4

Figure 4 shows the design for a logo that is to be displayed on the side of a large building. The logo consists of three rectangles, C, D and E, each of which is in contact with two horizontal parallel lines l_1 and l_2. Rectangle D touches rectangles C and E as shown in Figure 4.

Rectangles C, D and E each have length 4 m and width 2 m. The acute angle θ between the line l_2 and the longer edge of each rectangle is shown in Figure 4.

Given that l_1 and l_2 are 4 m apart,

(b) show that

$$2\sin\theta + \cos\theta = 2$$

(2)

Given also that $0 < \theta < 45°$,

(c) solve the equation

$$2\sin\theta + \cos\theta = 2$$

giving the value of θ to 1 decimal place. **(3)**

Rectangles C and D and rectangles D and E touch for a distance h m as shown in Figure 4.

Using your answer to part (c), or otherwise,

(d) find the value of h, giving your answer to 2 significant figures. **(3)**

Leave blank

Question 13 continued

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14. Relative to a fixed origin O, the line l has vector equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

where λ is a scalar parameter.

Points A and B lie on the line l, where A has coordinates $(1, a, 5)$ and B has coordinates $(b, -1, 3)$.

(a) Find the value of the constant a and the value of the constant b.
(3)

(b) Find the vector $\overrightarrow{AB}$.
(2)

The point C has coordinates $(4, -3, 2)$

(c) Show that the size of the angle CAB is $30°$
(3)

(d) Find the exact area of the triangle CAB, giving your answer in the form $k\sqrt{3}$, where k is a constant to be determined.
(2)

The point D lies on the line l so that the area of the triangle CAD is twice the area of the triangle CAB.

(e) Find the coordinates of the two possible positions of D.
(4)

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Question 14 continued

Q14

(Total 14 marks)

TOTAL FOR PAPER: 125 MARKS

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